NAME:_____FULL SOLUTION_______________ **PUID:**_______________________

ECE 305 – Fall 2017

Final Exam (Exam 5) – Wednesday, December 13, 2017

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).
The exam is designed to be taken in 60 minutes (or less). However, the full two hours will be allowed, if you need them. Be sure to fill in your name and Purdue student ID at the top of the page. DO NOT open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)
II) 30 points
III) 30 points

Course policy

If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: ____

____

Signature: ____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

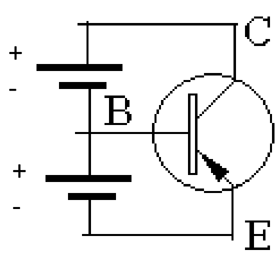
1 (8 points). Which of the following factors directly increases the common emitter gain β_{DC} in a bipolar junction transistor (BJT)?

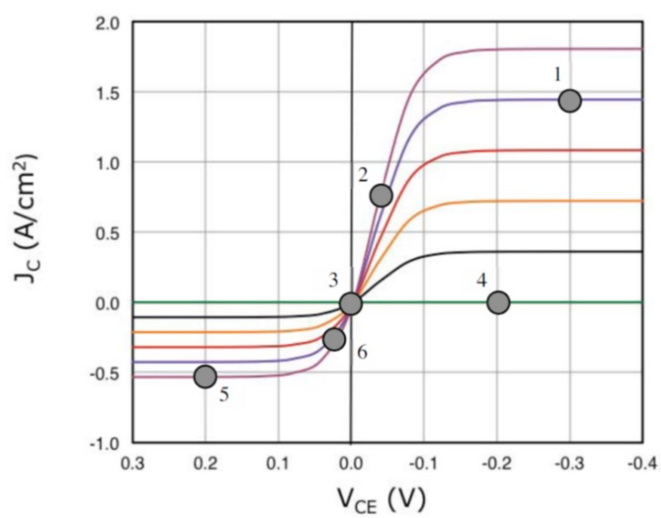
a. Decreasing the collector doping
b. Decreasing the emitter doping
c. Increasing the emitter doping
d. Increasing the base doping
e. Increasing the collector doping

2 (8 points). What is the greatest advantage of an npn heterojunction bipolar transistor (HBT), compared to a BJT ?

a. Electron current through the emitter is larger
b. Hole current through the emitter is larger
c. Electron current through the emitter is smaller
d. Hole current through the emitter is smaller
e. Electron current through the collector is larger

3 (8 points). Which of the label numbers in the diagram below on the right below corresponds to the operating regime depicted in the diagram below on the left?



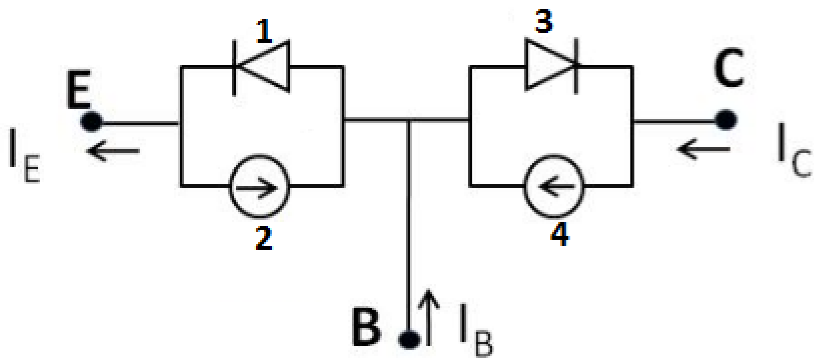


a. 1
b. 2
c. 4
d. 5
e. 6

4 (8 points). Which of the following would decrease the magnitude of the Early effect in a BJT?

a. Decreasing the collector doping
b. Decreasing the emitter doping
c. Increasing the emitter doping
d. Decreasing the base doping
e. Increasing the collector doping

5 (8 points). Which of the following components of the Ebers-Moll model can be neglected in the forward active mode, depicted below?



a. 1, 4
b. 1, 3
c. 1
d. 2, 4
e. 2, 3

Part II (Free Response, 30 points)

Consider an *npn* BJT made of silicon, depicted below. Assume that $I_{Ep} = 15$ µA, $I_{En} = 2.5$ mA, $I_{Cp} =$ 130 nA, and $I_{Cn} = 2.38$ mA. Assume that recombination within the BJT is small (i.e., diffusion lengths are much larger than the thicknesses of each layer).

a. What is the base transport factor α_T?

$$\alpha_T = \frac{I_{cn}}{I_{En}} = \frac{2.38\text{ mA}}{2.5\text{ mA}} = 0.952$$

b. What is the emitter injection efficiency γ_F?

$$\gamma_F = \frac{I_{En}}{I_{En} + I_{Ep}} = \frac{2.5\text{ mA}}{2.5 + 0.015\text{ mA}} = 0.994$$

c. What are the total emitter, collector, and base currents I_E, I_C, and I_B?

$$I_E = I_{En} + I_{Ep} = 2.5 + 0.015\text{ mA} = 2.515\text{ mA}$$

$$I_C = I_{Cn} + I_{Cp} = 2.38 + 0.00013\text{ mA} = 2.38013\text{ mA}$$
$$I_B = I_E - I_C = 2.515 - 2.38013\text{ mA} = 134.87\text{ µA}$$

d. What is the base current gain α_{dc} and emitter current gain β_{dc}?

$$\alpha_{DC} = \gamma_F \alpha_T = 0.994 \cdot 0.952 = 0.946$$
$$\beta_{DC} = \frac{\alpha_{DC}}{1 - \alpha_{DC}} = \frac{0.946}{1 - 0.946} = 17.5$$

e. What would be the new value of I_{En} if the base region silicon were replaced by a hypothetical material with a bandgap of only 0.9 eV, but otherwise identical properties?

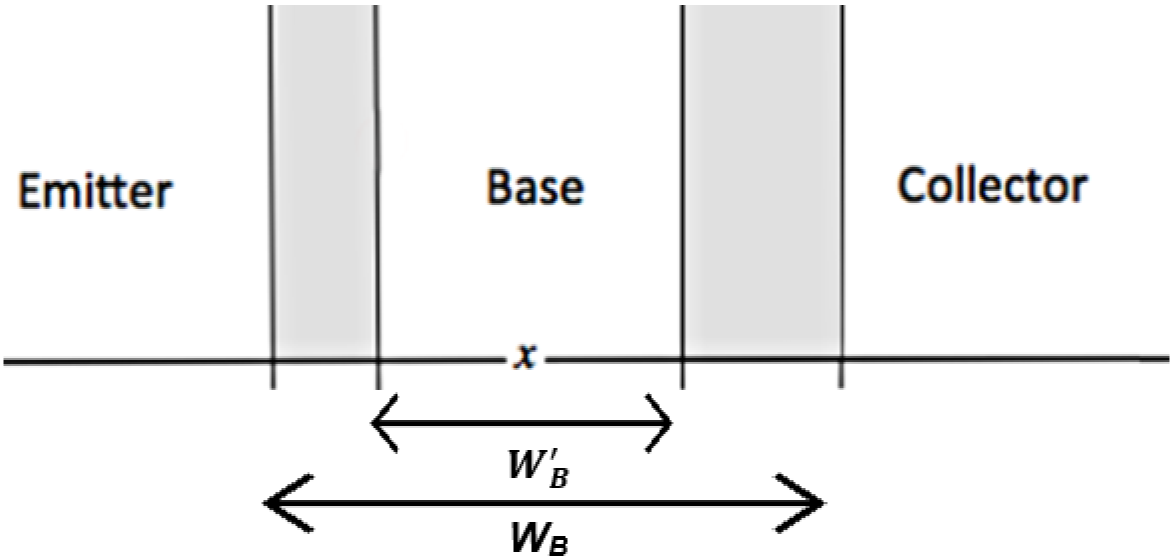
$$I_{En \propto n_{i,B}^2}$$

$$I_{En,new} = I_{En,old} \times e^{-(E_{g,new} - E_{g,old})/kT}$$

$$= (2.5\text{ mA}) \times e^{-(0.9-1.12)/(0.0259)} = 12.2\text{ A}$$

Part III (Free Response, 30 points)

Consider an ideal silicon *pnp* BJT at room temperature. The doping concentrations at emitter, base and collector are respectively $N_{A,E} = 10^{17}$ cm^{-3}, $N_{D,B} = 10^{15}$ cm^{-3}, and $N_{A,C} = 10^{14}$ cm^{-3}. The base width is W_B. Assume that the minority carrier diffusion lengths $L_{n,E} = 40\ W_B$, $L_{p,B} = 5\ W_B$, and $L_{n,C} = 200\ W_B$. The total and quasi-neutral base widths depicted below are schematic, and may not be to scale.



a. Write an expression for the quasi-neutral base width W_B' as a function of W_B, V_{EB}, & V_{CB}. You may use the one-sided junction approximation.

$$W_B' = W_B - x_{n,BE} - x_{n,BC}$$

$$= W_B - \sqrt{\frac{2K_s\epsilon_o(V_{bi,1} - V_{EB})}{q}\left[\frac{N_{A,E}}{(N_{A,E} + N_{D,B})N_{D,B}}\right]} - \sqrt{\frac{2K_s\epsilon_o(V_{bi,2} - V_{CB})}{q}\left[\frac{N_{A,C}}{(N_{A,C} + N_{D,B})N_{D,B}}\right]}$$

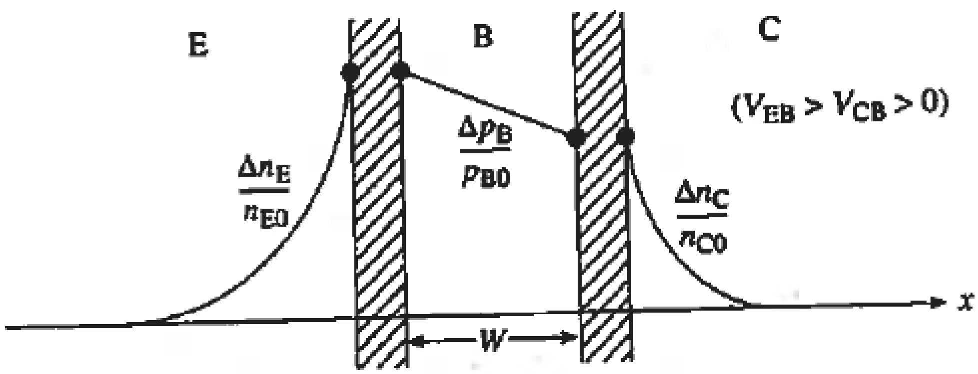
$$= W_B - \sqrt{\frac{2K_s\epsilon_o(V_{bi,1} - V_{EB})}{qN_{D,B}}} - \sqrt{\frac{2K_s\epsilon_o(V_{bi,2} - V_{CB})}{q}\left[\frac{N_{A,C}}{N_{D,B}^2}\right]}$$

$$\approx W_B - \sqrt{\frac{2K_s\epsilon_o(V_{bi,1} - V_{EB})}{qN_{D,B}}} = W_B - (807\text{ nm})\sqrt{V_{bi,1} - V_{EB}}$$

where $V_{bi,1} = \frac{kT}{q}\ln\left(\frac{N_E N_B}{n_i^2}\right) = 0.716$ V and $V_{bi,2} = \frac{kT}{q}\ln\left(\frac{N_C N_E}{n_i^2}\right) = 0.534$ V

b. Sketch the excess minority carrier concentration across the emitter, base, and collector quasi-neutral regions, if V_{EB}=0.5 V and V_{CB}=0.3 V. Please indicate any assumptions about the emitter and collector thicknesses. What is the operating regime of the BJT under this condition?

For a pnp BJT, if $V_{EB} > 0$ and $V_{CB}>0$, we are in the saturation regime. The sketch will generally have the following form:



c. Given that the emitter injection efficiency γ_F is 0.99, find the emitter current gain β_{dc}. Assume that $W_B' = 0.8\,W_B$ in this operating condition.

$$\alpha_T = \frac{1}{1+\frac{1}{2}\left(\frac{W_B'}{L_{p,B}}\right)^2} = \frac{1}{1+\frac{1}{2}\left(\frac{0.8W_B}{5W_B}\right)^2} = 0.9874$$

Now $\alpha_{DC} = \gamma_F \alpha_T = 0.99 \cdot 0.9874 = 0.9775$

$$\beta_{DC} = \frac{\alpha_{DC}}{1-\alpha_{DC}} = \frac{0.9775}{1-0.9775} = 43.4$$

ECE 305 Exam 5 Formula Sheet (Fall 2017)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	Silicon parameters ($T = 300$ K)
$h/2\pi = \hbar = 1.055 \times 10^{-34}$ J·s	$N_C = 3.23 \times 10^{19}$ cm^{-3}
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm^{-3}
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm^{-3}
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n^*)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4}\left(\frac{2m_n^* kT}{\pi\hbar^2}\right)^{3/2}$ $\quad N_V = \frac{1}{4}\left(\frac{2m_p^* kT}{\pi\hbar^2}\right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $\quad p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_i-E_F)/kT}$

Space charge neutrality: $p - n + N_D^+ - N_A^- = 0$

Law of Mass Action: $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $\quad p = N_V e^{(E_V-F_P)/kT}$ $\quad np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_n = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations: $J_n = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx}$ $\quad \frac{D_n}{\mu_n} = \frac{kT}{q}$

$J_p = pq\mu_p \mathcal{E}_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx}$ $\quad \frac{D_p}{\mu_p} = \frac{kT}{q}$

Carrier conservation equations: $\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q}\right) + G_n - R_n$

$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q}\right) + G_p - R_p$

Poisson's equation: $\nabla \cdot (\epsilon \mathcal{E}) = \rho$

SRH carrier recombination: $R = \Delta n/\tau_n$ or $R = \Delta p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$ $\quad L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right)$ $\quad \frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_s \epsilon_o}$

$W = \sqrt{\frac{2K_s\epsilon_o V_{bi}}{q}\left(\frac{N_A+N_D}{N_A N_D}\right)}$ $\quad x_n = \left(\frac{N_A}{N_A+N_D}\right)W$ $\quad x_p = \left(\frac{N_D}{N_A+N_D}\right)W$ $\quad \mathcal{E}(0) = \sqrt{\frac{2qV_{bi}}{K_s\epsilon_o}\left(\frac{N_A N_D}{N_A+N_D}\right)}$

PN diode current: $\Delta n(0) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)$ $\quad \Delta p(0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $\quad J_o = q\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)$ *(long)* $\quad J_o = q\left(\frac{D_n}{W_p}\frac{n_i^2}{N_A} + \frac{D_p}{W_n}\frac{n_i^2}{N_D}\right)$ *(short)*

Non-ideal diodes: $I = I_o\left(e^{q(V_A - IR_s)/kT} - 1\right)$ $\quad J_{gen} = -q\frac{n_i}{2\tau_o}W$

Photovoltaics: $V_{oc} = \frac{nkT}{q}\ln\left(\frac{J_{sc}}{J_o}\right)$ $\quad J_{PV} = J_o\left(e^{qV_A/kT} - 1\right) - J_{sc}$

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $\quad C_J(V_R) = \frac{K_s\epsilon_o A}{\sqrt{\frac{2K_s\epsilon_o V_{bi}}{qN_A}}} = A\sqrt{\frac{qK_s\epsilon_o N_A}{2V_{bi}}}$ $\quad C_D = G_d\tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\quad \Phi_{BP} = \chi + E_G - \Phi_M$ $\quad \Phi_{BN} = \Phi_M - \chi$

$J_D = J_o\left(e^{qV_A/kT} - 1\right)$ $\quad J_o = A^* T^2 e^{-\Phi_B/kT}$ $\quad A^* = \frac{4\pi q m^* k_B^2}{h^3} = 120\frac{m^*}{m_o}\ \frac{\mathrm{A}}{\mathrm{cm^2 \cdot K^2}}$

MOS capacitors: $W = \sqrt{\frac{2K_s\epsilon_o\phi_s}{qN_A}}$ cm $\quad \mathcal{E}_s = \sqrt{\frac{2qN_A\phi_s}{K_s\epsilon_o}}\ \frac{\mathrm{V}}{\mathrm{cm}}$

$$Q_B = -qN_A W(\phi_s) = -\sqrt{2qK_s\epsilon_o N_A\phi_s}\ \frac{\mathrm{C}}{\mathrm{cm^2}}$$

$$V_G = V_{FB} + \phi_s + \Delta\phi_{ox} = V_{FB} + \phi_s - \frac{Q_s(\phi_s)}{C_{ox}}$$

$C_{ox} = K_o\epsilon_o/x_o$ $\quad V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$

$C = C_{ox}/\left[1 + \frac{K_o W(\phi_s)}{K_s x_o}\right]$ $\quad V_T = -Q_B(2\phi_F)/C_{ox} + 2\phi_F$ $\quad Q_n = -C_{ox}(V_G - V_T)$

MOSFETs: $I_D = -WQ_n(y=0)\langle v_y(y=0)\rangle$

$I_D = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_T)V_{DS}$ $\quad I_D = WC_{ox}v_{sat}(V_{GS} - V_T)$

Square Law (for $V_{GS} \geq V_T$): $I_D = \begin{cases} \frac{W}{L}\mu_n C_{ox}[(V_{GS} - V_T)V_{DS} - V_{DS}^2/2], & 0 \leq V_{DS} \leq V_{GS} - V_T \\ \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2, & V_{DS} \geq V_{GS} - V_T \end{cases}$

Bipolar transistors: (assuming NPN, short emitter, base, and collector)

Ebers-Moll Equations:

$$I_C\left(V_{BE},V_{BC}\right)=\alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E\left(V_{BE},V_{BC}\right)=I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-\alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_{F0}=qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}}+\frac{D_{pE}}{W_E}\frac{n_i^2}{N_{DE}}\right)$$

$$I_{R0}=qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}}+\frac{D_{pC}}{W_C}\frac{n_i^2}{N_{DC}}\right)$$

$$\alpha_F=\gamma_F\alpha_T$$

$$\alpha_R=\gamma_R\alpha_T$$

$$\alpha_F I_{F0}=\alpha_R I_{R0}$$

$$\gamma_F=\frac{I_{En}}{I_{En}+I_{Ep}}=\frac{1}{1+\frac{D_{pE}}{D_{nB}}\frac{W_B}{W_E}\frac{N_{AB}}{N_{DE}}}$$

$$\alpha_T=\frac{I_{Cn}}{I_{En}}=\frac{1}{1+\frac{1}{2}\left(\frac{W_B}{L_{nB}}\right)^2}$$

$$\beta_F=\frac{\alpha_F}{1-\alpha_F}$$

$$\alpha_F=\frac{\beta_F}{1+\beta_F}$$