NAME:__________________________________ **PUID:**________________________

ECE 305 – Fall 2016

Exam 4 – Monday, November 21, 2016

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).
The exam is designed to be taken in 50 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)
II) 30 points
III) 30 points

Course policy

If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: ____

____

Signature: ____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

1 (8 points). What is the minimum subthreshold swing value for a MOSFET at room temperature?

a. 26 mV/decade
b. 60 mV/decade
c. 90 mV/decade
d. 150 mV/decade
e. 260 mV/decade

2 (8 points). The on-current for an n-type MOSFET occurs at what bias values?

a. $V_{GS} = V_T, V_{DS} = 0$
b. $V_{GS} = V_{DD}, V_{DS} = 0$
c. $V_{GS} = V_T, V_{DS} = V_{DD}$
d. $V_{GS} = V_{DD}, V_{DS} = V_T$
e. $V_{GS} = V_{DD}, V_{DS} = V_{DD}$

3 (8 points). How does exposure to radiation affect MOS devices?

a. Changes the electron affinity of the oxide
b. Changes the bandgap of the semiconductor
c. Changes the doping near the oxide-semiconductor interface
d. Introduces traps in the oxide and at the oxide-semiconductor interface
e. Changes the metal workfunction

4 (8 points).For a short-channel MOSFET biased at a high drain voltage V_{DS}, how does $I_{D,sat}$ vary with the gate voltage V_{GS}?

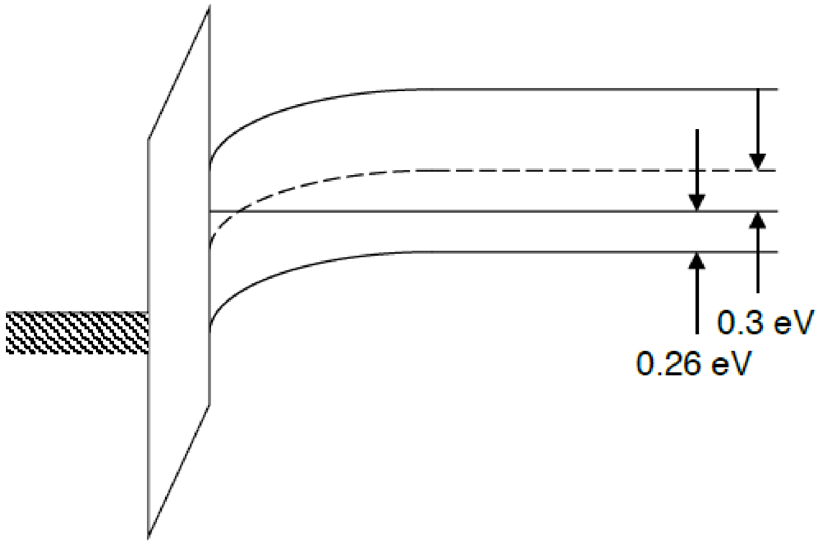
a. $(V_{GS} - V_T)^2$
b. $(V_{GS} - V_T)^{1.5}$
c. $(V_{GS} - V_T)^{1.0}$
d. $(V_{GS} - V_T)^{0.5}$
e. $(V_{GS} - V_T)^{-2}$

5 (8 points). When virtually no free carriers are found near the oxide-Si interface of a MOS device, what is the bias condition?

a. Depletion
b. Inversion
c. Flatband
d. Deep inversion
e. Accumulation

Part II (Free Response, 30 points)

Consider an MOS capacitor made of crystalline silicon described by the band diagram below, on the threshold of inversion. Note that unmarked values may not be to scale. Assume that $\mathcal{E}_{ox} = 3 \cdot 10^6$ V/cm, $t_{ox} = 1.0$ nm, $K_{ox} = 3.9$, $K_{Si} = 11.8$, and $E_g = 1.12$ eV..



a. What is the numerical value of the surface potential?

$$\phi_S = 2\phi_F = 2 \cdot (0.3\ V) = 0.6\ V$$

b. What is the level of p-type doping N_A in the semiconductor?

$$N_A = n_i e^{\phi_F/kT} = 1.1 \cdot 10^{10}\ e^{0.3/.026} = 1.1 \cdot 10^{15}\ \text{cm}^{-3}$$

c. What is the numerical value of the electric field in the semiconductor near the oxide ($x = 0^+$)?

$$K_{ox}\mathcal{E}_{ox} = K_{Si}\mathcal{E}_{Si}$$
$$\mathcal{E}_{Si} = 10^6\ \text{V/cm}$$

d. What is the depletion width (in the semiconductor)?

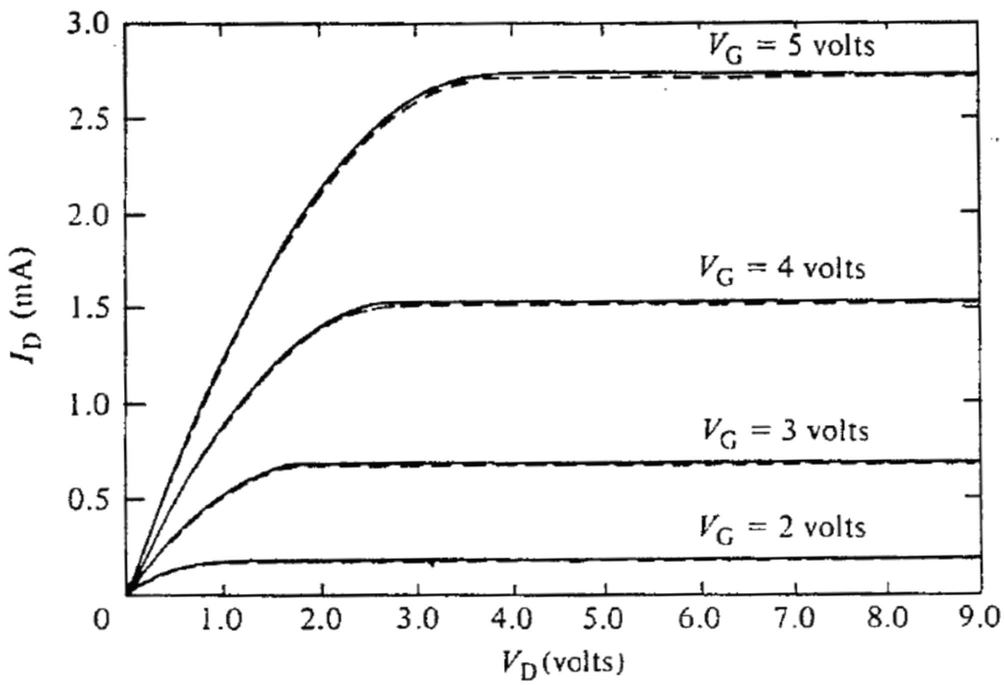
$$W = \sqrt{\frac{2K_s\epsilon_o\phi_s}{qN_A}} = \sqrt{\frac{2(11.8)(8.854 \cdot 10^{-14}\ \text{F/cm})(0.6\ V)}{(1.6 \cdot 10^{-19}\text{C})(1.1 \cdot 10^{15}\ \text{cm}^{-3})}} = 0.856\ \mu\text{m}$$

e. What is the electrostatic potential voltage of the gate electrode with respect to the semiconductor?

$$V_G = \Delta V_{ox} + \phi_s = (3 \cdot 10^6\ \text{V/cm})(1\ \text{nm}) + 0.6\ \text{V} = 0.9\ \text{V}$$

Part III (Free Response, 30 points)

Consider a MOSFET that produces the experimental data shown in the figure below.



a. Using the square law relations and the plotted data, what is the approximate threshold voltage V_T? Justify your answer.
When $V_G = 5$ V, we observe saturation at $V_D = 3.5\ V = V_G - V_T$. Thus, $V_T = 1.5$ V.
We can also obtain similar results for $V_G = 4$ V, and $V_G = 3$ V.

b. If the channel width $= 1\ \mu\text{m}$, mobility $\mu_n = 1400\ \text{cm}^2/\text{V}\cdot\text{s}$, and oxide capacitance $C_{ox} = 10\ \text{nF/cm}^2$, estimate the channel length from the data given.
Since $I_D = \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2$, we can rearrange to write:
$L = \frac{W}{2I_D}\mu_n C_{ox}(V_{GS} - V_T)^2$. If we use the data for $V_G = 5$ V, we obtain:

$$L = \frac{(1\ \mu\text{m})}{2\cdot(2.7\ \text{mA})}(1400\ \text{cm}^2/\text{V}\cdot\text{s})(10\ \text{nF/cm}^2)(3.5\ \text{V})^2 = 32\ \text{nm}$$

c. Using the plot, estimate the transconductance g_m when $V_G = 5$ V and $V_D = 5$ V.
Recalling that transconductance $g_m = \frac{\Delta I_D}{\Delta V_G}$, we can use the square law in the saturation regime to estimate $g_m = \frac{W}{2L}\mu_n C_{ox}\frac{d}{dV_{GS}}(V_{GS} - V_T)^2 = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)$
Thus, we have $g_m = \frac{2I_D}{(V_{GS}-V_T)} = \frac{2\cdot 2.7\ \text{mA}}{3.5\ \text{V}} = 1.54\ \text{mS}$

ECE 305 Exam 4 Formula Sheet (Fall 2016)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	**Silicon parameters ($T = 300$ K)**
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}\ \text{cm}^{-3}$
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}\ \text{cm}^{-3}$
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}\ \text{cm}^{-3}$
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V}e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\quad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $\quad p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_F-E_i)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $\quad p = N_V e^{(E_V-F_P)/kT}$ $\quad np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations:

$$J_n = nq\mu_n\mathcal{E}_x + qD_n\frac{dn}{dx} = n\mu_n\frac{dF_n}{dx} \qquad \frac{D_n}{\mu_n} = \frac{kT}{q}$$

$$J_p = pq\mu_p\mathcal{E}_x - qD_p\frac{dp}{dx} = p\mu_p\frac{dF_p}{dx} \qquad \frac{D_p}{\mu_p} = \frac{kT}{q}$$

Carrier conservation equations:

$$\frac{\partial n}{\partial t} = +\nabla\cdot\left(\frac{J_n}{q}\right) + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\nabla\cdot\left(\frac{J_p}{q}\right) + G_p - R_p$$

Poisson's equation: $\nabla\cdot(\epsilon\mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n/\tau_n$ or $R = \Delta p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial\Delta n}{\partial t} = D_n\frac{\partial^2\Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $\quad L_{D,n} = \sqrt{D_n\tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\quad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s\epsilon_o}$

$$W = \sqrt{\frac{2K_s\epsilon_o V_{bi}}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)} \qquad x_n = \left(\frac{N_A}{N_A+N_D}\right)W \qquad x_p = \left(\frac{N_D}{N_A+N_D}\right)W \qquad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\epsilon_o}\left(\frac{N_A N_D}{N_A+N_D}\right)}$$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$ $\Delta p(0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = q\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)$ *(long)* $J_o = q\left(\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right)$ *(short)*

Non-ideal diodes: $I = I_o\left(e^{q(V_A - IR_S)/kT} - 1\right)$ $J_{gen} = -q\frac{n_i}{2\tau_o}W$

Photovoltaics: $V_{oc} = \frac{nkT}{q}\ln\left(\frac{J_{sc}}{J_o}\right)$ $J_{PV} = J_o\left(e^{qV_A/kT} - 1\right) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $C_J(V_R) = \frac{K_s\epsilon_o A}{\sqrt{\frac{2K_s\epsilon_o V_{bi}}{qN_A}}} = A\sqrt{\frac{qK_s\epsilon_o N_A}{2V_{bi}}}$ $C_D = G_d\tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\Phi_{BP} = \chi + E_G - \Phi_M$ $\Phi_{BN} = \Phi_M - \chi$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = A^* T^2 e^{-\Phi_B/kT}$ $A^* = \frac{4\pi q m^* k_B^2}{h^3} = 120\frac{m^*}{m_o}\ \frac{\text{A}}{\text{cm}^2\cdot\text{K}^2}$

MOS capacitors: $W = \sqrt{\frac{2K_s\epsilon_o\phi_s}{qN_A}}$ cm $\mathcal{E}_s = \sqrt{\frac{2qN_A\phi_s}{K_s\epsilon_o}}\ \frac{\text{V}}{\text{cm}}$

$$Q_B = -qN_A W(\phi_s) = -\sqrt{2qK_s\epsilon_o N_A\phi_s}\ \frac{\text{C}}{\text{cm}^2}$$

$$V_G = V_{FB} + \phi_s + \Delta\phi_{ox} = V_{FB} + \phi_s - \frac{Q_s(\phi_s)}{C_{ox}}$$

$C_{ox} = K_o\epsilon_o/x_o$ $V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$

$C = C_{ox}/\left[1 + \frac{K_o W(\phi_s)}{K_s x_o}\right]$ $V_T = -Q_B(2\phi_F)/C_{ox} + 2\phi_F$ $Q_n = -C_{ox}(V_G - V_T)$

MOSFETs: $I_D = -WQ_n(y=0)\langle v_y(y=0)\rangle$

$I_D = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}$ $I_D = WC_{ox}v_{sat}(V_{GS} - V_T)$

Square Law (for $V_{GS} \geq V_T$): $I_D = \begin{cases} \frac{W}{L}\mu_n C_{ox}[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2], & 0 \leq V_{DS} \leq V_{GS} - V_T \\ \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2, & V_{DS} \geq V_{GS} - V_T \end{cases}$