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PUID: _____

ECE 305 – Fall 2015

Final Exam (Exam 5) – Wednesday, December 16, 2015

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI---30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 50 minutes (or less). However, the full two hours will be allowed, if you need them. Be sure to fill in your name and Purdue student ID at the top of the page. **DO NOT** open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

- I) 40 points (8 points per question)
- II) 30 points
- III) 30 points

Course policy

If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: _____

Signature: _____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

1 (8 points). Which of the following would increase the collector breakdown voltage in a BJT?

- a. Increasing the emitter doping
- b. Increasing the base doping
- c. Increasing the collector doping
- d. Decreasing the collector width
- e. Decreasing the collector doping

2 (8 points). How are the PN junctions biased in the forward active region of an NPN BJT?

- a. Base-collector: forward biased Emitter-base: forward biased
- b. Base-collector: forward biased Emitter-base: reverse biased
- c. Base-collector: reverse biased Emitter-base: forward biased
- d. Base-collector: reverse biased Emitter-base: reverse biased
- e. Base-collector: forward biased Emitter-base: biased in breakdown

3 (8 points). What would be considered good values for α_{dc} and β_{dc} , respectively, in a BJT?

- a. 0.1 and 0.11
- b. 0.5 and 1
- c. 0.99 and 99
- d. 3 and 1.5
- e. 20 and 1.052

4 (8 points). For large gain in a bipolar transistor, the emitter doping must be ?

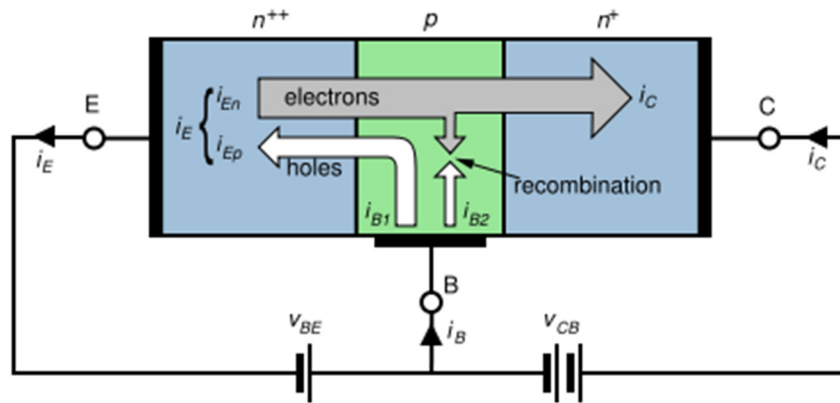
- a. Larger than the base doping only
- b. Smaller than the base doping only
- c. Larger than the collector doping only
- d. Larger than both base and collector doping
- e. It does not matter

5 (8 points). How can a heterojunction bipolar transistor be designed to improve β_{dc} ?

- a. Low bandgap material in the base region
- b. Low bandgap material in the emitter region
- c. High electron affinity material in the base region
- d. Low breakdown voltage material in the collector region
- e. High bandgap material in the base region

Part II (Free Response, 30 points)

Consider the NPN BJT made of silicon, depicted below. Assume that it is in the forward active region, with $I_C = 10 \mu\text{A}$, doping concentrations $N_{D,E}=10^{18}/\text{cm}^3$, $N_{A,B}=10^{17}/\text{cm}^3$, and $N_{D,C}= 10^{16} /\text{cm}^3$; thicknesses $W_E=0.5 \mu\text{m}$, $W_B=0.25 \mu\text{m}$, and $W_C=2 \mu\text{m}$; and diffusion constants $D_{p,E}=2 \text{ cm}^2/\text{s}$, $D_{n,B}=20 \text{ cm}^2/\text{s}$, and $D_{p,C}=12 \text{ cm}^2/\text{s}$, and $L_b=1 \mu\text{m}$. The cross-sectional area $A=1 \mu\text{m}^2$. Assume that recombination within the BJT is small (i.e., diffusion lengths are much larger than the thicknesses of each layer).

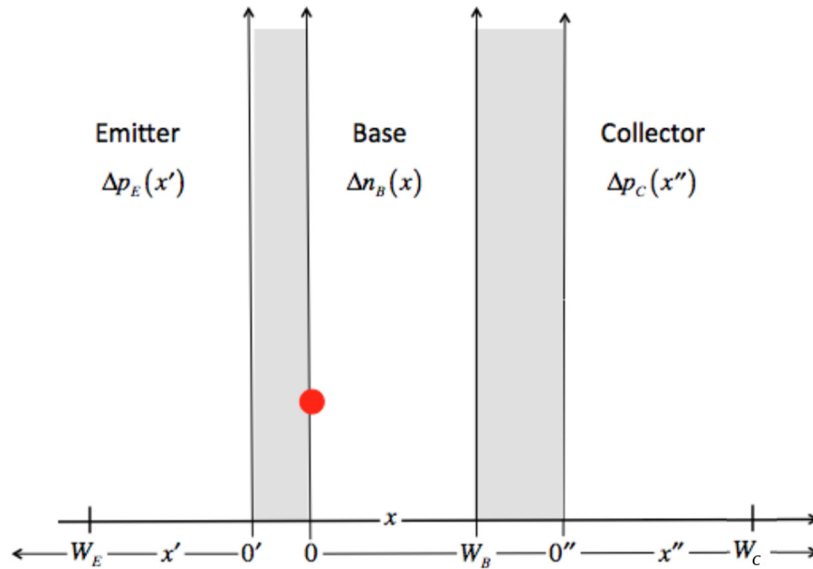


- What is the emitter injection efficiency γ_F ?
- What is the base transport factor α_T ?
- What is the common base current gain α_{DC} ?
- What is the emitter gain β_F (aka β_{dc})?
- What is the forward saturation current density I_{F0} ?

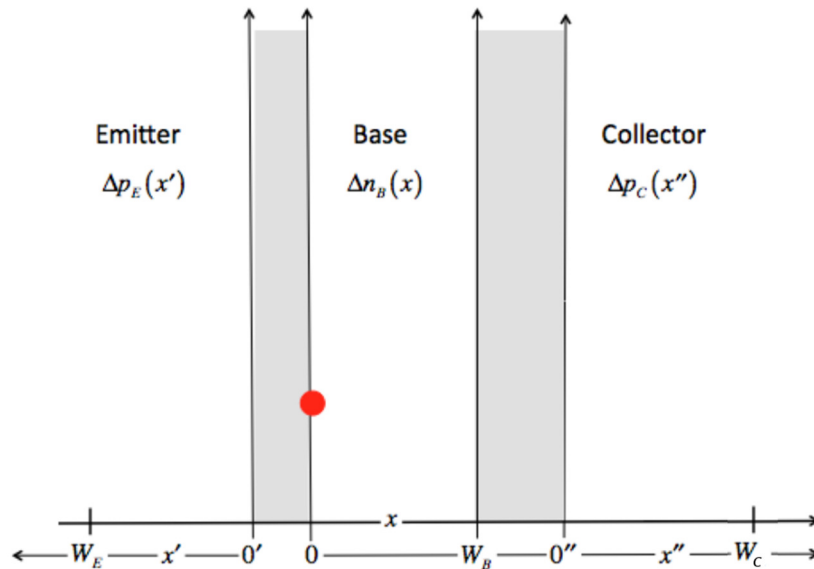
Part III (Free Response, 30 points)

Consider the crystalline silicon-based bipolar junction transistor (BJT) depicted below. The shaded regions represent the depletion regions, while white regions may be treated as having zero electric field (in the depletion approximation). Assume that the doping concentrations $N_{D,E}=10^{18}/\text{cm}^3$, $N_{A,B}=10^{17}/\text{cm}^3$, and $N_{D,C}=10^{16}/\text{cm}^3$. Assume $L_D \gg W_E$ and $L_D \gg W_C$.

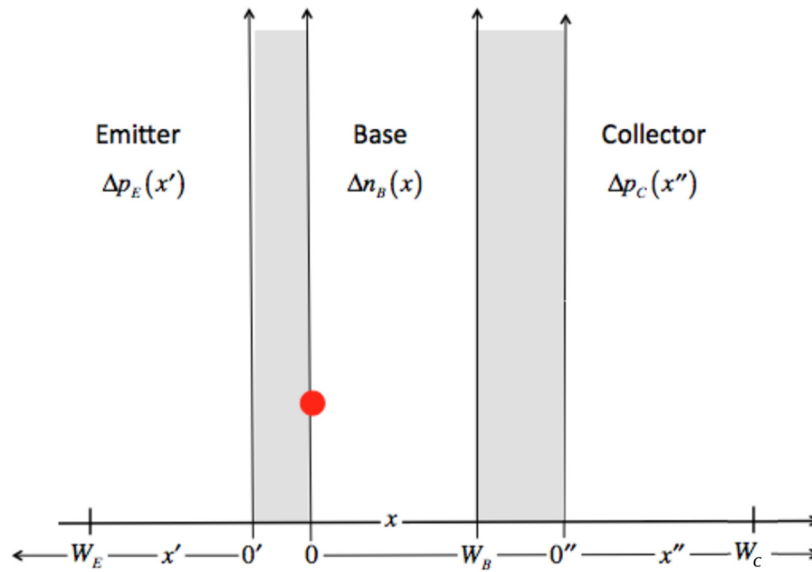
- a. Sketch the excess minority carrier concentrations (in all white regions) in **forward** active mode biasing.



- b. Sketch the excess minority carrier concentrations (in all white regions) in inverted mode biasing.



- c. Sketch the excess minority carrier concentrations (in all white regions) in cutoff mode biasing.



(Questions d and e are based on the following information) The minority carrier concentrations in the quasineutral regions of a pnp BJT are given in the diagram below.

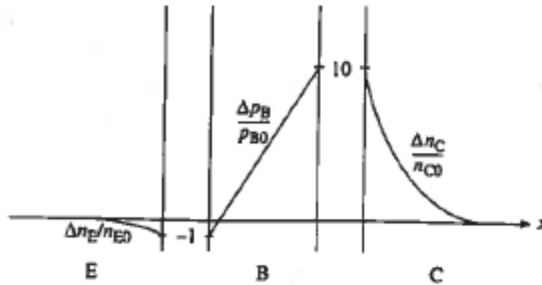


Figure P11.8

- d. Estimate the sign of the collector-base and emitter-base biases, and deduce the operating mode of this BJT.
- e. Calculate the magnitude of the collector-base bias V_{CB} .

ECE 305 Exam 5 Formula Sheet (Fall 2015)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	Silicon parameters ($T = 300$ K)
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm ⁻³
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm ⁻³
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm ⁻³
$q = 1.602 \times 10^{-19}$ C	$K_S = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2} \sqrt{2(E-E_C)}}{\pi^2 \hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4} \left(\frac{2m_n^* kT}{\pi \hbar^2} \right)^{3/2}$ $N_V = \frac{1}{4} \left(\frac{2m_p^* kT}{\pi \hbar^2} \right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_F-E_i)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$ **Law of Mass Action:** $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $p = N_V e^{(E_V-F_P)/kT}$ $np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_p = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations: $J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx}$ $\frac{D_n}{\mu_n} = \frac{kT}{q}$

$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx}$ $\frac{D_p}{\mu_p} = \frac{kT}{q}$

Carrier conservation equations: $\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q} \right) + G_n - R_n$

$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q} \right) + G_p - R_p$

Poisson's equation: $\nabla \cdot (\epsilon \mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n / \tau_n$ or $R = \Delta p / \tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q} \ln \left(\frac{N_D N_A}{n_i^2} \right)$ $\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S \epsilon_0}$

$W = \sqrt{\frac{2K_S \epsilon_0 V_{bi}}{q} \left(\frac{N_A + N_D}{N_A N_D} \right)}$ $x_n = \left(\frac{N_A}{N_A + N_D} \right) W$ $x_p = \left(\frac{N_D}{N_A + N_D} \right) W$ $\mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_S \epsilon_0} \left(\frac{N_A N_D}{N_A + N_D} \right)}$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1)$ $\Delta p(0) = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1)$

$J_D = J_o (e^{qV_A/kT} - 1)$ $J_o = q \left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right)$ (long) $J_o = q \left(\frac{D_n n_i^2}{W_p N_A} + \frac{D_p n_i^2}{W_n N_D} \right)$ (short)

Non-ideal diodes: $I = I_o (e^{q(V_A - IR_s)/kT} - 1)$ $J_{gen} = -q \frac{n_i}{2\tau_o} W$

Photovoltaics: $V_{oc} = \frac{nkT}{q} \ln \left(\frac{J_{sc}}{J_o} \right)$ $J_{PV} = J_o (e^{qV_A/kT} - 1) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $C_J(V_R) = \frac{K_s \epsilon_o A}{\sqrt{\frac{2K_s \epsilon_o V_{bi}}{q N_A}}} = A \sqrt{\frac{q K_s \epsilon_o N_A}{2V_{bi}}}$ $C_D = G_d \tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\Phi_{BP} = \chi + E_G - \Phi_M$ $\Phi_{BN} = \Phi_M - \chi$

$J_D = J_o (e^{qV_A/kT} - 1)$ $J_o = A^* T^2 e^{-\Phi_B/kT}$ $A^* = \frac{4\pi q m^* k_B^2}{h^3} = 120 \frac{m^*}{m_o} \frac{A}{\text{cm}^2 \cdot \text{K}^2}$

MOS capacitors: $W = \sqrt{\frac{2K_s \epsilon_o \phi_s}{q N_A}} \text{ cm}$ $\epsilon_s = \sqrt{\frac{2q N_A \phi_s}{K_s \epsilon_o}} \frac{V}{\text{cm}}$

$Q_B = -q N_A W (\phi_s) = -\sqrt{2q K_s \epsilon_o N_A \phi_s} \frac{C}{\text{cm}^2}$

$V_G = V_{FB} + \phi_s + \Delta\phi_{ox} = V_{FB} + \phi_s - \frac{Q_s(\phi_s)}{C_{ox}}$

$C_{ox} = K_o \epsilon_o / x_o$ $V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$

$C = C_{ox} / \left[1 + \frac{K_o W (\phi_s)}{K_s x_o} \right]$ $V_T = -Q_B(2\phi_F)/C_{ox} + 2\phi_F$ $Q_n = -C_{ox}(V_G - V_T)$

MOSFETs: $I_D = -W Q_n(y=0) \langle v_y(y=0) \rangle$

$I_D = \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_T) V_{DS}$ $I_D = W C_{ox} v_{sat} (V_{GS} - V_T)$

Square Law (for $V_{GS} \geq V_T$): $I_D = \begin{cases} \frac{W}{L} \mu_n C_{ox} [(V_{GS} - V_T) V_{DS} - V_{DS}^2/2], & 0 \leq V_{DS} \leq V_{GS} - V_T \\ \frac{W}{2L} \mu_n C_{ox} (V_{GS} - V_T)^2, & V_{DS} \geq V_{GS} - V_T \end{cases}$

Bipolar transistors: (assuming NPN, short emitter, base, and collector)

Ebers-Moll Equations:

$$I_C(V_{BE}, V_{BC}) = \alpha_F I_{F0} \left(e^{qV_{BE}/k_B T} - 1 \right) - I_{R0} \left(e^{qV_{BC}/k_B T} - 1 \right)$$

$$I_E(V_{BE}, V_{BC}) = I_{F0} \left(e^{qV_{BE}/k_B T} - 1 \right) - \alpha_R I_{R0} \left(e^{qV_{BC}/k_B T} - 1 \right)$$

$$I_{F0} = qA \left(\frac{D_{nB}}{W_B} \frac{n_i^2}{N_{AB}} + \frac{D_{pE}}{W_E} \frac{n_i^2}{N_{DE}} \right)$$

$$I_{R0} = qA \left(\frac{D_{nB}}{W_B} \frac{n_i^2}{N_{AB}} + \frac{D_{pC}}{W_C} \frac{n_i^2}{N_{DC}} \right)$$

$$\alpha_F = \gamma_F \alpha_T$$

$$\alpha_R = \gamma_R \alpha_T$$

$$\alpha_F I_{F0} = \alpha_R I_{R0}$$

$$\gamma_F = \frac{I_{En}}{I_{En} + I_{Ep}} = \frac{1}{1 + \frac{D_{pE}}{D_{nB}} \frac{W_B}{W_E} \frac{N_{AB}}{N_{DE}}}$$

$$\alpha_T = \frac{I_{Cn}}{I_{En}} = \frac{1}{1 + \frac{1}{2} \left(\frac{W_B}{L_{nB}} \right)^2}$$

$$\beta_F = \frac{\alpha_F}{1 - \alpha_F}$$

$$\alpha_F = \frac{\beta_F}{1 + \beta_F}$$