NAME:_______________________________ **PUID:**_____________________

ECE 305 – Fall 2016

Final Exam (Exam 5) – Monday, December 12, 2016

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).
The exam is designed to be taken in 50 minutes (or less). However, the full two hours will be allowed, if you need them. Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)
II) 30 points
III) 30 points

Course policy

If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: ____

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Signature: ____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

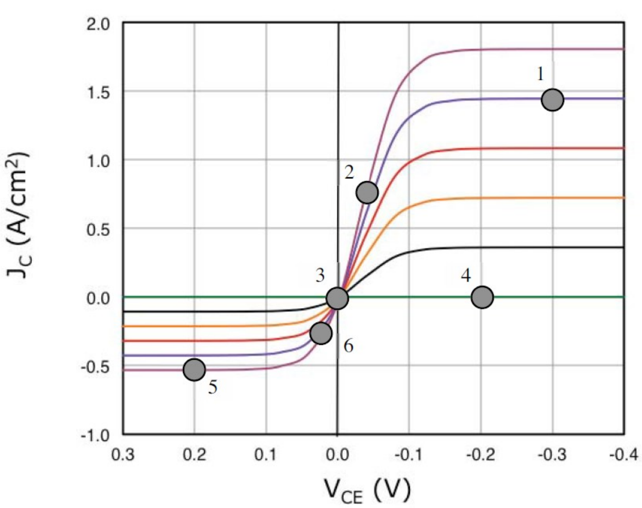
1 (8 points). Which of the following would reduce the emitter current crowding in a BJT?

a. Decreasing the base doping
b. Decreasing the emitter doping
c. Increasing the emitter doping
d. Increasing the base doping
e. Increasing the collector doping

2 (8 points). If a BJT is hooked up in the inverted active mode, which of the following is true, compared to the forward active mode?

a. The emitter injection efficiency is lower
b. The emitter injection efficiency is higher
c. The base transport factor is lower
d. The base transport factor is higher
e. The base transit time is longer

3 (8 points). Based on the diagram of a BJT below, which of the label numbers below corresponds to the cutoff regime?



a. 1
b. 2
c. 4
d. 5
e. 6

4 (8 points). What is the classical Shockley ordering of doping (from high to low)?

a. Base, collector, emitter
b. Base, emitter, collector
c. Collector, emitter, base
d. Emitter, collector, base
e. Emitter, base, collector

5 (8 points). Which of the following factors can improve β_{dc} in a BJT?

a. High electron affinity material in the base region
b. Relatively high bandgap material in the base region
c. Relatively high bandgap materials in the emitter and collector regions
d. Low breakdown voltage material in the collector region
e. Relatively high doping in the collector region

Part II (Free Response, 30 points)

Consider an *npn* BJT made of silicon, depicted below. Assume that $I_{Ep} = 10\ \mu\text{A}$, $I_{En} = 2\ \text{mA}$, $I_{Cp} = 100\ \text{nA}$, and $I_{Cn} = 1.98\ \text{mA}$. Assume that recombination within the BJT is small (i.e., diffusion lengths are much larger than the thicknesses of each layer).

a. What is the base transport factor α_T?

$$\alpha_T = \frac{I_{cn}}{I_{En}} = \frac{1.98\ \text{mA}}{2\ \text{mA}} = 0.99$$

b. What is the emitter injection efficiency γ_F?

$$\gamma_F = \frac{I_{En}}{I_{En} + I_{Ep}} = \frac{2.00\ \text{mA}}{2.00 + 0.01\ \text{mA}} = 0.995$$

c. What are the total emitter, collector, and base currents I_E, I_C, and I_B?

$$I_E = I_{En} + I_{Ep} = 2.00 + 0.01\ \text{mA} = 2.01\ \text{mA}$$

$$I_C = I_{Cn} + I_{Cp} = 1.98 + 0.0001\ \text{mA} = 1.9801\ \text{mA}$$

$$I_B = I_E - I_C = 2.01 - 1.9801\ \text{mA} = 29.9\ \mu\text{A}$$

d. What is the base current gain α_{dc} and emitter current gain β_{dc}?

$$\alpha_{DC} = \gamma \alpha_T = 0.995 \cdot 0.99 = 0.985$$

$$\beta_{DC} = \frac{\alpha_{DC}}{1 - \alpha_{DC}} = \frac{0.985}{1 - 0.985} = 65.9$$

e. What are the resulting currents I_{CB0} if one leaves the emitter lead open ~~I_{CB0}?~~, and I_{CE0}? if one leaves the base lead open ? ~~I_{CE0}?~~

$$I_{CB0} = I_C - \alpha I_E = 1.9801 - 0.985 \cdot 2.01 = 0.00025\ \text{mA}$$

$$I_{CE0} = \frac{I_{CB0}}{1 - \alpha} = \frac{0.00025\ \text{mA}}{1 - 0.985} = 0.0167\ \text{mA}$$

Part III (Free Response, 30 points)

Consider an ideal silicon *pnp* BJT that is biased to the forward active mode at room temperature. The amplification factor $\beta_{DC} = 100$. Assume that $D_E = D_B$, $I_{Cp} = I_{Ep}$, $L_E = 40W$, $L_B = 10W$, and $L_C = 200W$, where W is the width of the quasi-neutral region in the base. Please answer the following questions:

a. What is the value of the emitter to base doping ratio N_E/N_B?

$$\beta_{DC} = \frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B} = 100 = \frac{40W}{10W}\cdot\frac{D_B}{D_E}\cdot 1\cdot\frac{N_E}{N_B} = 4\frac{N_E}{N_B}$$

Thus, $\frac{N_E}{N_B} = 25.$

b. Assume that $I_{F0} = 10$ nA and $I_{R0} = 9$ nA. Write down the Ebers-Moll equations for the collector and emitter current of this system as a function of V_{BE} and V_{BC}. All other values in these equations should be provided numerically.

$$I_C = \alpha_F I_{F0}\left(e^{qV_{BE}/kT} - 1\right) - I_{R0}\left(e^{qV_{BC}/kT} - 1\right)$$

Since $\alpha_F = \frac{\beta_{dc}}{\beta_{dc}+1} = \frac{100}{100+1} = 0.99$,

$$I_C = 9.9\left(e^{qV_{BE}/kT} - 1\right) - 9\left(e^{qV_{BC}/kT} - 1\right)\ \text{nA}$$

$$I_E = I_{F0}\left(e^{qV_{BE}/kT} - 1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/kT} - 1\right)$$

Since $\alpha_R I_{R0} = \alpha_F I_{F0} = 9.9$ nA, we obtain:

$$I_E = 10\left(e^{qV_{BE}/kT} - 1\right) - 9.9\left(e^{qV_{BC}/kT} - 1\right)\ \text{nA}$$

c. If the base region silicon were substituted with a hypothetical material with a smaller bandgap of 0.8 eV but otherwise identical properties, calculate the new value of the amplification factor β_{DC}.

Now $\beta'_{DC} = \frac{D_n}{W_B}\frac{W_E}{D_p}\frac{n_{i,B}^2}{n_{i,E}^2}\frac{N_E}{N_B} = \beta_{DC}\frac{n_{i,B}^2}{n_{i,E}^2} = 100\cdot\frac{N_C N_V e^{-\frac{0.8}{.026}}}{N_C N_V e^{-\frac{1.12}{.026}}} = 100\cdot e^{\left(\frac{0.32}{.026}\right)} = 2.21\cdot 10^7$

ECE 305 Exam 5 Formula Sheet (Fall 2016)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	**Silicon parameters ($T = 300$ K)**
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm^{-3}
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm^{-3}
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm^{-3}
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\quad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $\quad p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_F-E_i)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $\quad p = N_V e^{(E_V-F_P)/kT}$ $\quad np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations:

$$J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx} \qquad \frac{D_n}{\mu_n} = \frac{kT}{q}$$

$$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx} \qquad \frac{D_p}{\mu_p} = \frac{kT}{q}$$

Carrier conservation equations:

$$\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q}\right) + G_n - R_n$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q}\right) + G_p - R_p$$

Poisson's equation: $\nabla \cdot (\epsilon \mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n/\tau_n$ or $R = \Delta p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $\quad L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\quad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s \epsilon_o}$

$$W = \sqrt{\frac{2K_s\epsilon_o V_{bi}}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)} \qquad x_n = \left(\frac{N_A}{N_A+N_D}\right)W \qquad x_p = \left(\frac{N_D}{N_A+N_D}\right)W \qquad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\epsilon_o}\left(\frac{N_A N_D}{N_A+N_D}\right)}$$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$ $\Delta p(0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = q\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)$ *(long)* $J_o = q\left(\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right)$ *(short)*

Non-ideal diodes: $I = I_o\left(e^{q(V_A - IR_s)/kT} - 1\right)$ $J_{gen} = -q\frac{n_i}{2\tau_o}W$

Photovoltaics: $V_{oc} = \frac{nkT}{q}\ln\left(\frac{J_{sc}}{J_o}\right)$ $J_{PV} = J_o\left(e^{qV_A/kT} - 1\right) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $C_J(V_R) = \frac{K_s\epsilon_o A}{\sqrt{\frac{2K_s\epsilon_o V_{bi}}{qN_A}}} = A\sqrt{\frac{qK_s\epsilon_o N_A}{2V_{bi}}}$ $C_D = G_d\tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\Phi_{BP} = \chi + E_G - \Phi_M$ $\Phi_{BN} = \Phi_M - \chi$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $J_o = A^* T^2 e^{-\Phi_B/kT}$ $A^* = \frac{4\pi q m^* k_B^2}{h^3} = 120\frac{m^*}{m_o}\ \frac{\mathrm{A}}{\mathrm{cm}^2\cdot\mathrm{K}^2}$

MOS capacitors: $W = \sqrt{\frac{2K_s\epsilon_o\phi_s}{qN_A}}$ cm $\mathcal{E}_s = \sqrt{\frac{2qN_A\phi_s}{K_s\epsilon_o}}\ \frac{\mathrm{V}}{\mathrm{cm}}$

$$Q_B = -qN_A W(\phi_s) = -\sqrt{2qK_s\epsilon_o N_A\phi_s}\ \frac{\mathrm{C}}{\mathrm{cm}^2}$$

$$V_G = V_{FB} + \phi_s + \Delta\phi_{ox} = V_{FB} + \phi_s - \frac{Q_s(\phi_s)}{C_{ox}}$$

$C_{ox} = K_o\epsilon_o/x_o$ $V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$

$C = C_{ox}/\left[1 + \frac{K_o W(\phi_s)}{K_s x_o}\right]$ $V_T = -Q_B(2\phi_F)/C_{ox} + 2\phi_F$ $Q_n = -C_{ox}(V_G - V_T)$

MOSFETs: $I_D = -WQ_n(y=0)\langle v_y(y=0)\rangle$

$I_D = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}$ $I_D = WC_{ox}v_{sat}(V_{GS} - V_T)$

Square Law (for $V_{GS} \geq V_T$): $I_D = \begin{cases} \frac{W}{L}\mu_n C_{ox}[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2], & 0 \leq V_{DS} \leq V_{GS} - V_T \\ \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2, & V_{DS} \geq V_{GS} - V_T \end{cases}$

Bipolar transistors: (assuming NPN, short emitter, base, and collector)

Ebers-Moll Equations:

$$I_C(V_{BE},V_{BC}) = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right) - I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E(V_{BE},V_{BC}) = I_{F0}\left(e^{qV_{BE}/k_BT}-1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_{F0} = qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}} + \frac{D_{pE}}{W_E}\frac{n_i^2}{N_{DE}}\right)$$

$$I_{R0} = qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}} + \frac{D_{pC}}{W_C}\frac{n_i^2}{N_{DC}}\right)$$

$$\alpha_F = \gamma_F \alpha_T$$

$$\alpha_R = \gamma_R \alpha_T$$

$$\alpha_F I_{F0} = \alpha_R I_{R0}$$

$$\gamma_F = \frac{I_{En}}{I_{En}+I_{Ep}} = \frac{1}{1+\frac{D_{pE}}{D_{nB}}\frac{W_B}{W_E}\frac{N_{AB}}{N_{DE}}}$$

$$\alpha_T = \frac{I_{Cn}}{I_{En}} = \frac{1}{1+\frac{1}{2}\left(\frac{W_B}{L_{nB}}\right)^2}$$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F}$$

$$\alpha_F = \frac{\beta_F}{1+\beta_F}$$