NAME:____FULL SOLUTION____ PUID:________________________

ECE 305 – Spring 2018

Exam 4 – Thursday, April 12, 2018

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 60 minutes (or less). Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)
II) 30 points
III) 30 points

Course policy

If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: ____

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Signature: ____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

1 (8 points). Gradually introducing a limited number of mobile positive charges into the gate oxide of a MOS-capacitor would:

a. Make the oxide highly conductive
b. Erode the oxide over time, leaving an empty gap between the gate and channel
c. Dope the silicon substrate
d. Change the threshold voltage over time
e. Change the flatband voltage with time

2 (8 points). For a polycrystalline Si gate MOS capacitor, changing the doping in the crystalline Si substrate would change the threshold voltage as

a. V_{FB} changes
b. Φ_F changes
c. ΔV_{OX} changes
d. a. and c. only
e. All of the above

3 (8 points). When V_G=V_T for an n-MOSFET, what is the value of the semiconductor potential Φ_S?

a. 0.6 Φ_F
b. 1.2 Φ_F
c. 1.5 Φ_F
d. 2 Φ_F
e. 2.4 Φ_F

4 (8 points) How does the saturated current of a n-channel MOSFET **in the long-channel regime** vary with the gate voltage V_{GS}?

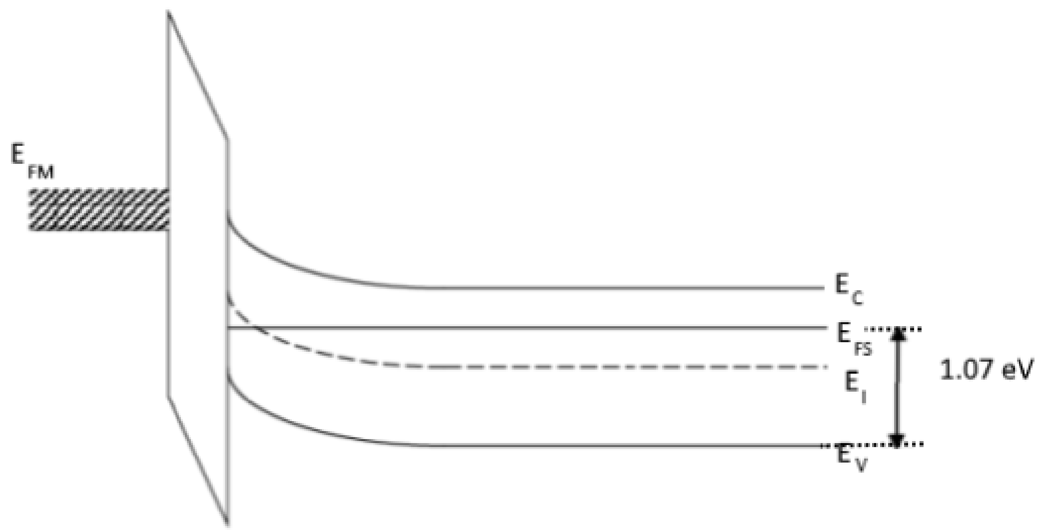
a. $(V_{GS} - V_T)^2$
b. $(V_{GS} - V_T)^{1.5}$
c. $(V_{GS} - V_T)^{1.0}$
d. $(V_{GS} - V_T)^{0.5}$
e. $(V_{GS} - V_T)^{-2}$

5 (8 points). When minority free carriers are found near the oxide-Si interface of a MOS device, what is the bias condition?

a. Depletion
b. Inversion
c. Flatband
d. Deep accumulation
e. Accumulation

Part II (Free Response, 30 points)

Consider an MOS capacitor made of crystalline silicon described by the band diagram below. Note that unmarked values may not be to scale. Assume that $|\Delta V_{OX}|$ = 0.9 V, $t_{ox} = 3.0$ nm, $K_{ox} = 3.9$, $K_{Si} = 11.7$, and $E_g = 1.12$ eV.



a. What is the magnitude of the electric field in the **semiconductor** right next to the oxide ($x = 0^+$)?

$$|\mathcal{E}_s| = \frac{K_{ox}}{K_s}\frac{\Delta V_{ox}}{t_{ox}} = \frac{3.9}{11.8} \cdot \frac{0.9 \text{ V}}{3 \times 10^{-7} \text{ cm}} = 1 \times 10^6 \text{ V/cm}$$

b. What is the magnitude of the electrostatic potential voltage of the gate electrode with respect to the semiconductor, assuming the flat band voltage V_{FB} is 0? Assume that $N_D = 4.7 \cdot 10^{18}$ cm^{-3}.

$$W = \frac{K_S \epsilon_o |\mathcal{E}_s|}{qN_D} = \frac{11.8 \cdot (8.854 \cdot 10^{-14} \text{ F/cm})(1 \times 10^6 \text{ V/cm})}{(1.602 \cdot 10^{-19} \text{ C}) \cdot (4.7 \cdot 10^{18} \text{ cm}^{-3})} = 13.9 \text{ nm}$$

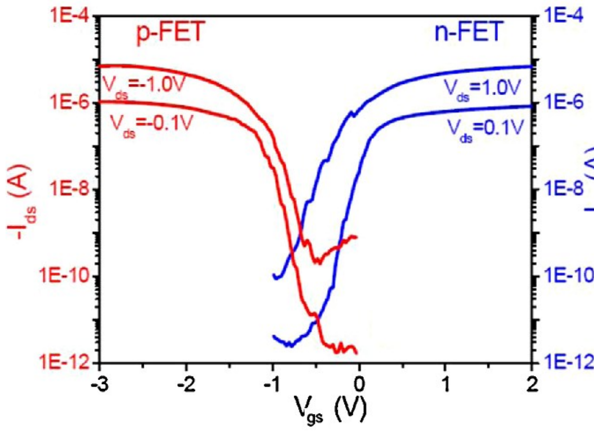
$$|V_G| = |\Delta V_{ox} + \phi_s| = \left|\Delta V_{ox} + \frac{1}{2}\mathcal{E}_s W\right| = 0.9 \text{ V} + \frac{1}{2}(1 \times 10^6 \text{ V/cm})(13.9 \text{ nm}) = 1.59 \text{ V}$$

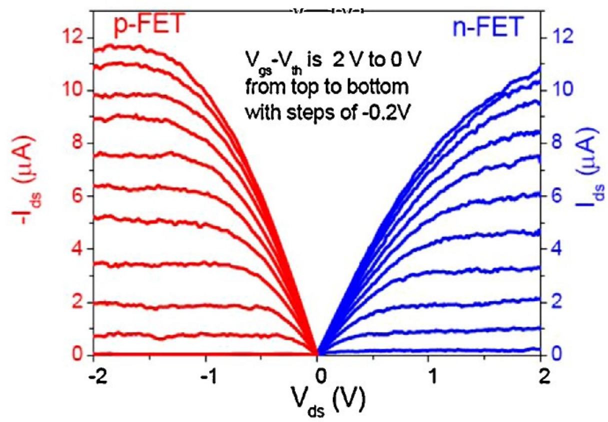
c. For the above diagram, which regime is the capacitor operating in? If the gate voltage is kept constant, but the oxide thickness is reduced gradually, eventually the operating regime changes. What would be the new operating regime?

It appears to be in the depletion regime, since $\phi_s < 2\phi_F$. However, a thinner gate oxide would increase ϕ_s, eventually pushing the system into inversion.

Part III (Free Response, 30 points)

Consider a pair of p-channel (left) and n-channel (right) carbon nanotube-based MOSFETs, characterized in terms of drain current as a function of gate voltage and drain voltage.





a. Estimate the saturation threshold voltage ($V_{T,SAT}$) for both types of logic. Justify your answer.

$$V_{T,SAT}(\text{p FET}) \approx -0.9 \text{ V to} -1 \text{ V}$$

$$V_{T,SAT}(\text{n FET}) \approx -0.25 \text{ V to} + 0.1 \text{ V}$$

b. Estimate the drain-induced barrier lowering for both types of logic.

DIBL(p-FET) ≈ 100 mV / 0.9 V = 110 mV/V

DIBL(n-FET) ≈ 600 mV / 0.9 V = 670 mV/V

c. How can the asymmetry in the output characteristics for comparable $V_G - V_T$ values be explained?

The carrier mobilities, channel lengths, and gate oxide thicknesses might differ.

d. Find and mention the best performing device here in terms of subthreshold slope. State the subthreshold slope and the operating conditions to obtain that optimum performance.

Clearly, p-FETs have the best subthreshold slope. The value is about 100 mV/decade. The value for n-FETs is about 200 to 250 mV/decade.

e. Is this carbon nanotube MOSFET superior to a conventional crystalline Si MOSFET? Justify your answer.

Not in terms of performance, since c-Si MOSFETs have higher on-off ratios and saturation currents, given sufficient gate widths. C-Si MOSFETs also generally have less noisy output characteristics.

The main advantage is in terms of size – CNT MOSFETs can be much smaller. They also can have higher mobilities than conventional Si, and thus may have higher saturation current densities.

ECE 305 Exam 4 Formula Sheet (Spring 2018)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	**Silicon parameters ($T = 300$ K)**
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm^{-3}
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm^{-3}
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm^{-3}
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\quad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $\quad p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_i-E_F)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $\quad p = N_V e^{(E_V-F_P)/kT}$ $\quad np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations:

$$J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx} \qquad \frac{D_n}{\mu_n} = \frac{kT}{q}$$

$$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx} \qquad \frac{D_p}{\mu_p} = \frac{kT}{q}$$

Carrier conservation equations:

$$\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q}\right) + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q}\right) + G_p - R_p$$

Poisson's equation: $\nabla \cdot (\epsilon \mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n/\tau_n$ or $R = \Delta p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $\quad L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\quad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s\epsilon_o}$

$$W = \sqrt{\frac{2K_s\epsilon_o V_{bi}}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)} \quad x_n = \left(\frac{N_A}{N_A+N_D}\right)W \quad x_p = \left(\frac{N_D}{N_A+N_D}\right)W \quad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\epsilon_o}\left(\frac{N_A N_D}{N_A+N_D}\right)}$$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$ $\Delta p(0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = q\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)$ *(long)* $J_o = q\left(\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right)$ *(short)*

Non-ideal diodes: $I = I_o\left(e^{q(V_A - IR_S)/kT} - 1\right)$ $J_{gen} = -q\frac{n_i}{2\tau_o}W$

Photovoltaics: $V_{oc} = \frac{nkT}{q}\ln\left(\frac{J_{sc}}{J_o}\right)$ $J_{PV} = J_o\left(e^{qV_A/kT} - 1\right) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $C_J(V_R) = \frac{K_s\epsilon_o A}{\sqrt{\frac{2K_s\epsilon_o V_{bi}}{qN_A}}} = A\sqrt{\frac{qK_s\epsilon_o N_A}{2V_{bi}}}$ $C_D = G_d\tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\Phi_{BP} = \chi + E_G - \Phi_M$ $\Phi_{BN} = \Phi_M - \chi$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = A^* T^2 e^{-\Phi_B/kT}$ $A^* = \frac{4\pi q m^* k_B^2}{h^3} = 120\frac{m^*}{m_o}\ \frac{\text{A}}{\text{cm}^2\cdot\text{K}^2}$

MOS capacitors: $W = \sqrt{\frac{2K_s\epsilon_o\phi_s}{qN_A}}$ $\mathcal{E}_s = \sqrt{\frac{2qN_A\phi_s}{K_s\epsilon_o}}$ (for p-type substrates)

$$Q_B = -qN_A W(\phi_s) = -\sqrt{2qK_s\epsilon_o N_A\phi_s}\ \frac{\text{C}}{\text{cm}^2}$$

$$V_G = V_{FB} + \phi_s + \Delta\phi_{ox} = V_{FB} + \phi_s - \frac{Q_s(\phi_s)}{C_{ox}}$$

$$C_{ox} = K_o\epsilon_o/x_o \qquad V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$$

$C = C_{ox}/\left[1 + \frac{K_o W(\phi_s)}{K_s x_o}\right]$ $V_T = -Q_B(2\phi_F)/C_{ox} + 2\phi_F$ $Q_n = -C_{ox}(V_G - V_T)$

MOSFETs: $I_D = -WQ_n(y=0)\langle v_y(y=0)\rangle$

$$I_D = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS} \qquad I_D = WC_{ox}v_{sat}(V_{GS} - V_T)$$

Square Law (for $V_{GS} \geq V_T$): $I_D = \begin{cases} \frac{W}{L}\mu_n C_{ox}[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2], & 0 \leq V_{DS} \leq V_{GS} - V_T \\ \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2, & V_{DS} \geq V_{GS} - V_T \end{cases}$