**NAME:**______________________________ **PUID:** ______________________________

# ECE 305 Exam 6 SOLUTIONS Spring 2015

**May 7, 2015**

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This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

There are three equally weighted questions. To receive full credit, you must **show your work**.

The exam is designed to be taken in 50 minutes – just like Exams 1-5, but you will have the entire 2 hours to complete it.
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Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

**75 points possible, 10 per question**

1) 25 points (5 point per part)

2) 25 points (5 points per part)

3) 25 points (5 points per part)

------------------------- Course policy -------------------------

I understand that if I am caught cheating in this course, I will earn an F for the course and be reported to the Dean of Students.

Read and understood: ____________________________________
signature

Answer the **multiple choice questions** below by **circling** the **one, best answer**.

1a) Which of the following would reduce "base width modulation" (i.e. the Early effect) and, therefore, increase the output resistance?

a) Increasing the emitter doping.
b) Increasing the collector doping.
**c) Increasing the base width.**
d) Increase the emitter thickness.
e) Decrease the base doping.

1b) If the emitter injection efficiency is 0.97, what is beta? (Assume active region of operation and a base transport factor of one.)

a) 0.97.
b) 0.03.
**c) 32.**
d) 323.
e) 97.

1c) What is the order of highest doping density, next highest, and lowest doping density in a conventional BJT?

**a) Emitter, base, collector.**
b) Emitter, collector, base
c) Base, emitter, collector.
d) Base, collector, emitter.
e) Collector, base, emitter.

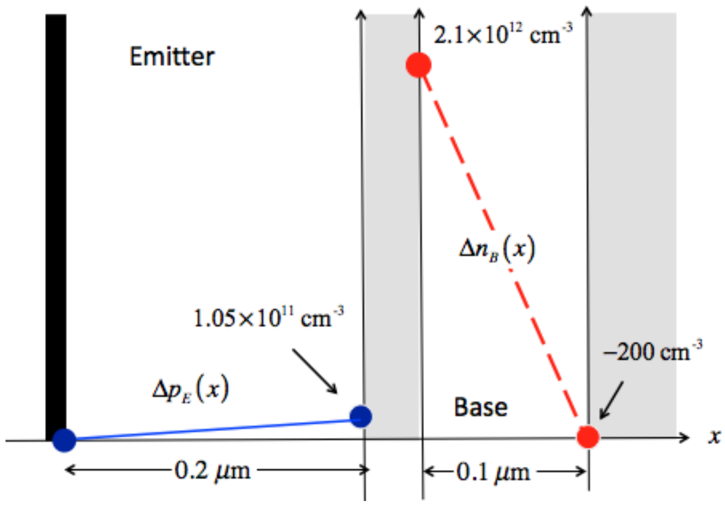
1d) How are the PN junctions biased in the saturation region of a PNP BJT?

**a) Emitter-base: forward-biased, Base-collector: forward-biased.**
b) Emitter-base: forward-biased, Base-collector: reverse-biased.
c) Emitter-base: reverse-biased, Base-collector: forward-biased.
d) Emitter-base: reverse-biased, Base-collector: reverse-biased.
e) Emitter-base: reverse-biased, Base-collector: zero-biased.

1e) If the emitter injection efficiency is 1.00 and the base transport factor is 0.98, the collector current is $100\ \mu\text{A}$, what is the base current? (Assume active region of operation.)

a) $1\ \mu\text{A}$.
**b) $2\ \mu\text{A}$.**
c) $3\ \mu\text{A}$.
d) $4\ \mu\text{A}$.
e) $5\ \mu\text{A}$.

2) The figure below is a sketch of the excess minority carrier concentrations in the quasi-neutral emitter and base regions of a bipolar transistor. The shaded areas are the depletion regions and the black rectangle is the emitter contact. You may ignore recombination and assume a transistor area of $10\ \mu\text{m} \times 10\ \mu\text{m} = 100 \times 10^{-8}\ \text{cm}^2$. The diffusion coefficient for electrons is $D_n = 10\ \text{cm}^2/\text{s}$ and for holes, $D_p = 2\ \text{cm}^2/\text{s}$. The temperature is 300 K ($k_B T/q = 0.026$ V).




Answer the following questions.

2a) What region of operation is this transistor biased in? Explain your answer.

**Solution:**

**(Forward) Active region.**

We see that there are excess carriers injected in the quasi-neutral base and emitter sides of the EB PN junction, so it is forward biased. There are no excess carriers injected into the base from the collector, so it is reverse biased (or zero biased). These are the conditions for the forward active region of operation.

2b) What is the collector current in Amperes for this transistor?

**Solution:**
The electron current density (in the +x direction) in the base is

$$J_{En} = -q\frac{D_n}{W_B}\Delta n_B(0) = 1.6\times10^{-19}\times\frac{10}{0.1\times10^{-4}}\times2.1\times10^{12}\text{ A/cm}^2 = -0.34\text{ A/cm}^2$$

The electron profile in the base is linear, so the base transport factor is close to one. (Since we are told that there is no recombination in the transistors, the base transport factor is exactly one.) Assuming that all of this current comes out the collector, we multiply by the emitter area and find:

$$I_C = -A_E\times J_{En} = 100\times10^{-8}\times0.34\text{ A/cm}^2 = 0.34\times10^{-6}\text{ A}$$

$$\boxed{I_C = 0.35\ \mu\text{A}}$$

The positive sign indicates that current flows into the collector terminal.

2c) What is the base current in Amperes for this transistor?

**Solution:**
The hole current density (in the +x direction) injected into the emitter is

$$J_{Ep} = -q\frac{D_p}{W_E}\Delta p_E(0') = 1.6\times10^{-19}\times\frac{2}{0.2\times10^{-4}}\times1.05\times10^{11}\text{ A/cm}^2 = -1.68\times10^{-3}\text{ A/cm}^2$$

We multiply by the emitter area and find:

$$I_B = -A_E\times J_{Ep} = 100\times10^{-8}\times1.68\times10^{-3}\text{ A/cm}^2 = 1.7\times10^{-9}\text{ A}$$

$$\boxed{I_B = 1.7\text{ nA}}$$

The positive sign indicates that current flows into the base terminal.

2d) What is the emitter injection efficiency of this BJT?

**Solution:**

$$\gamma = \frac{|J_{En}|}{|J_{En}|+|J_{Ep}|} = \frac{I_C}{I_C+I_B} = \frac{350\text{ nA}}{350\text{ nA}+1.7\text{ nA}} = 0.995$$

$$\boxed{\gamma = 0.995}$$

Exam 6 ECE 305 Spring 2015

2e) What is the ratio of the emitter doping density to the base doping density? i.e.: $\frac{N_{DE}}{N_{AB}} = ?$

**Solution:**

According to the Law of the Junction:

$$\Delta n_B(0) = \frac{n_i^2}{N_{AB}} e^{qV_{BE}/k_BT}$$

$$\Delta p_E(0') = \frac{n_i^2}{N_{DE}} e^{qV_{BE}/k_BT}$$

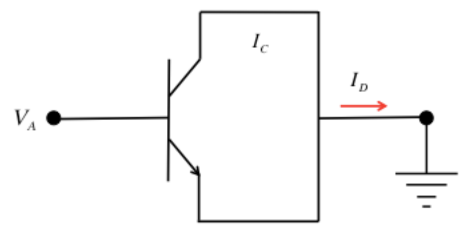
$$\frac{\Delta n_B(0)}{\Delta p_E(0')} = \frac{N_{DE}}{N_{AB}}$$

Reading from the plot:

$$\frac{\Delta n_B(0)}{\Delta p_E(0')} = \frac{N_{DE}}{N_{AB}} = \frac{2.1\times10^{12}}{1.05\times10^{11}} = 20$$

$$\boxed{\frac{N_{DE}}{N_{AB}} = 20}$$

3) This problem is about the transistor show below.




3a) What is the region of operation for this transistor if $V_A > 0$ ? Explain your answer.

**Solution:**
$V_C = V_E = 0$, so when a positive voltage is put on the base, $V_{BE} = V_{BC} = V_A > 0$. Both junctions are forward biased, so the region of operation is **saturation.**

3b) Use the Ebers-Moll equations to derive an expression for $I_D(V_A)$.

**Solution:**
From the formula sheet:

$$I_C(V_{BE,}V_{BC}) = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$
$$I_E(V_{BE},V_{BC}) = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

From the figure, we see: $I_D = I_B$, so

$$I_D = I_B = I_E - I_C = (1-\alpha_F)I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) + (1-\alpha_R)I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$
$V_{BE} = V_{BC} = V_A$, so

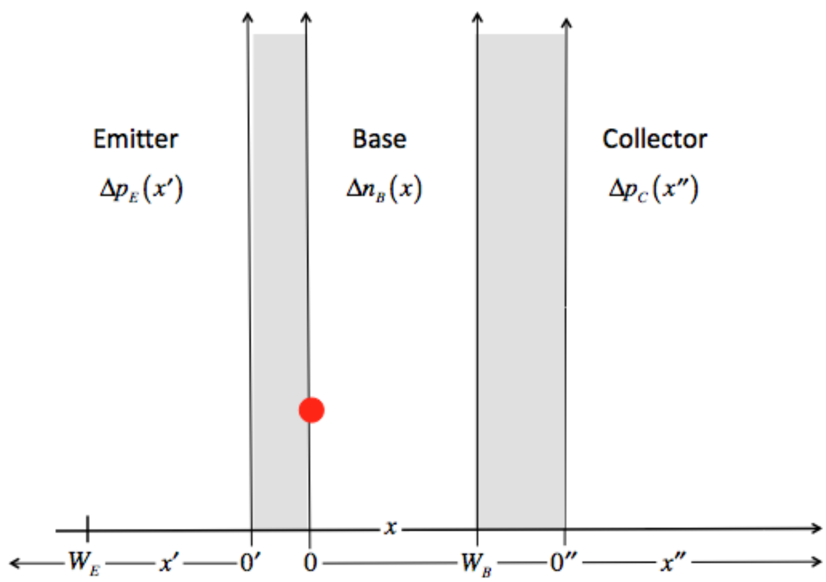
$$I_D(V_A) = (1-\alpha_F)I_{F0}\left(e^{qV_A/k_BT} - 1\right) + (1-\alpha_R)I_{R0}\left(e^{qV_A/k_BT} - 1\right)$$
$$I_D(V_A) = \left((1-\alpha_F)I_{F0} + (1-\alpha_R)I_{R0}\right)\left(e^{qV_A/k_BT} - 1\right)$$

This could be the answer, but we can also remember "reciprocity": $\alpha_F I_{F0} = \alpha_R I_{R0}$ and write the above equation as:

$$\boxed{I_D(V_A) = 2(1-\alpha_F)I_{F0}\left(e^{qV_A/k_BT} - 1\right)}$$

The device behaves like a diode.

The next three parts of question 3 are about the same transistor biased as shown in the figure above, but this time you are asked to plot some internal quantities. Assume that the emitter and base regions are "short" and that the collector is "long" and that $N_{DE} = 4\times10^{18}$ cm$^{-3}$, $N_{AB} = 2\times10^{18}$ cm$^{-3}$, and $N_{DC} = 4\times10^{17}$ cm$^{-3}$. Assume Si at room temperature (300 K and $k_BT/q = 0.026$ V) and that $V_A = 0.7$ V. Your answers to questions 3c), 3d), and 3e) should be plotted on the figure below.




3c) The concentration of excess minority carrier electrons at the beginning of the base, $\Delta n_B(0)$, is indicated by the filled circle on the figure above. Determine numerical values of the excess minority electron density at the two ends of the base, $\Delta n_B(0)$ and $\Delta n_B(W_B)$ and sketch $\Delta n_B(x)$ within the base.

Numerical values of:

$\Delta n_B(0) = ?$ cm$^{-3}$

$\Delta n_B(W_B)?$ cm$^{-3}$

Also, plot $\Delta n_B(x)$ on the figure above. **(Make it clear whether your plot is linear or curved.)**

**Solution:**
Use the law of the junction:

$$\Delta n_B(0) = \frac{n_i^2}{N_{AB}}\left(e^{qV_{BE}/k_BT} - 1\right)\text{ cm}^{-3} = \frac{10^{20}}{2\times10^{18}}\left(e^{qV_A/k_BT} - 1\right) = 50\left(e^{qV_A/k_BT} - 1\right) = 50\left(e^{0.7/0.026} - 1\right)$$
$$= 2.5\times10^{13}\text{ cm}^{-3}$$

The base-collector junction has exactly the same forward bias, so
$\Delta n_B(W_B) = \Delta n_B(0)\text{ cm}^{-3}$

$$\boxed{\Delta n_B(0) = \Delta n_B(W_B) = 2.5\times10^{13}\text{ cm}^{-3}}$$

The base is short, so in between, we have a straight line for $\Delta n_B(x)$. See the plot below for the result.

3d) Find the concentration of excess minority carrier holes, $\Delta p_E(0')$ at the beginning of the emitter. You may assume that $\Delta p_E(W_E) = 0$ at the emitter contact, $x' = W_E$. Determine the numerical value of $\Delta p_E(0')$ and plot $\Delta p_E(x')$ in the emitter. Be sure that the scale is consistent with the minority electron profile in the base.

Numerical value of:
$\Delta p_E(0') = ?\text{ cm}^{-3}$
Also, plot $\Delta p_E(x')$ on the figure above. **(Make it clear whether your plot is linear or curved.)**

**Solution:**
Use the law of the junction:

$$\Delta p_E(0') = \frac{n_i^2}{N_{DE}}\left(e^{qV_{BE}/k_BT} - 1\right)\text{ cm}^{-3} = \frac{10^{20}}{4\times10^{18}}\left(e^{qV_A/k_BT} - 1\right) = 25\left(e^{qV_A/k_BT} - 1\right) = 25\left(e^{0.7/0.026} - 1\right)$$
$$= 1.25\times10^{12}\text{ cm}^{-3}$$

$$\boxed{\Delta p_E(0') = 1.25\times10^{12}\text{ cm}^{-3}}$$

This is two times smaller than $\Delta n_B(0)$ because the emitter is doped two times heavier.

The emitter is short compared to a diffusion length, so we connect $\Delta p_E(0')$ and $\Delta p_E(W_E)$ with a straight line. See plot below for the result.



3e) Find the concentration of excess minority carrier holes, $\Delta p_C(0'')$ at the beginning of the collector. Determine the numerical value of $\Delta p_C(0'')$ and plot $\Delta p_C(x'')$ in the collector. Be sure that the scale is consistent with the minority electron profile in the base.

Numerical value of
$\Delta p_C(0'') = ?\ \text{cm}^{-3}$
Also, plot $\Delta p_C(x'')$ on the figure above. **(Make it clear whether your plot is linear or curved.)**

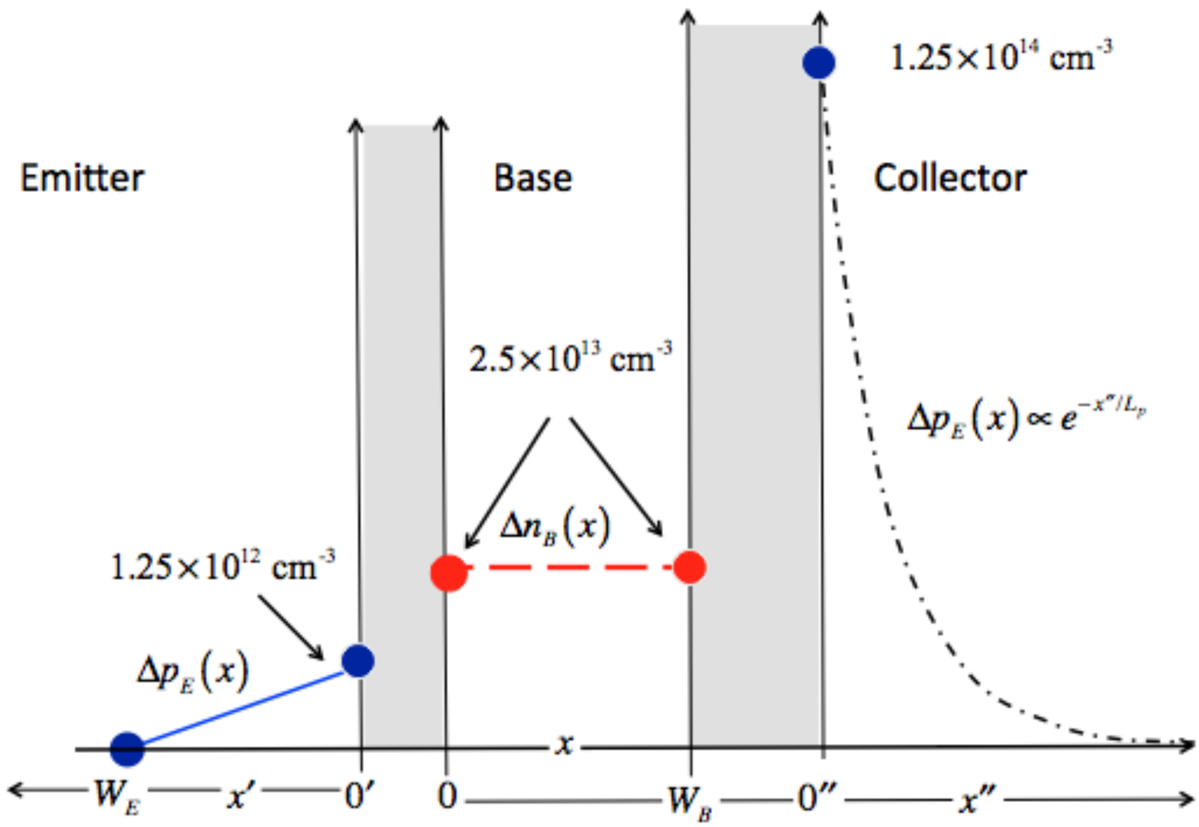
**Solution:**
Use the law of the junction:

$$\Delta p_E(0'') = \frac{n_i^2}{N_{DC}}\left(e^{qV_{BE}/k_BT} - 1\right)\text{cm}^{-3} = \frac{10^{20}}{4\times10^{17}}\left(e^{qV_A/k_BT} - 1\right) = 250\left(e^{qV_A/k_BT} - 1\right) = 250\left(e^{0.7/0.026} - 1\right)$$
$$= 1.25\times10^{14}\ \text{cm}^{-3}$$

$$\boxed{\Delta p_E(0'') = 1.25\times10^{14}\ \text{cm}^{-3}}$$

This is five times larger than $\Delta n_B(W_B)$ because the collector is doped five times lighter.

The collector is **long** compared to a diffusion length, so $\Delta p_C(x'')$ begins at $\Delta p_C(0'')$ and then decays as $\Delta p_C(x'') = \Delta p_C(0'')e^{-x''/L_p}$. See plot below for the result.


1.25×10^14 cm^-3
Emitter
Base
Collector
2.5×10^13 cm^-3
Δp_E(x) ∝ e^{-x''/L_p}
Δn_B(x)
1.25×10^12 cm^-3
Δp_E(x)
x
W_E
x′
0′
0
W_B
0″
x″