NAME:______FULL SOLUTION___________ **PUID:**_______________________

ECE 305 – Fall 2017

Exam 4 – Thursday, November 16, 2017

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 60 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)
II) 30 points
III) 30 points

Course policy

If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: ____

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Signature: ____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

1 (8 points). Which statement is true about the capacitance of a MOS device at high frequencies?

a. It is a parallel combination of C_{ox} and C_{depl} in the depletion regime only
b. It is a series combination of C_{ox} and C_{depl} in the accumulation regime only
c. It is a series combination of C_{ox} and C_{depl} in the depletion and inversion regimes
d. It is a parallel combination of C_{ox} and C_{depl} at in the accumulation regime only
e. It is a parallel combination of C_{ox} and C_{depl} in the inversion regime only

2 (8 points). A linear $I_D - V_{DS}$ relation can be observed around what combination of biases for an n-channel MOS device?

a. $V_{GS} = 4V_T, V_{DS} = -2V_T$
b. $V_{GS} = V_T, V_{DS} = -2V_T$
c. $V_{GS} = 4V_T, V_{DS} = 2V_T$
d. $V_{GS} = V_T, V_{DS} = 4V_T$
e. $V_{GS} = -2V_T, V_{DS} = -2V_T$

3 (8 points). How does exposure to radiation affect MOS devices?

a. Changes the electron affinity of the oxide
b. Changes the bandgap of the semiconductor
c. Changes the doping near the oxide-semiconductor interface
d. Introduces traps in the oxide and at the oxide-semiconductor interface
e. Changes the metal workfunction

4 (8 points) How does the saturated current of an ideal MOSFET device governed by the square law vary with the gate voltage V_{GS}?

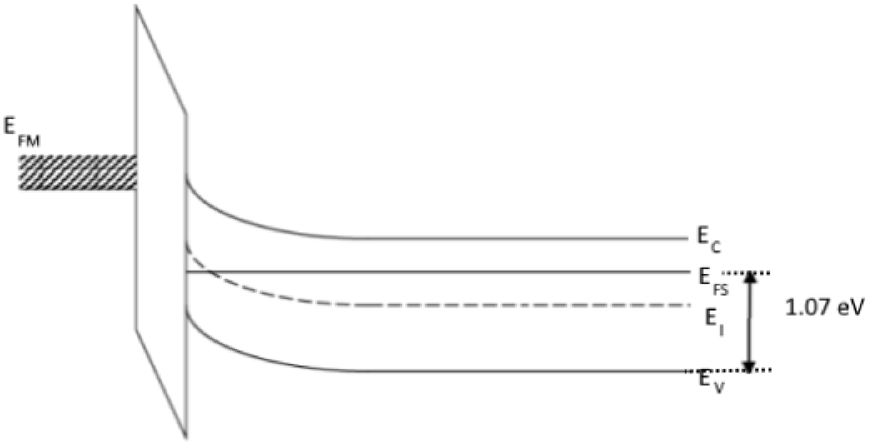
a. $(V_{GS} - V_T)^2$
b. $(V_{GS} - V_T)^{1.5}$
c. $(V_{GS} - V_T)^{1.0}$
d. $(V_{GS} - V_T)^{0.5}$
e. $(V_{GS} - V_T)^{-2}$

5 (8 points). In which regime are excess free carriers of type opposite to those expected by the semiconductor doping are found near the oxide-Si interface of a MOS device?

a. Depletion
b. Inversion
c. Flatband
d. Deep inversion
e. Accumulation

Part II (Free Response, 30 points)

Consider an MOS capacitor made of crystalline silicon described by the band diagram below. Note that unmarked values may not be to scale. Assume that $\mathcal{E}_{ox} = 2.7 \cdot 10^6$ V/cm, $t_{ox} = 2.0$ nm, $K_{ox} = 3.9$, $K_{Si} = 11.8$, and $E_g = 1.12$ eV.



a. What is the level of n-type doping N_D in the semiconductor?

$N_D=N_C*\exp[(E_{FS}-E_C)/kT]=3.23\cdot10^{19} * \exp(-0.05/0.029)=4.7\cdot10^{18}\ \text{cm}^{-3}$

b. What is the electric field amplitude in the semiconductor right next to the oxide?

$K_s\mathcal{E}_s = K_{ox}\mathcal{E}_{ox}$, so $\mathcal{E}_s = \left(\frac{K_{ox}}{K_s}\right)\mathcal{E}_{ox} = \left(\frac{3.9}{11.8}\right)\cdot 2.7\cdot 10^6\,\frac{\text{V}}{\text{cm}} = 8.9\cdot 10^5\,\frac{\text{V}}{\text{cm}}$.

$\mathcal{E}_s = \sqrt{\frac{2qN_A|\phi_s|}{K_s\epsilon_o}}$ so $|\phi_s| = \frac{K_s\epsilon_o\mathcal{E}_s^2}{2qN_A} = \frac{11.8\cdot8.854\cdot10^{-14}\text{F/cm}\cdot(8.9\cdot10^5\ \text{V/cm})^2}{2\cdot(1.6\cdot10^{-19}\text{C})\cdot(4.7\cdot10^{18}\ \text{cm}^{-3})} = 0.55\ \text{V}$

c. Assume that $|\phi_s| = 0.55$ V. What is the depletion width (in the semiconductor)?

$$W = \sqrt{\frac{2K_s\epsilon_o|\phi_s|}{qN_A}} = \sqrt{\frac{2(11.8)(8.854\cdot10^{-14}\text{F/cm})(0.55\ \text{V})}{(1.6\cdot10^{-19}\text{C})(4.7\cdot10^{18}\ \text{cm}^{-3})}} = 1.23\cdot10^{-6}\ \text{cm}$$

d. What is the electrostatic potential voltage of the gate electrode with respect to the semiconductor (assuming the flat band voltage $V_{FB} = 0$)?

$|V_G| = |V_{FB} + \phi_s + \Delta\phi_{ox}| = 0 + 0.55 + 0.54 = 1.09$ V (negative value is OK).

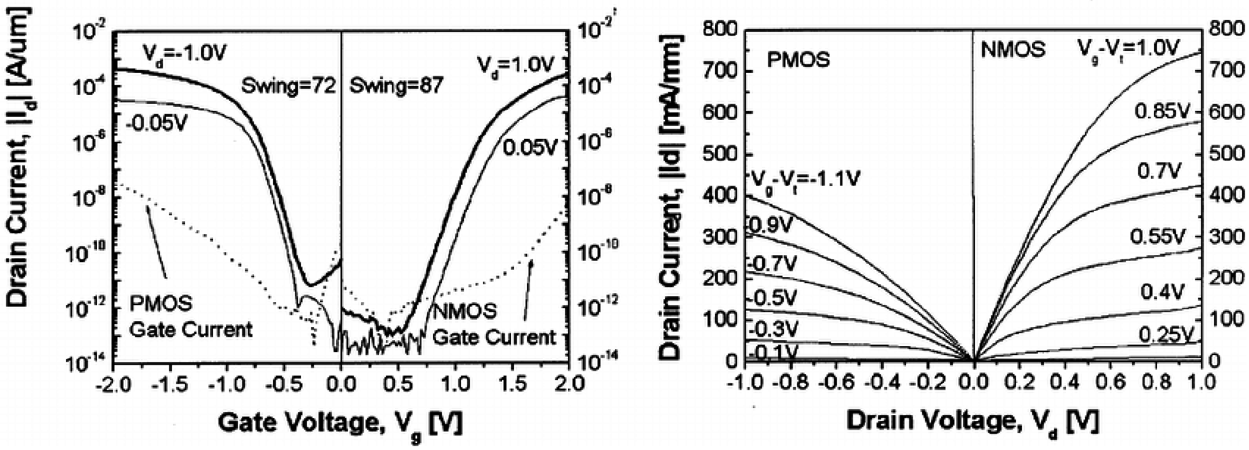
e. Assume $\phi_F = 0.52$ V. What is the minimum change in gate voltage required to drive the MOS capacitor to the inversion regime?

$\phi_s = 2\phi_F = 2\cdot 0.52 = 1.04$ V; Thus $\mathcal{E}'_{ox} = \mathcal{E}_{ox}\sqrt{2\phi_F/\phi_s} = 3.7$ V/cm.

Thus, $V_G = V_{FB} + \phi_s + \Delta\phi_{ox} = 0 + 1.04 + 3.7 * 0.2 = 1.78$ V. Therefore, $|\Delta V_G| = 1.78 - 1.09 = 0.69$ V.

Part III (Free Response, 30 points)

Consider a pair of p-channel (left) and n-channel (right) crystalline silicon-based MOSFETs, characterized in terms of drain current as a function of gate voltage and drain voltage.



a. Estimate the saturation threshold voltage ($V_{T,SAT}$) for both types of logic. Justify your answer.

PMOS: $V_{T,SAT} = -0.65$ V

NMOS: $V_{T,SAT} = +1.1$ V

b. Estimate the drain-induced barrier lowering for both types of logic.

PMOS: DIBL ≈ 50 mV/(1-0.05 V)=53 mV/V

NMOS: DIBL ≈ 200 mV/(1-0.05 V)=211 mV/V

c. How can the asymmetry in the output characteristics for comparable $V_G - V_T$ values be explained?

N-type channels in crystalline silicon have higher mobility. The channel lengths also might differ.

d. If the mobility $\mu_n = 1400 \text{ cm}^2/\text{V}\cdot\text{s}$, and oxide capacitance $C_{ox} = 86 \text{ nF/cm}^2$, estimate the NMOS channel length from the data given.

Since $I_D = \frac{W}{2L}\mu_n C_{ox}(V_G - V_T)^2$, we can rearrange to write:
$L = \frac{\mu_n C_{ox}}{2I_D/W}(V_G - V_T)^2$. If we use the data for $V_G - V_T = 1$ V, we obtain:
$L = \frac{(1400 \text{ cm}^2/\text{V}\cdot\text{s})(86 \text{ nF/cm}^2)}{2\cdot(750 \text{ mA/mm})}(1 \text{ V})^2 = 80 \text{ nm}.$

e. Given part d, what is the inversion layer charge density (in nC/cm^2) in the NMOS semiconductor, if $V_G - V_T = 0.7$ V?

|Q| = C$_{ox}$ $(V_G - V_T)$ = 60 nC/cm^2. A negative value is acceptable here.

ECE 305 Exam 4 Formula Sheet (Fall 2017)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	**Silicon parameters ($T = 300$ K)**
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}\ \text{cm}^{-3}$
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}\ \text{cm}^{-3}$
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}\ \text{cm}^{-3}$
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_i-E_F)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $p = N_V e^{(E_V-F_P)/kT}$ $np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q\left(n\mu_n + p\mu_p\right) = 1/\rho$

Drift-diffusion current equations: $J_n = nq\mu_n\mathcal{E}_x + qD_n\frac{dn}{dx} = n\mu_n\frac{dF_n}{dx}$ $\frac{D_n}{\mu_n} = \frac{kT}{q}$

$J_p = pq\mu_p\mathcal{E}_x - qD_p\frac{dp}{dx} = p\mu_p\frac{dF_p}{dx}$ $\frac{D_p}{\mu_p} = \frac{kT}{q}$

Carrier conservation equations: $\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q}\right) + G_n - R_n$

$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q}\right) + G_p - R_p$

Poisson's equation: $\nabla \cdot (\epsilon\mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n/\tau_n$ or $R = \Delta p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n\frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $L_{D,n} = \sqrt{D_n\tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s\epsilon_o}$

$W = \sqrt{\frac{2K_s\epsilon_o V_{bi}}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)}$ $x_n = \left(\frac{N_A}{N_A+N_D}\right)W$ $x_p = \left(\frac{N_D}{N_A+N_D}\right)W$ $\mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\epsilon_o}\left(\frac{N_A N_D}{N_A+N_D}\right)}$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$ $\quad \Delta p(0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $\quad J_o = q\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)$ *(long)* $\quad J_o = q\left(\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right)$ (*short*)

Non-ideal diodes: $I = I_o\left(e^{q(V_A - IR_S)/kT} - 1\right)$ $\quad J_{gen} = -q\frac{n_i}{2\tau_o}W$

Photovoltaics: $V_{oc} = \frac{nkT}{q}\ln\left(\frac{J_{sc}}{J_o}\right)$ $\quad J_{PV} = J_o\left(e^{qV_A/kT} - 1\right) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $\quad C_J(V_R) = \frac{K_S\epsilon_o A}{\sqrt{\frac{2K_S\epsilon_o V_{bi}}{qN_A}}} = A\sqrt{\frac{qK_S\epsilon_o N_A}{2V_{bi}}}$ $\quad C_D = G_d\tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\quad \Phi_{BP} = \chi + E_G - \Phi_M$ $\quad \Phi_{BN} = \Phi_M - \chi$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $\quad J_o = A^* T^2 e^{-\Phi_B/kT}$ $\quad A^* = \frac{4\pi q m^* k_B^2}{h^3} = 120\frac{m^*}{m_o}\ \frac{\mathrm{A}}{\mathrm{cm^2 \cdot K^2}}$

MOS capacitors: $W = \sqrt{\frac{2K_S\epsilon_o\phi_S}{qN_A}}$ cm $\quad \mathcal{E}_S = \sqrt{\frac{2qN_A\phi_S}{K_S\epsilon_o}}\ \frac{\mathrm{V}}{\mathrm{cm}}$

$$Q_B = -qN_A W(\phi_S) = -\sqrt{2qK_S\epsilon_o N_A\phi_S}\ \frac{\mathrm{C}}{\mathrm{cm^2}}$$

$$V_G = V_{FB} + \phi_S + \Delta\phi_{ox} = V_{FB} + \phi_S - \frac{Q_S(\phi_S)}{C_{ox}}$$

$$C_{ox} = K_o\epsilon_o/x_o \qquad V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$$

$C = C_{ox}/\left[1 + \frac{K_o W(\phi_S)}{K_S x_o}\right]$ $\quad V_T = -Q_B(2\phi_F)/C_{ox} + 2\phi_F$ $\quad Q_n = -C_{ox}(V_G - V_T)$

MOSFETs: $I_D = -WQ_n(y=0)\langle v_y(y=0)\rangle$

$$I_D = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS} \qquad I_D = WC_{ox}v_{sat}(V_{GS} - V_T)$$

Square Law (for $V_{GS} \geq V_T$): $I_D = \begin{cases} \frac{W}{L}\mu_n C_{ox}\left[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2\right], & 0 \leq V_{DS} \leq V_{GS} - V_T \\ \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2, & V_{DS} \geq V_{GS} - V_T \end{cases}$