

NAME: **FULL SOLUTION**

PUID: _____

ECE 305 – Spring 2018

Final Exam (Exam 5) – Thursday, May 3, 2018

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 60 minutes (or less). However, the full two hours will be allowed, if you need them. Be sure to fill in your name and Purdue student ID at the top of the page. **DO NOT** open the exam until told to do so, and stop working immediately when time is called. The last 2 pages are equation sheets, which you may remove, if you want.

100 points possible,

I) 40 points (8 points per question)

II) 30 points

III) 30 points

Course policy

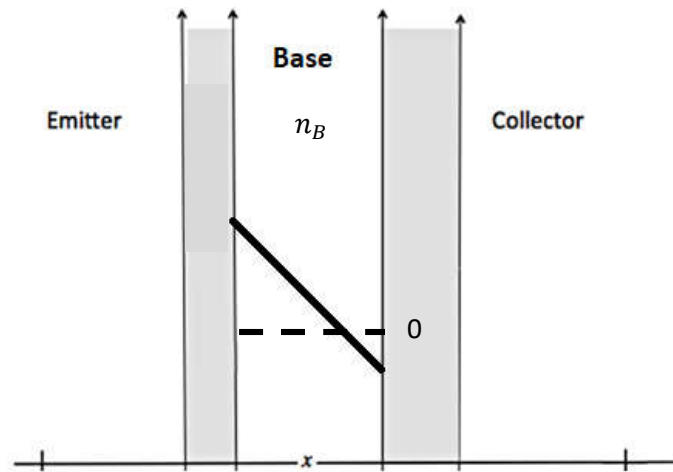
If I am caught cheating, I will earn an F in the course & be reported to the Dean of Students.

I repeat: _____

Signature: _____

Part I: Answer the 5 multiple choice questions below by entering them on your IDP-15 Scantron.

1 (8 points). Which is true for the npn BJT given the information below?



- a. The BJT is operating in the saturation mode
- b. The BJT has negligible recombination at base
- c. The BJT is operating in the forward active mode
- d. Both a. and b.
- e. Both b. and c.

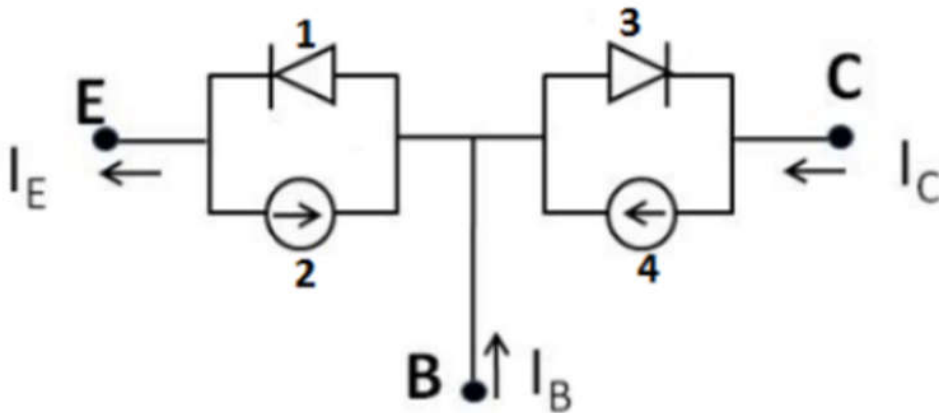
2 (8 points). Maximizing which of the following ratios increases the gain in a bipolar junction transistor?

- a. Ratio of emitter to base doping
- b. Ratio of emitter to collector doping
- c. Ratio of base to emitter band gap
- d. Ratio of base to emitter width
- e. None of the above

3 (8 points). Compared to forward active mode, ____ in the inverted active mode.

- a. the emitter injection efficiency is typically higher
- b. the emitter injection efficiency is typically lower
- c. the base transport factor is typically lower
- d. the base transport factor is typically higher
- e. the base transit time is typically longer

4 (8 points). Which of the following components of the Ebers-Moll model depicted below cannot be neglected in describing the reverse active mode?

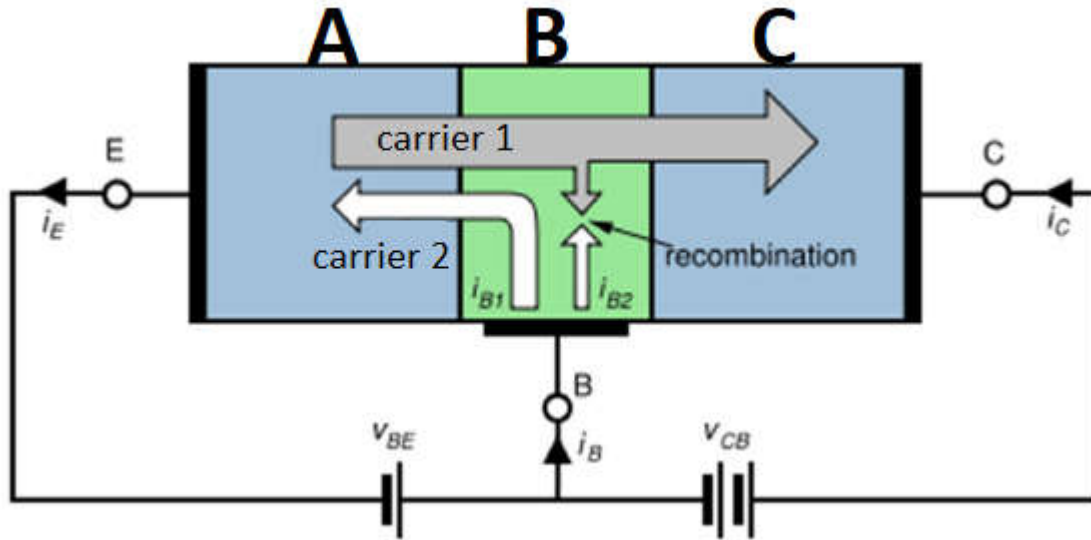


- a. 1,4
- b. 1,3
- c. 1
- d. 2,4
- e. 2,3

5 (8 points). The Early effect in a BJT is caused by which phenomenon?

- a. Quantum mechanical tunneling
- b. Impact ionization
- c. SRH recombination
- d. The decreasing width of the quasi neutral (flat-band) region in the base
- e. Current crowding

Part II (Free Response, 30 points)



The schematics of a silicon BJT is shown above, where i_E , i_C , and i_B are emitter current, collector current, and base current respectively, with the arrows showing the current directions. The three regions A, B, and C in the schematics have dopant concentrations of 10^{17} cm^{-3} , 10^{16} cm^{-3} , and 10^{15} cm^{-3} respectively.

- a. Name the type of carrier represented by carrier 1 in the diagram. From that, state the dopant type (donor or acceptor) for region B.

Electrons; p-type

- b. If the silicon in region B is replaced with another semiconductor with a larger bandgap (e.g., gallium arsenide), and the emitter current is held constant, what do you expect to happen to i_B ?

Base current must increase to compensate for the reduction in the amplification factor β_{DC} .

Assume that the BJT is operating in the forward active region, with $I_C=8\mu\text{A}$; thicknesses $W_E=0.6 \mu\text{m}$, $W_B=0.3 \mu\text{m}$, and $W_C=2.2 \mu\text{m}$; minority carrier diffusion constants $D_A=2 \text{ cm}^2/\text{s}$, $D_B=20 \text{ cm}^2/\text{s}$, $D_C=12 \text{ cm}^2/\text{s}$ for regions A, B, and C; and minority carrier diffusion length $L_{nB}=1.2 \mu\text{m}$ for region B.

- c. What is the emitter injection efficiency γ_F ?

$$\gamma_F = \frac{1}{1 + \frac{D_{pE} W_B N_{AB}}{D_{nB} W_E N_{DE}}} = \frac{1}{1 + \frac{2 \cdot 0.3 \cdot 10^{16}}{20 \cdot 0.6 \cdot 10^{17}}} = \frac{1}{1.005} \approx 0.995$$

Continued on next page....

Part II (continued)

d. What is the base transport factor α_T ?

$$\alpha_T = \frac{1}{1 + \frac{1}{2} \left(\frac{W_B}{L_{nB}} \right)^2} = \frac{1}{1.03125} \approx 0.9697$$

e. What is the common base current gain α_{dc} ?

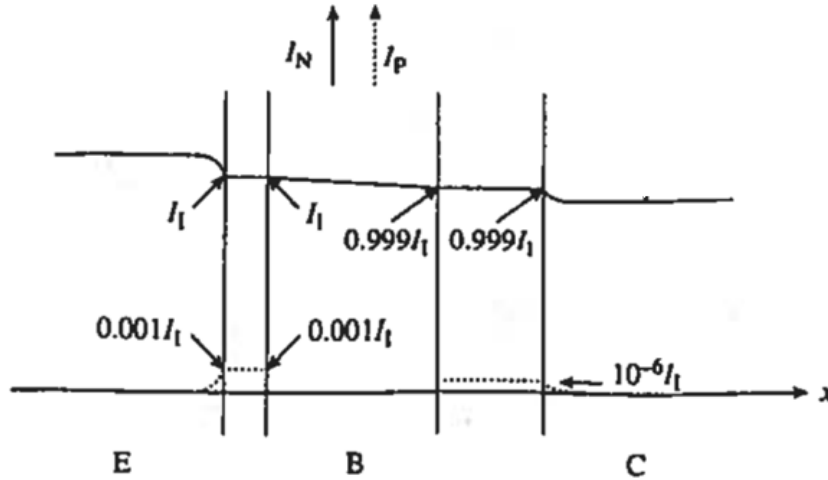
$$\alpha_{dc} = \alpha_T \gamma_F \approx 0.9648$$

f. What is the emitter gain β_{dc} ?

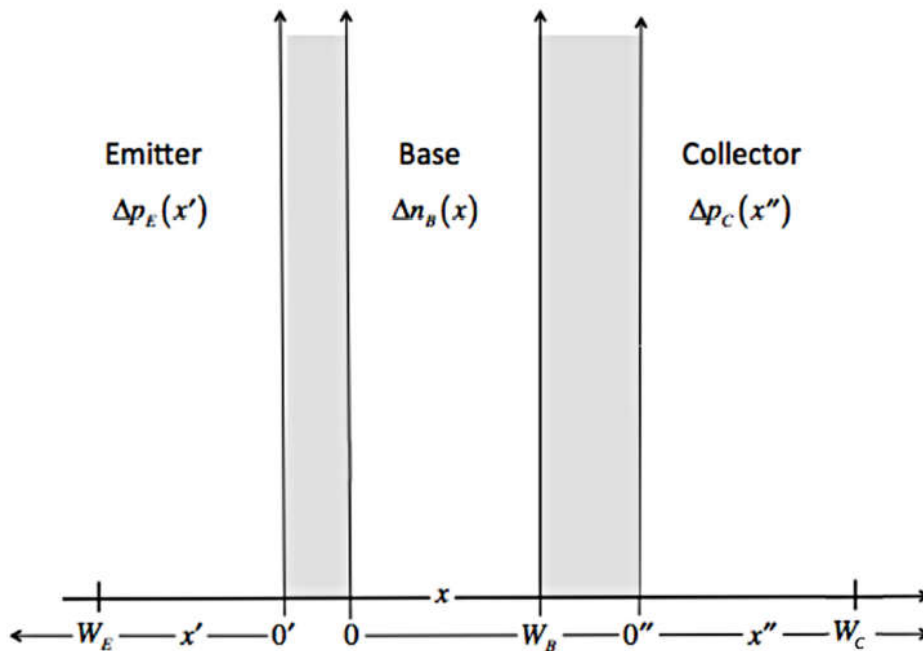
$$\beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} \approx 27.45$$

Part III (Free Response, 30 points)

Consider the electron and hole currents I_N and I_P for a npn BJT plotted below. The recombination currents can be neglected everywhere.

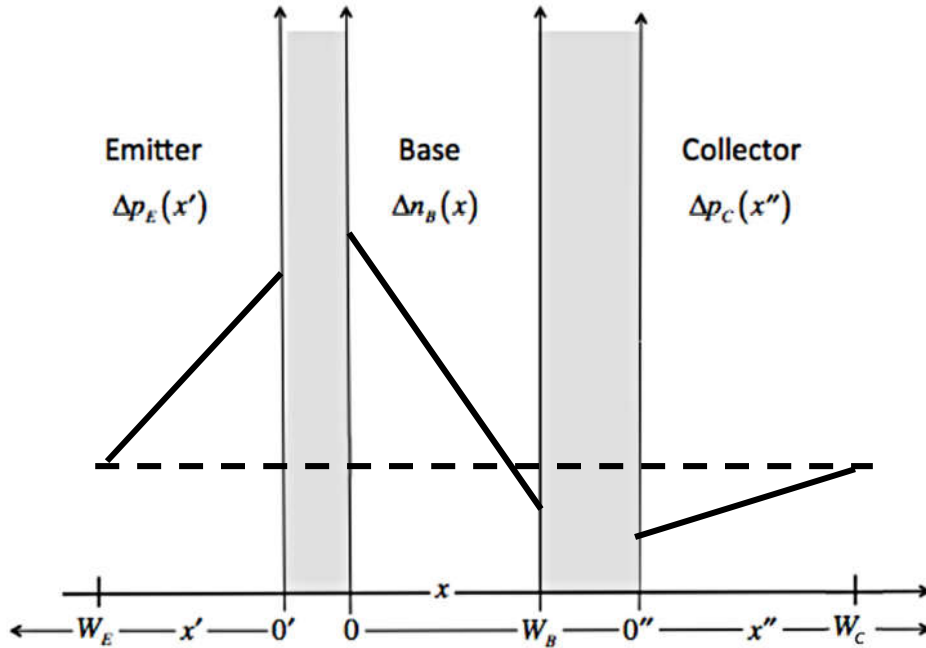


- a. Sketch the corresponding excess carrier concentration in all the white regions below. Justify your sketch.



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Part III (continued)



This BJT is operating in forward active mode; this creates forward bias at the emitter-base junction; and reverse bias at the base-collector junction.

- b. Write down the base current due to excess minority carriers in terms of $\Delta n_B(0)$, $\Delta n_B(W_B)$, and W_B (and other relevant constants). How is this related to I_1 from the diagram above?

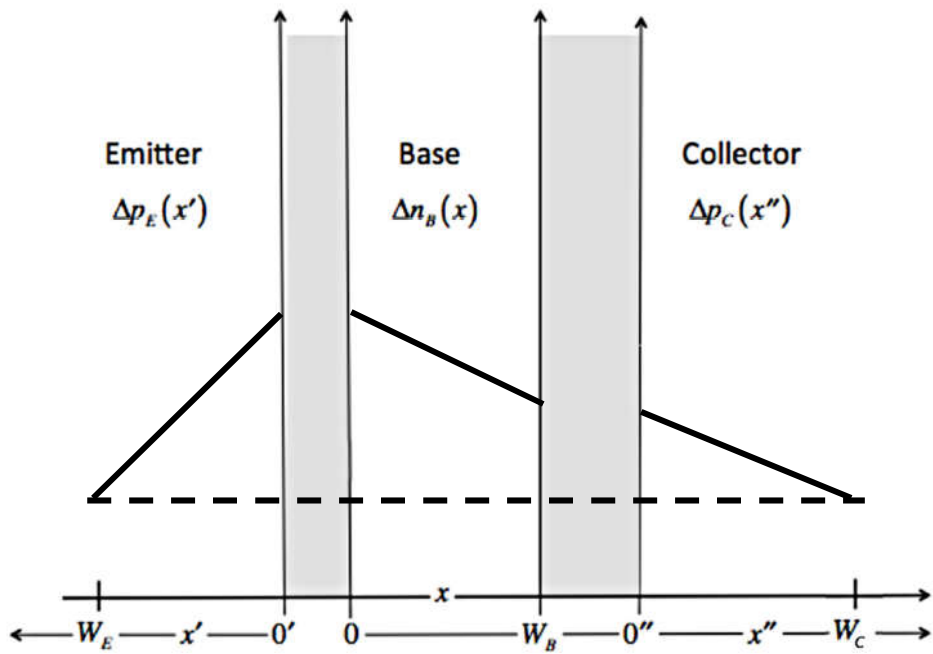
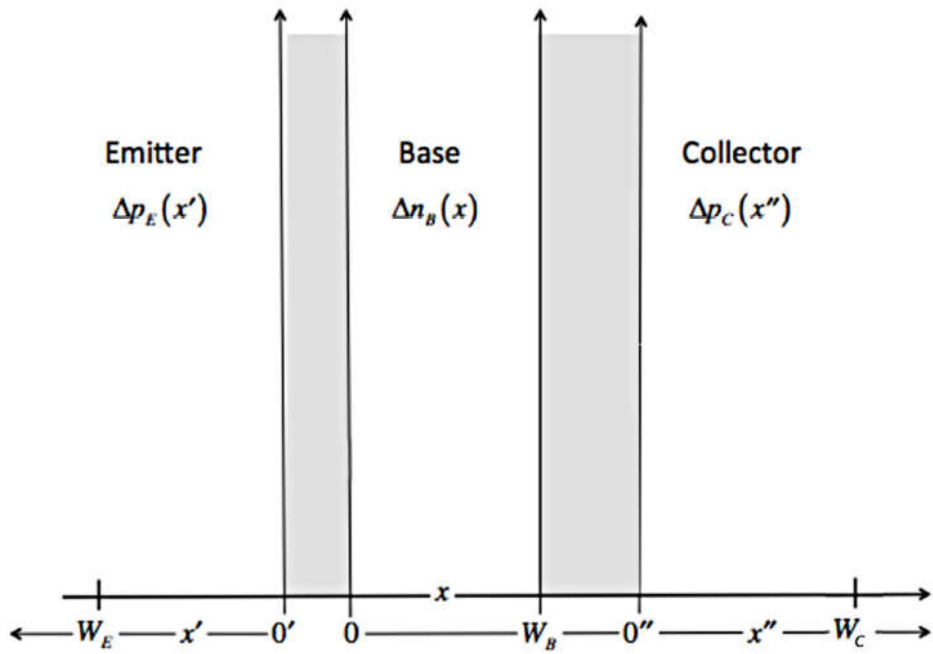
$$I_{Bn} = qD_n \frac{d n_B}{dx} = \frac{qD_n}{W_B} [n_B(W_B) - n_B(0)]$$

This expression is functionally equivalent to the average of the base current in the quasi-neutral region of the base. Thus, it is equal to $0.9995I_1$.

- c. Sketch the excess carrier concentration in all the white regions if V_{CB} flips its sign (i.e., is inverted), but everything else remains the same. What is the name of this mode of operation of BJT?

Saturation region

Part III (continued)



ECE 305 Exam 5 Formula Sheet (Spring 2018)

You may remove these pages from the exam packet, and take them with you.

Physical Constants	Silicon parameters ($T = 300$ K)
$/2\pi = 1.055 \times 10^{-34}$ J s	$N_C = 3.23 \times 10^{19}$ cm ⁻³
$m_0 = 9.109 \times 10^{-31}$ kg	$N_V = 1.83 \times 10^{19}$ cm ⁻³
$k_B = 1.38 \times 10^{-23}$ J/K	$n_i = 1.1 \times 10^{10}$ cm ⁻³
$q = 1.602 \times 10^{-19}$ C	$K_s = 11.8$
$\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$E_g = 1.12$ eV; $\chi = 4.03$ eV

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Density of states $g_C(E) = \frac{(m_n)^{3/2} \sqrt{2(E-E_C)}}{\pi^2 \hbar^3}$

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Intrinsic carrier concentration $n_i = \sqrt{N_C N_V} e^{-E_g/2kT}$

Equilibrium carrier densities: $N_C = \frac{1}{4} \left(\frac{2m_n kT}{\pi \hbar^2} \right)^{3/2}$ $N_V = \frac{1}{4} \left(\frac{2m_p kT}{\pi \hbar^2} \right)^{3/2}$

$n_0 = N_C e^{(E_F-E_C)/kT} = n_i e^{(E_F-E_i)/kT}$ $p_0 = N_V e^{(E_V-E_F)/kT} = n_i e^{(E_F-E_i)/kT}$

Space charge neutrality: $p + n + N_D^+ - N_A^- = 0$ **Law of Mass Action:** $n_0 p_0 = n_i^2$

Non-equilibrium carriers: $n = N_C e^{(F_N-E_C)/kT}$ $p = N_V e^{(E_V-F_P)/kT}$ $np = n_i^2 e^{(F_N-F_P)/kT}$

Conductivity/resistivity: $\sigma = \sigma_n + \sigma_p = q(n\mu_n + p\mu_p) = 1/\rho$

Drift-diffusion current equations: $J_n = nq\mu_n E_x + qD_n \frac{dn}{dx} = n\mu_n \frac{dF_n}{dx}$ $\frac{D_n}{\mu_n} = \frac{kT}{q}$

$J_p = pq\mu_p E_x - qD_p \frac{dp}{dx} = p\mu_p \frac{dF_p}{dx}$ $\frac{D_p}{\mu_p} = \frac{kT}{q}$

Carrier conservation equations: $\frac{\partial n}{\partial t} = +\nabla \cdot \left(\frac{J_n}{q} \right) + G_n - R_n$

$\frac{\partial p}{\partial t} = -\nabla \cdot \left(\frac{J_p}{q} \right) + G_p - R_p$

Poisson's equation: $\nabla \cdot (\epsilon) = \rho$

SRH carrier recombination: $R = n/\tau_n$ or $R = p/\tau_p$

Minority carrier diffusion equation: $\frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} - \frac{n}{\tau_n} + G_L$ $L_{D,n} = \sqrt{D_n \tau_n}$

PN homojunction electrostatics: $V_{bi} = \frac{kT}{q} \ln \left(\frac{N_D N_A}{n_i^2} \right)$ $\frac{d}{dx} = \frac{\rho(x)}{K_s \epsilon_0}$

$W = \sqrt{\frac{2K_s \epsilon_0 V_{bi}}{q} \left(\frac{N_A + N_D}{N_A N_D} \right)}$ $x_n = \left(\frac{N_A}{N_A + N_D} \right) W$ $x_p = \left(\frac{N_D}{N_A + N_D} \right) W$ $(0) = \sqrt{\frac{2qV_{bi}}{K_s \epsilon_0} \left(\frac{N_A N_D}{N_A + N_D} \right)}$

PN diode current: $n(0) = \frac{n_i^2}{N_A} (e^{qV_A/kT} - 1)$ $p(0) = \frac{n_i^2}{N_D} (e^{qV_A/kT} - 1)$

$J_D = J_o (e^{qV_A/kT} - 1)$ $J_o = q \left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right)$ (long) $J_o = q \left(\frac{D_n n_i^2}{W_p N_A} + \frac{D_p n_i^2}{W_n N_D} \right)$ (short)

Non-ideal diodes: $I = I_o (e^{q(V_A - IR_s)/kT} - 1)$ $J_{gen} = q \frac{n_i}{2\tau_o} W$

Photovoltaics: $V_{oc} = \frac{nkT}{q} \ln \left(\frac{J_{sc}}{J_o} \right)$ $J_{PV} = J_o (e^{qV_A/kT} - 1)$ J_{sc}

Small signal model: $G_d = \frac{I_D + I_o}{kT/q}$ $C_J(V_R) = \frac{K_s \epsilon_o A}{\sqrt{\frac{2K_s \epsilon_o V_{bi}}{q N_A}}} = A \sqrt{\frac{q K_s \epsilon_o N_A}{2V_{bi}}}$ $C_D = G_d \tau_n$

MS diode properties: $qV_{bi} = |\Phi_M - \Phi_S|$ $\Phi_{BP} = \chi + E_G$ Φ_M $\Phi_{BN} = \Phi_M - \chi$

$J_D = J_o (e^{qV_A/kT} - 1)$ $J_o = A T^2 e^{-\Phi_B/kT}$ $A = \frac{4\pi q m k_B^2}{h^3} = 120 \frac{m}{m_o} \frac{A}{cm^2 K^2}$

MOS capacitors: $W = \sqrt{\frac{2K_s \epsilon_o \phi_s}{q N_A}} \text{ cm}$ $s = \sqrt{\frac{2q N_A \phi_s}{K_s \epsilon_o}} \frac{V}{\text{cm}}$

$Q_B = q N_A W (\phi_s) = \sqrt{2q K_s \epsilon_o N_A \phi_s} \frac{C}{cm^2}$

$V_G = V_{FB} + \phi_s + \phi_{ox} = V_{FB} + \phi_s + \frac{Q_s(\phi_s)}{C_{ox}}$

$C_{ox} = K_o \epsilon_o / x_o$ $V_{FB} = \Phi_{ms}/q - Q_F/C_{ox}$

$C = C_{ox} / \left[1 + \frac{K_o W (\phi_s)}{K_s x_o} \right]$ $V_T = Q_B(2\phi_F)/C_{ox} + 2\phi_F$ $Q_n = C_{ox}(V_G - V_T)$

MOSFETs: $I_D = W Q_n(y=0) v_y(y=0)$

$I_D = \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_T) V_{DS}$ $I_D = W C_{ox} v_{sat} (V_{GS} - V_T)$

Square Law (for $V_{GS} \geq V_T$): $I_D = \begin{cases} \frac{W}{L} \mu_n C_{ox} [(V_{GS} - V_T) V_{DS} - V_{DS}^2/2], & 0 \leq V_{DS} \leq V_{GS} - V_T \\ \frac{W}{2L} \mu_n C_{ox} (V_{GS} - V_T)^2, & V_{DS} \geq V_{GS} - V_T \end{cases}$

Bipolar transistors: (assuming NPN, short emitter, base, and collector)

Ebers-Moll Equations:

$$I_C(V_{BE}, V_{BC}) = \alpha_F I_{F0} \left(e^{qV_{BE}/k_B T} - 1 \right) - I_{R0} \left(e^{qV_{BC}/k_B T} - 1 \right)$$

$$I_E(V_{BE}, V_{BC}) = I_{F0} \left(e^{qV_{BE}/k_B T} - 1 \right) - \alpha_R I_{R0} \left(e^{qV_{BC}/k_B T} - 1 \right)$$

$$I_{F0} = qA \left(\frac{D_{nB}}{W_B} \frac{n_i^2}{N_{AB}} + \frac{D_{pE}}{W_E} \frac{n_i^2}{N_{DE}} \right)$$

$$I_{R0} = qA \left(\frac{D_{nB}}{W_B} \frac{n_i^2}{N_{AB}} + \frac{D_{pC}}{W_C} \frac{n_i^2}{N_{DC}} \right)$$

$$\alpha_F = \gamma_F \alpha_T$$

$$\alpha_R = \gamma_R \alpha_T$$

$$\alpha_F I_{F0} = \alpha_R I_{R0}$$

$$\gamma_F = \frac{I_{En}}{I_{En} + I_{Ep}} = \frac{1}{1 + \frac{D_{pE}}{D_{nB}} \frac{W_B}{W_E} \frac{N_{AB}}{N_{DE}}}$$

$$\alpha_T = \frac{I_{Cn}}{I_{En}} = \frac{1}{1 + \frac{1}{2} \left(\frac{W_B}{L_{nB}} \right)^2}$$

$$\beta_F = \frac{\alpha_F}{1 - \alpha_F}$$

$$\alpha_F = \frac{\beta_F}{1 + \beta_F}$$