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## ECE 305 - Fall 2016

Exam 4 - Monday, November 21, 2016
This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator must be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must show your work (scratch paper is attached).
The exam is designed to be taken in 50 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,
I) 40 points ( 8 points per question)
II) 30 points
III) 30 points

## Course policy

If I am caught cheating, I will earn an F in the course \& be reported to the Dean of Students.
I repeat: $\qquad$
$\qquad$
Signature: $\qquad$

## Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

1 (8 points). What is the minimum subthreshold swing value for a MOSFET at room temperature?
a. $\quad 26 \mathrm{mV} /$ decade
b. $60 \mathrm{mV} /$ decade
c. $\quad 90 \mathrm{mV} /$ decade
d. $150 \mathrm{mV} /$ decade
e. $260 \mathrm{mV} /$ decade

2 (8 points). The on-current for an n-type MOSFET occurs at what bias values?
a. $\quad V_{G S}=V_{T}, V_{D S}=0$
b. $\quad V_{G S}=V_{D D}, V_{D S}=0$
c. $\quad V_{G S}=V_{T}, V_{D S}=V_{D D}$
d. $\quad V_{G S}=V_{D D}, V_{D S}=V_{T}$
e. $V_{G S}=V_{D D}, V_{D S}=V_{D D}$

3 (8 points). How does exposure to radiation affect MOS devices?
a. Changes the electron affinity of the oxide
b. Changes the bandgap of the semiconductor
c. Changes the doping near the oxide-semiconductor interface
d. Introduces traps in the oxide and at the oxide-semiconductor interface
e. Changes the metal workfunction

4 (8 points).For a short-channel MOSFET biased at a high drain voltage $V_{D S}$, how does $I_{D, s a t}$ vary with the gate voltage $V_{G S}$ ?
a. $\left(V_{G S}-V_{T}\right)^{2}$
b. $\left(V_{G S}-V_{T}\right)^{1.5}$
c. $\left(V_{G S}-V_{T}\right)^{1.0}$
d. $\left(V_{G S}-V_{T}\right)^{0.5}$
e. $\left(V_{G S}-V_{T}\right)^{-2}$

5 (8 points). When virtually no free carriers are found near the oxide-Si interface of a MOS device, what is the bias condition?
a. Depletion
b. Inversion
c. Flatband
d. Deep inversion
e. Accumulation

## Part II (Free Response, 30 points)

Consider an MOS capacitor made of crystalline silicon described by the band diagram below, on the threshold of inversion. Note that unmarked values may not be to scale. Assume that $\varepsilon_{o x}=3 \cdot 10^{6}$ $\mathrm{V} / \mathrm{cm}, t_{o x}=1.0 \mathrm{~nm}, K_{o x}=3.9, K_{S i}=11.8$, and $E_{g}=1.12 \mathrm{eV} .$.

a. What is the numerical value of the surface potential?
b. What is the level of $p$-type doping $N_{A}$ in the semiconductor?
c. What is the numerical value of the electric field in the semiconductor near the oxide ( $x=0^{+}$)?
d. What is the depletion width (in the semiconductor)?
e. What is the electrostatic potential voltage of the gate electrode with respect to the semiconductor?

## Part III (Free Response, 30 points)

Consider a MOSFET that produces the experimental data shown in the figure below.

a. Using the square law relations and the plotted data, what is the approximate threshold voltage $V_{T}$ ? Justify your answer.
b. If the channel width $=1 \mu \mathrm{~m}$, mobility $\mu_{n}=1400 \mathrm{~cm}^{2} / \mathrm{V} \cdot \mathrm{s}$, and oxide capacitance $C_{o x}=10 \mathrm{nF} / \mathrm{cm}^{2}$, estimate the channel length from the data given.
c. Using the plot, estimate the transconductance $g_{m}$ when $V_{G}=5 \mathrm{~V}$ and $V_{D}=5 \mathrm{~V}$.

## ECE 305 Exam 4 Formula Sheet (Fall 2016)

You may remove these pages from the exam packet, and take them with you.

| Physical Constants | Silicon parameters $(\boldsymbol{T}=\mathbf{3 0 0} \mathbf{K})$ |
| :---: | :---: |
| $h / 2 \pi=\hbar=1.055 \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}$ | $N_{C}=3.23 \times 10^{19} \mathrm{~cm}^{-3}$ |
| $m_{0}=9.109 \times 10^{-31} \mathrm{~kg}$ | $N_{V}=1.83 \times 10^{19} \mathrm{~cm}^{-3}$ |
| $k_{B}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ | $n_{i}=1.1 \times 10^{10} \mathrm{~cm}^{-3}$ |
| $q=1.602 \times 10^{-19} \mathrm{C}$ | $K_{S}=11.8$ |
| $\epsilon_{0}=8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$ | $E_{g}=1.12 \mathrm{eV} ; \chi=4.03 \mathrm{eV}$ |

Miller Indices: (hkl) \{hkl\} [hkl] <hkl>
Density of states $g_{C}(E)=\frac{\left(m_{n}^{*}\right)^{3 / 2} \sqrt{2\left(E-E_{C}\right)}}{\pi^{2} \hbar^{3}}$
Fermi function $f(E)=\frac{1}{1+e^{\left(E-E_{F}\right) / k T}} \quad$ Intrinsic carrier concentration $n_{i}=\sqrt{N_{C} N_{V}} e^{-E_{g} / 2 k T}$
Equilibrium carrier densities: $N_{C}=\frac{1}{4}\left(\frac{2 m_{n}^{*} k T}{\pi \hbar^{2}}\right)^{3 / 2} \quad N_{V}=\frac{1}{4}\left(\frac{2 m_{p}^{*} k T}{\pi \hbar^{2}}\right)^{3 / 2}$
$n_{0}=N_{C} e^{\left(E_{F}-E_{C}\right) / k T}=n_{i} e^{\left(E_{F}-E_{i}\right) / k T}$
$p_{0}=N_{V} e^{\left(E_{V}-E_{F}\right) / k T}=n_{i} e^{\left(E_{F}-E_{i}\right) / k T}$
Space charge neutrality: $p-n+N_{D}^{+}-N_{A}^{-}=0$
Law of Mass Action: $n_{0} p_{0}=n_{i}^{2}$
Non-equilibrium carriers: $\quad n=N_{C} e^{\left(F_{N}-E_{C}\right) / k T} \quad p=N_{V} e^{\left(E_{V}-F_{P}\right) / k T} \quad n p=n_{i}^{2} e^{\left(F_{N}-F_{P}\right) / k T}$
Conductivity/resistivity: $\sigma=\sigma_{n}+\sigma_{n}=q\left(n \mu_{n}+p \mu_{p}\right)=1 / \rho$
Drift-diffusion current equations: $\quad J_{n}=n q \mu_{n} \varepsilon_{x}+q D_{n} \frac{d n}{d x}=n \mu_{n} \frac{d F_{n}}{d x} \quad \frac{D_{n}}{\mu_{n}}=\frac{k T}{q}$

$$
J_{p}=p q \mu_{p} \varepsilon_{x}-q D_{p} \frac{d p}{d x}=p \mu_{p} \frac{d F_{p}}{d x} \quad \frac{D_{p}}{\mu_{p}}=\frac{k T}{q}
$$

Carrier conservation equations: $\quad \frac{\partial n}{\partial t}=+\nabla \cdot\left(\frac{J_{n}}{q}\right)+G_{n}-R_{n}$

$$
\frac{\partial p}{\partial t}=-\nabla \cdot\left(\frac{J_{p}}{q}\right)+G_{p}-R_{p}
$$

Poisson's equation:
$\nabla \cdot(\epsilon \varepsilon)=\rho$
SRH carrier recombination: $\quad R=\Delta n / \tau_{n} \quad$ or $\quad R=\Delta p / \tau_{p}$
Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t}=D_{n} \frac{\partial^{2} \Delta n}{\partial x^{2}}-\frac{\Delta n}{\tau_{n}}+G_{L} \quad L_{D, n}=\sqrt{D_{n} \tau_{n}}$
PN homojunction electrostatics: $\quad V_{b i}=\frac{k T}{q} \ln \left(\frac{N_{D} N_{A}}{n_{i}^{2}}\right) \quad \frac{d \varepsilon}{d x}=\frac{\rho(x)}{K_{s} \epsilon_{o}}$

$$
W=\sqrt{\frac{2 K_{s} \epsilon_{0} V_{b i}}{q}\left(\frac{N_{A}+N_{D}}{N_{A} N_{D}}\right)} \quad x_{n}=\left(\frac{N_{A}}{N_{A}+N_{D}}\right) W \quad x_{p}=\left(\frac{N_{D}}{N_{A}+N_{D}}\right) W \quad \varepsilon(0)=\sqrt{\frac{2 q V_{b i}}{K_{s} \epsilon_{o}}\left(\frac{N_{A} N_{D}}{N_{A}+N_{D}}\right)}
$$

PN diode current: $\quad \Delta n(0)=\frac{n_{i}^{2}}{N_{A}}\left(e^{q V_{A} / k T}-1\right) \quad \Delta p(0)=\frac{n_{i}^{2}}{N_{D}}\left(e^{q V_{A} / k T}-1\right)$
$J_{D}=J_{o}\left(e^{q V_{A} / k T}-1\right) \quad J_{o}=q\left(\frac{D_{n}}{L_{n}} \frac{n_{i}^{2}}{N_{A}}+\frac{D_{p}}{L_{p}} \frac{n_{i}^{2}}{N_{D}}\right)$ (long) $\quad J_{o}=q\left(\frac{D_{n}}{W_{p}} \frac{n_{i}^{2}}{N_{A}}+\frac{D_{p}}{W_{n}} \frac{n_{i}^{2}}{N_{D}}\right)$ (short)
Non-ideal diodes: $\quad I=I_{o}\left(e^{q\left(V_{A}-I R_{S}\right) / k T}-1\right) \quad J_{g e n}=-q \frac{n_{i}}{2 \tau_{o}} W$
Photovoltaics: $\quad V_{o c}=\frac{n k T}{q} \ln \left(\frac{J_{s c}}{J_{o}}\right) \quad J_{P V}=J_{o}\left(e^{q V_{A} / k T}-1\right)-J_{s c}$
Small signal model: $\quad G_{d}=\frac{I_{D}+I_{o}}{k T / q} \quad C_{J}\left(V_{R}\right)=\frac{K_{s} \epsilon_{o} A}{\sqrt{\frac{2 K_{s} \sigma_{0} V_{b i}}{q N_{A}}}}=A \sqrt{\frac{q K_{s} \epsilon_{0} N_{A}}{2 V_{b i}}} \quad C_{D}=G_{d} \tau_{n}$

MS diode properties: $q V_{b i}=\left|\Phi_{M}-\Phi_{S}\right| \quad \Phi_{B P}=\chi+E_{G}-\Phi_{M} \quad \Phi_{B N}=\Phi_{M}-\chi$ $J_{D}=J_{o}\left(e^{q V_{A} / k T}-1\right) \quad J_{o}=A^{*} T^{2} e^{-\Phi_{B} / k T} \quad A^{*}=\frac{4 \pi q m^{*} k_{B}^{2}}{h^{3}}=120 \frac{m^{*}}{m_{o}} \frac{\mathrm{~A}}{\mathrm{~cm}^{2} \cdot \mathrm{~K}^{2}}$

MOS capacitors: $\quad W=\sqrt{\frac{2 K_{s} \epsilon_{o} \phi_{s}}{q N_{A}}} \mathrm{~cm} \quad \varepsilon_{s}=\sqrt{\frac{2 q N_{A} \phi_{s}}{K_{s} \epsilon_{o}}} \frac{\mathrm{~V}}{\mathrm{~cm}}$

$$
\begin{gathered}
Q_{B}=-q N_{A} W\left(\phi_{s}\right)=-\sqrt{2 q K_{s} \epsilon_{o} N_{A} \phi_{s}} \frac{\mathrm{C}}{\mathrm{~cm}^{2}} \\
V_{G}=V_{F B}+\phi_{s}+\Delta \phi_{o x}=V_{F B}+\phi_{s}-\frac{Q_{s}\left(\phi_{s}\right)}{C_{o x}} \\
C_{o x}=K_{o} \epsilon_{o} / x_{o} \quad V_{F B}=\Phi_{m s} / q-Q_{F} / C_{o x}
\end{gathered}
$$

$C=C_{o x} /\left[1+\frac{K_{o} W\left(\phi_{s}\right)}{K_{s} x_{o}}\right]$

$$
V_{T}=-Q_{B}\left(2 \phi_{F}\right) / C_{o x}+2 \phi_{F}
$$

$$
Q_{n}=-C_{o x}\left(V_{G}-V_{T}\right)
$$

MOSFETs:

$$
I_{D}=-W Q_{n}(y=0)\left\langle v_{y}(y=0)\right\rangle
$$

$$
I_{D}=\frac{W}{L} \mu_{n} C_{o x}\left(V_{G S}-V_{T}\right) V_{D S} \quad I_{D}=W C_{o x} v_{s a t}\left(V_{G S}-V_{T}\right)
$$

Square Law (for $V_{G S} \geq V_{T}$ ): $\quad I_{D}=\left\{\begin{array}{lr}\frac{W}{L} \mu_{n} C_{o x}\left[\left(V_{G S}-V_{T}\right) V_{D S}-V_{D S}^{2} / 2\right], & 0 \leq V_{D S} \leq V_{G S}-V_{T} \\ \frac{W}{2 L} \mu_{n} C_{o x}\left(V_{G S}-V_{T}\right)^{2}, & V_{D S} \geq V_{G S}-V_{T}\end{array}\right.$

