ECE 305 Exam 5 Formula Sheet (Fall 2015)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	Silicon parameters ($T = 300$ K)
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm^{-3}
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm^{-3}
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm^{-3}
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\qquad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $\qquad p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_F-E_i)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $\quad p = N_V e^{(E_V-F_P)/kT}$ $\quad np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations: $J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx}$ $\qquad \frac{D_n}{\mu_n} = \frac{kT}{q}$

$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx}$ $\qquad \frac{D_p}{\mu_p} = \frac{kT}{q}$

Carrier conservation equations: $\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q}\right) + G_n - R_n$

$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q}\right) + G_p - R_p$

Poisson's equation: $\nabla \cdot (\epsilon \mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n/\tau_n$ or $R = \Delta p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $\qquad L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\qquad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s\epsilon_o}$

$W = \sqrt{\frac{2K_s\epsilon_o V_{bi}}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)}$ $\quad x_n = \left(\frac{N_A}{N_A+N_D}\right)W$ $\quad x_p = \left(\frac{N_D}{N_A+N_D}\right)W$ $\quad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\epsilon_o}\left(\frac{N_A N_D}{N_A+N_D}\right)}$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$ $\qquad \Delta p(0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $\qquad J_o = q\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)$ *(long)* $\qquad J_o = q\left(\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right)$ *(short)*

Non-ideal diodes: $I = I_o\left(e^{q(V_A - IR_S)/kT} - 1\right)$ $\qquad J_{gen} = -q\frac{n_i}{2\tau_o}W$

Photovoltaics: $V_{oc} = \frac{nkT}{q}\ln\left(\frac{J_{sc}}{J_o}\right)$ $\qquad J_{PV} = J_o\left(e^{qV_A/kT} - 1\right) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $\qquad C_J(V_R) = \frac{K_S\epsilon_o A}{\sqrt{\frac{2K_S\epsilon_o V_{bi}}{qN_A}}} = A\sqrt{\frac{qK_S\epsilon_o N_A}{2V_{bi}}}$ $\qquad C_D = G_d\tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\qquad \Phi_{BP} = \chi + E_G - \Phi_M$ $\qquad \Phi_{BN} = \Phi_M - \chi$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $\qquad J_o = A^* T^2 e^{-\Phi_B/kT}$ $\qquad A^* = \frac{4\pi q m^* k_B^2}{h^3} = 120\,\frac{m^*}{m_o}\ \frac{\mathrm{A}}{\mathrm{cm^2 \cdot K^2}}$

MOS capacitors: $W = \sqrt{\frac{2K_S\epsilon_o\phi_S}{qN_A}}$ cm $\qquad \mathcal{E}_S = \sqrt{\frac{2qN_A\phi_S}{K_S\epsilon_o}}\ \frac{\mathrm{V}}{\mathrm{cm}}$

$$Q_B = -qN_A W(\phi_S) = -\sqrt{2qK_S\epsilon_o N_A\phi_S}\ \ \frac{\mathrm{C}}{\mathrm{cm}^2}$$

$$V_G = V_{FB} + \phi_S + \Delta\phi_{ox} = V_{FB} + \phi_S - \frac{Q_S(\phi_S)}{C_{ox}}$$

$C_{ox} = K_o\epsilon_o/x_o$ $\qquad V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$

$C = C_{ox}/\left[1 + \frac{K_o W(\phi_S)}{K_S x_o}\right]$ $\qquad V_T = -Q_B(2\phi_F)/C_{ox} + 2\phi_F$ $\qquad Q_n = -C_{ox}(V_G - V_T)$

MOSFETs: $I_D = -WQ_n(y=0)\langle v_y(y=0)\rangle$

$I_D = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}$ $\qquad I_D = WC_{ox}v_{sat}(V_{GS} - V_T)$

Square Law (for $V_{GS} \geq V_T$): $I_D = \begin{cases} \frac{W}{L}\mu_n C_{ox}[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2], & 0 \leq V_{DS} \leq V_{GS} - V_T \\ \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2, & V_{DS} \geq V_{GS} - V_T \end{cases}$

Bipolar transistors: (assuming NPN, short emitter, base, and collector)

Ebers-Moll Equations:

$$I_C\left(V_{BE},V_{BC}\right)=\alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E\left(V_{BE},V_{BC}\right)=I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-\alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_{F0}=qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}}+\frac{D_{pE}}{W_E}\frac{n_i^2}{N_{DE}}\right)$$

$$I_{R0}=qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}}+\frac{D_{pC}}{W_C}\frac{n_i^2}{N_{DC}}\right)$$

$$\alpha_F=\gamma_F\alpha_T$$

$$\alpha_R=\gamma_R\alpha_T$$

$$\alpha_F I_{F0}=\alpha_R I_{R0}$$

$$\gamma_F=\frac{I_{En}}{I_{En}+I_{Ep}}=\frac{1}{1+\frac{D_{pE}}{D_{nB}}\frac{W_B}{W_E}\frac{N_{AB}}{N_{DE}}}$$

$$\alpha_T=\frac{I_{Cn}}{I_{En}}=\frac{1}{1+\frac{1}{2}\left(\frac{W_B}{L_{nB}}\right)^2}$$

$$\beta_F=\frac{\alpha_F}{1-\alpha_F}$$

$$\alpha_F=\frac{\beta_F}{1+\beta_F}$$