ECE-305: Spring 2015

BJTs: Ebers-Moll Model

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Pierret, Semiconductor Device Fundamentals (SDF)
pp. 403-407

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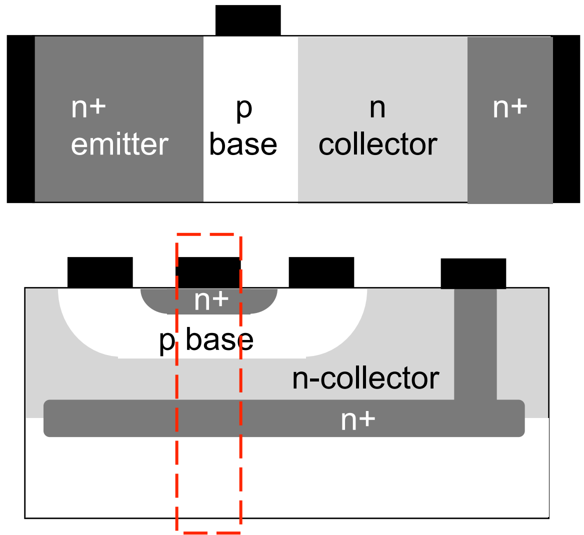
last lecture

1) Emitter injection efficiency
2) Base transport factor
3) Early effect
4) Speed (base transit time)
5) Effects of saturation
6) Gummel plots
7) Transconductance
8) HBTs
9) **Emitter crowding**

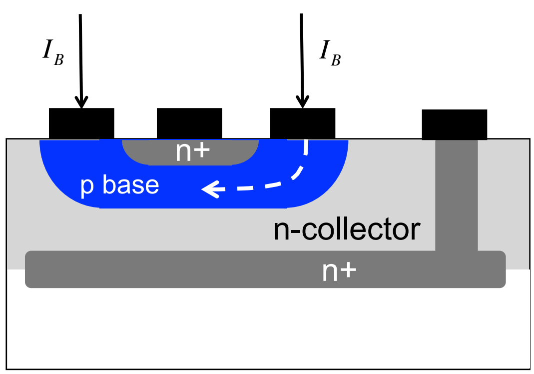
2D effects

double diffused BJT



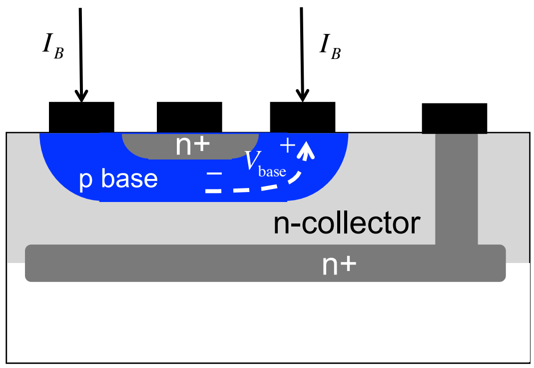
3

emitter current crowding

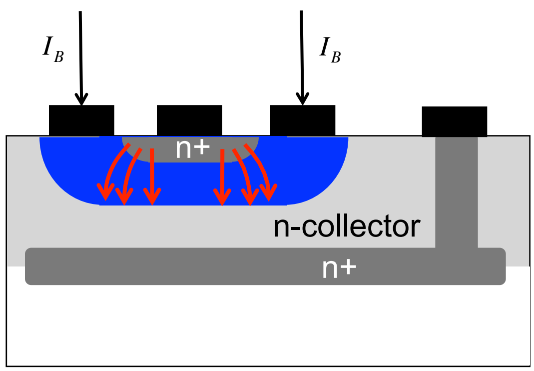


4

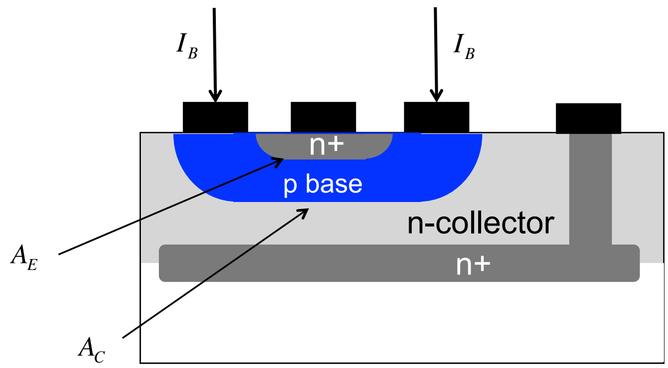
emitter current crowding



emitter current crowding

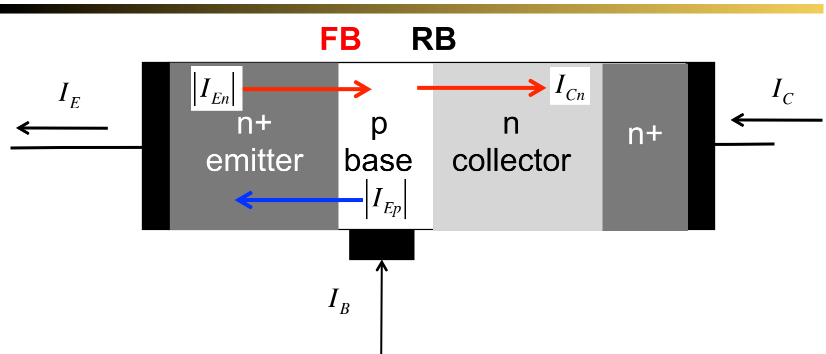


emitter and collector areas



$A_C >> A_E$

Q1) Which is the base transport factor?



a) $|I_{En}|/(|I_{En}|+|I_{Ep}|)$

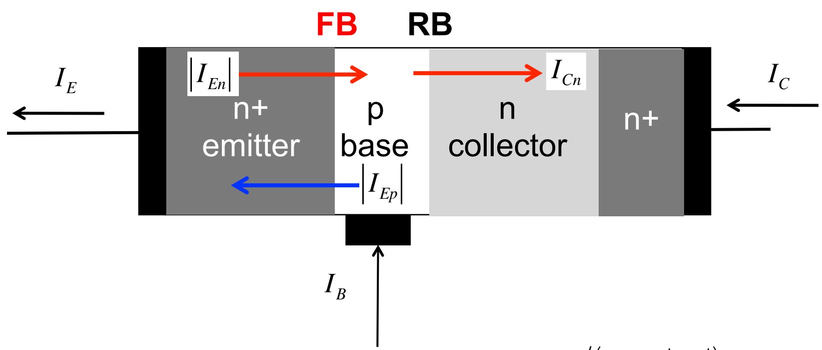
b) $|I_{Cn}|/(|I_{En}|+|I_{Ep}|)$

c) $|I_{Cn}|/|I_{En}|$

d) $|I_{Ep}|$

e) $|I_{Cn}|/|I_{Ep}|$

Q2) Which is beta_dc?

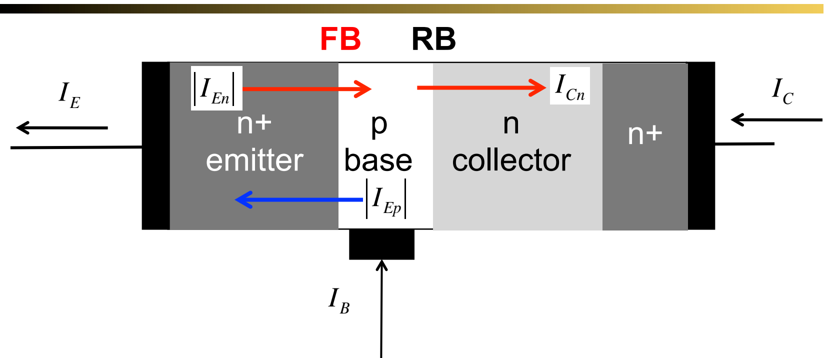


a) $|I_{En}|/(|I_{En}|+|I_{Ep}|)$ b) $|I_{Cn}|/(|I_{En}|+|I_{Ep}|)$

c) $|I_{Cn}|/|I_{En}|$ d) $|I_{Ep}|$

e) $|I_{Cn}|/|I_{Ep}|$

Q3) Which is alpha_dc?

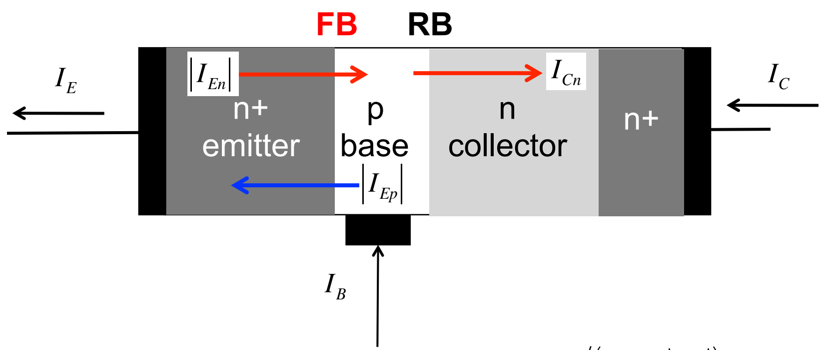


a) $|I_{En}|/(|I_{En}|+|I_{Ep}|)$ b) $|I_{Cn}|/(|I_{En}|+|I_{Ep}|)$

c) $|I_{Cn}|/|I_{En}|$ d) $|I_{Ep}|$

e) $|I_{Cn}|/|I_{Ep}|$

Q4) Which is the base current?



a) $|I_{En}|/(|I_{En}|+|I_{Ep}|)$

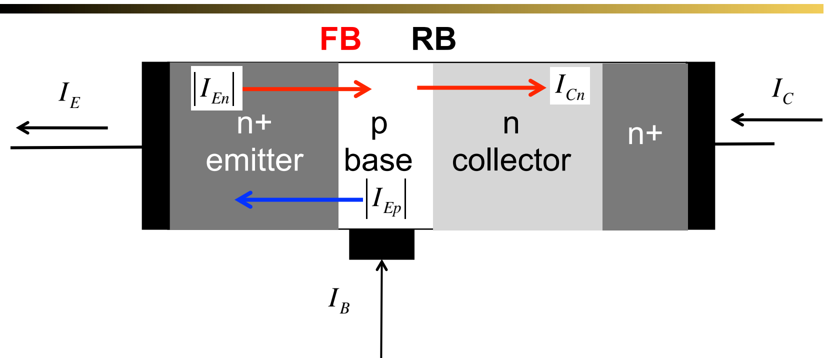
b) $|I_{Cn}|/(|I_{En}|+|I_{Ep}|)$

c) $|I_{Cn}|/|I_{En}|$

d) $|I_{Ep}|$

e) $|I_{Cn}|/|I_{Ep}|$

Q5) Which is the emitter injection efficiency?



a) $|I_{En}|/(|I_{En}|+|I_{Ep}|)$

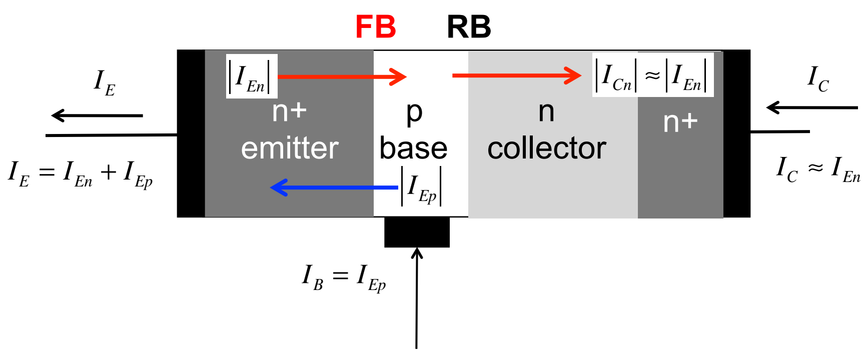
b) $|I_{Cn}|/(|I_{En}|+|I_{Ep}|)$

c) $|I_{Cn}|/|I_{En}|$

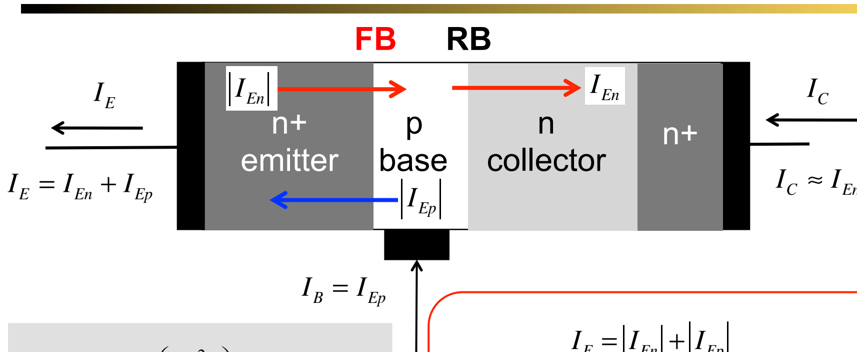
d) $|I_{Ep}|$

e) $|I_{Cn}|/|I_{Ep}|$

forward active region



emitter current: forward active region



$$|I_{En}| = qA_E\left(\frac{n_i^2}{N_{AB}}\right)\frac{D_n}{W_B}e^{qV_{BE}/k_BT}$$

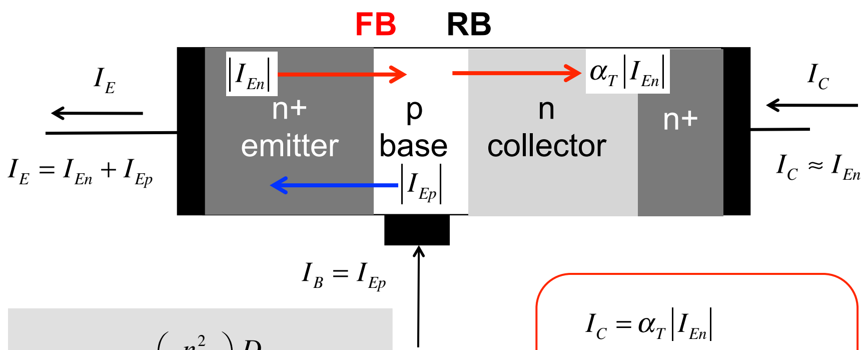
$$|I_{Ep}| = qA_E\left(\frac{n_i^2}{N_{DE}}\right)\frac{D_p}{W_E}e^{qV_{BE}/k_BT}$$

$$I_E = |I_{En}| + |I_{Ep}|$$

$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_{F0} = qA_E\left(\frac{n_i^2}{N_{AB}}\right)\frac{D_n}{W_B} + qA_E\left(\frac{n_i^2}{N_{DE}}\right)\frac{D_p}{W_E}$$

collector current: forward active region



$$|I_{En}| = qA_E\left(\frac{n_i^2}{N_{AB}}\right)\frac{D_n}{W_B}e^{qV_{BE}/k_BT}$$

$$|I_{Ep}| = qA_E\left(\frac{n_i^2}{N_{DE}}\right)\frac{D_p}{W_E}e^{qV_{BE}/k_BT}$$

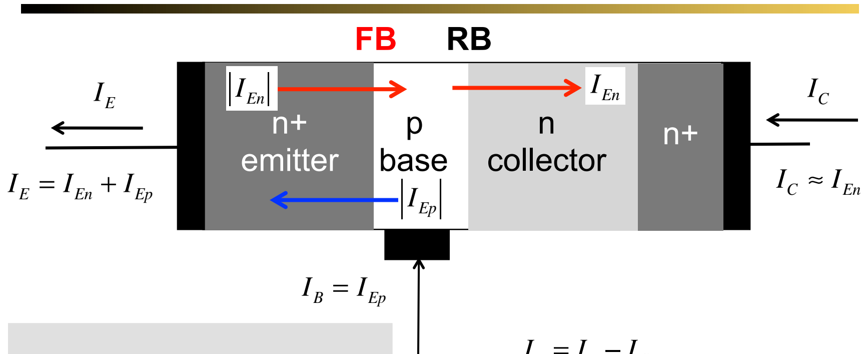
$$I_C = \alpha_T |I_{En}|$$

$$I_C = \alpha_T \gamma_F I_E$$

$$I_C = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$\alpha_F = \alpha_T \gamma_F$$

base current: forward active region



$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_C = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

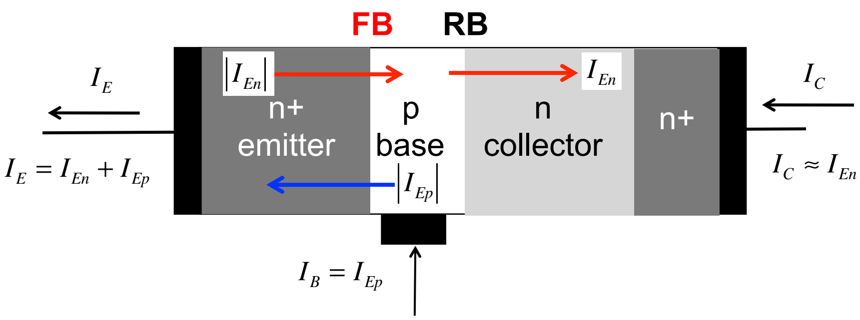
$$\alpha_F = \alpha_T \gamma_F$$

$$I_B = I_E - I_C$$

$$I_B = I_E - I_C$$

$$I_B = \left(1 - \alpha_F\right) I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

summary: forward active region



$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$\alpha_F = \alpha_T \gamma_F$$

$$I_C = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_{F0} = qA_E\left(\frac{n_i^2}{N_{AB}}\right)\frac{D_n}{W_B} + qA_E\left(\frac{n_i^2}{N_{DE}}\right)\frac{D_p}{W_E}$$

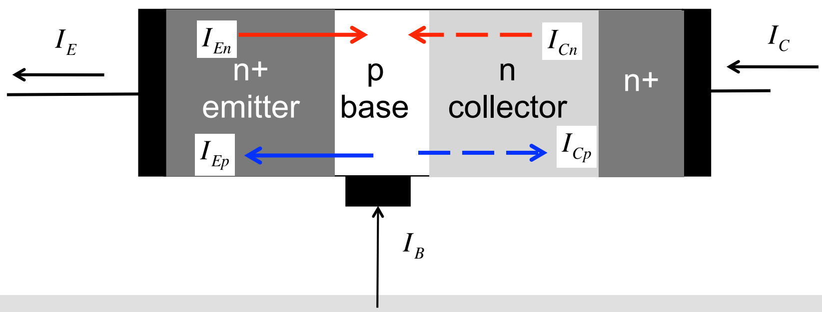
$$I_B = \left(1 - \alpha_F\right)I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

Ebers-Moll model

Question:

How do we describe the BJT in **any** region of operation?

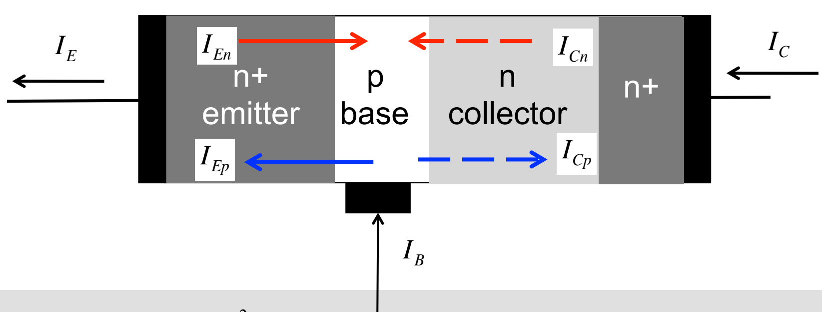
emitter-base junction (the **forward** diode)



$$I_{En} = -qA\frac{D_n}{W_B}\frac{n_i^2}{N_{AB}}\left(e^{qV_{BE}/k_BT} - 1\right) \qquad I_E\left(V_{BE}\right) = -I_{En}\left(V_{BE}\right) - I_{Ep}\left(V_{BE}\right)$$

$$I_{Ep} = -qA\frac{D_p}{W_E}\frac{n_i^2}{N_{DE}}\left(e^{qV_{BE}/k_BT} - 1\right) \qquad I_E\left(V_{BE}\right) = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

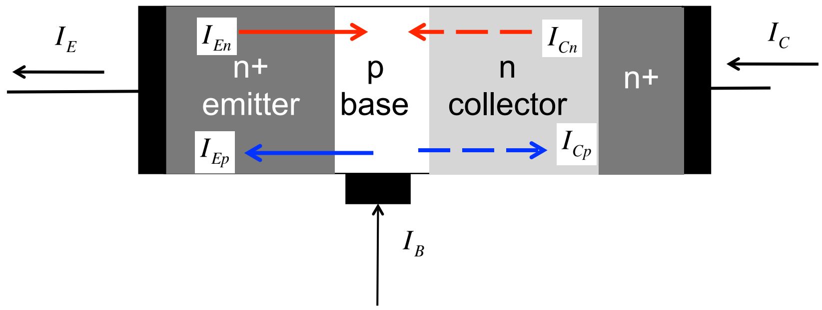
Base-collector junction (the **reverse** diode)



$$I_{Cn}\left(V_{BC}\right) = qA\frac{D_n}{W_B}\frac{n_i^2}{N_{AB}}\left(e^{qV_{BC}/k_BT} - 1\right) \qquad I_C\left(V_{BC}\right) = -\left[I_{Cn}\left(V_{BC}\right) + I_{Cp}\left(V_{BC}\right)\right]$$

$$I_{Cp}\left(V_{BC}\right) = qA\frac{D_p}{W_C}\frac{n_i^2}{N_{DC}}\left(e^{qV_{BC}/k_BT} - 1\right) \qquad I_C\left(V_{BC}\right) = -I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

Both junctions….

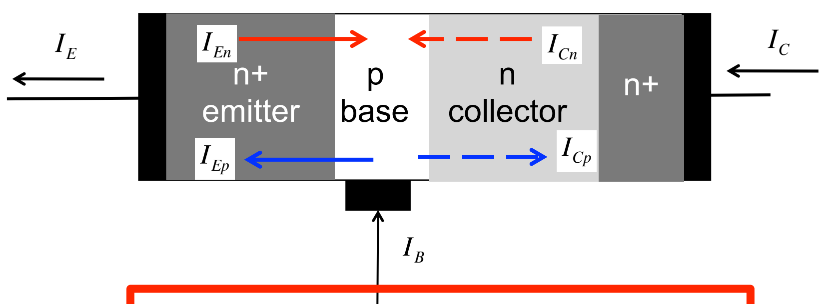


$$I_C\left(V_{BC}\right) = -I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_E\left(V_{BE}\right) = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

But….
The two junctions are coupled!

Ebers-Moll model



$$I_C\left(V_{BE}, V_{BC}\right) = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_E\left(V_{BE}, V_{BC}\right) = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

Ebers-Moll model

$$I_C\left(V_{BE,}V_{BC}\right)=\alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E\left(V_{BE},V_{BC}\right)=I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-\alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_B\left(V_{BE},V_{BC}\right)=I_E\left(V_{BE},V_{BC}\right)-I_C\left(V_{BE},V_{BC}\right)$$

See Pierret SDF, Chapter 11, sec. 11.1.4

Ebers-Moll model

$$I_C\left(V_{BE,}V_{BC}\right)=\alpha_F I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$I_E\left(V_{BE},V_{BC}\right)=I_{F0}\left(e^{qV_{BE}/k_BT}-1\right)-\alpha_R I_{R0}\left(e^{qV_{BC}/k_BT}-1\right)$$

$$\alpha_F=\alpha_T\gamma_F \qquad I_{F0}=qA_E\left(\frac{n_i^2}{N_{AB}}\right)\frac{D_n}{W_B}+qA_E\left(\frac{n_i^2}{N_{DE}}\right)\frac{D_p}{W_E}$$

$$\alpha_R=\alpha_T\gamma_R \qquad I_{R0}=qA_E\left(\frac{n_i^2}{N_{AB}}\right)\frac{D_n}{W_B}+qA_E\left(\frac{n_i^2}{N_{DC}}\right)\frac{D_p}{W_C}$$

$$\alpha_F I_{F0}=\alpha_R I_{R0}$$

Ebers-Moll model

$$I_C(V_{BE,}V_{BC}) = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_E(V_{BE}, V_{BC}) = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_B(V_{BE}, V_{BC}) = I_E(V_{BE}, V_{BC}) - I_C(V_{BE}, V_{BC})$$

$$\alpha_F I_{F0} = \alpha_R I_{R0}$$

See Pierret SDF, Chapter 11, sec. 11.1.4

Check active region

$$I_C(V_{BE,}V_{BC}) = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right) \rightarrow \alpha_F I_{F0} e^{qV_{BE}/k_BT} + I_{R0}$$

$$I_E(V_{BE}, V_{BC}) = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right) \rightarrow I_{F0} e^{qV_{BE}/k_BT} + \alpha_R I_{R0}$$

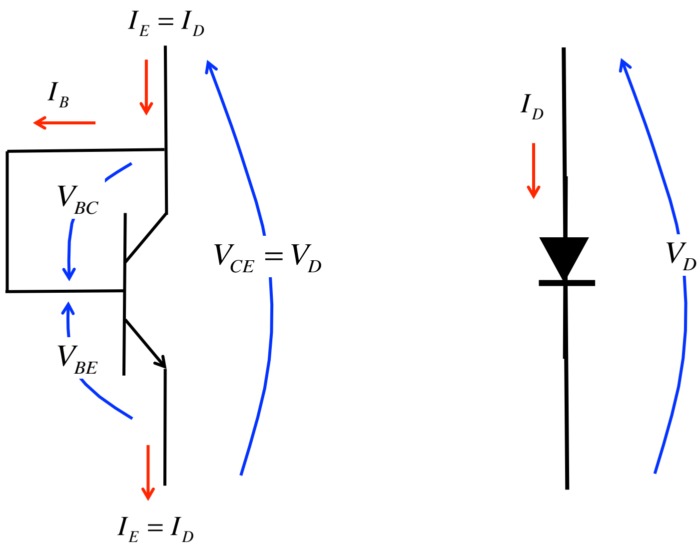
$$I_B(V_{BE}, V_{BC}) = I_E(V_{BE}, V_{BC}) - I_C(V_{BE}, V_{BC}) \rightarrow (1-\alpha_F) I_{F0} e^{qV_{BE}/k_BT} - (1-\alpha_R) I_{R0}$$

$$I_C \approx \alpha_F I_{F0} e^{qV_{BE}/k_BT}$$

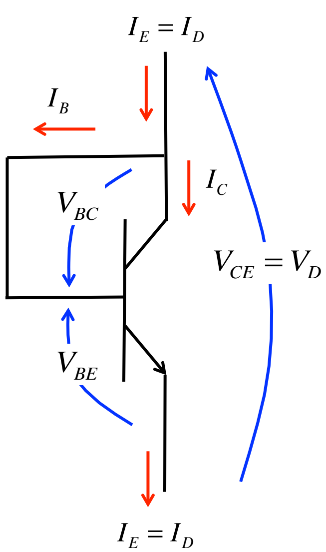
$$I_B \approx (1-\alpha_F) I_{F0} e^{qV_{BE}/k_BT} = \frac{(1-\alpha_F)}{\alpha_F} I_C = \frac{1}{\beta_{dc}} I_C$$

✔

What is I_D?



What is I_D?



$$I_C = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

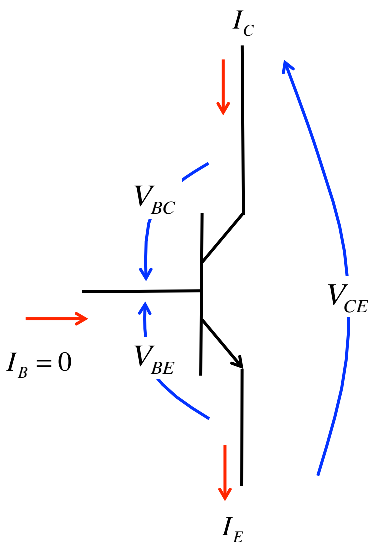
$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$V_{BC} = 0$$

$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right)$$

$$I_D = I_{F0}\left(e^{qV_D/k_BT} - 1\right)$$

What is I_C?



$$I_C = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_E = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

Ebers-Moll model

$$I_C\left(V_{BE,}V_{BC}\right) = \alpha_F I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_E\left(V_{BE},V_{BC}\right) = I_{F0}\left(e^{qV_{BE}/k_BT} - 1\right) - \alpha_R I_{R0}\left(e^{qV_{BC}/k_BT} - 1\right)$$

$$I_B\left(V_{BE},V_{BC}\right) = I_E\left(V_{BE},V_{BC}\right) - I_C\left(V_{BE},V_{BC}\right)$$

$$\alpha_F I_{F0} = \alpha_R I_{R0}$$

See Pierret SDF, Chapter 11, sec. 11.1.4

Ebers-Moll model (ii)

$$I_{F0} = qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}} + \frac{D_{pE}}{W_E}\frac{n_i^2}{N_{DE}}\right)$$

$$I_{R0} = qA\left(\frac{D_{nB}}{W_B}\frac{n_i^2}{N_{AB}} + \frac{D_{pC}}{W_C}\frac{n_i^2}{N_{DC}}\right)$$

$$\alpha_F = \gamma_F \alpha_T \qquad \alpha_R = \gamma_R \alpha_T$$

$$\alpha_F I_{F0} = \alpha_R I_{R0} \rightarrow \alpha_R = \alpha_F \frac{I_{F0}}{I_{R0}}$$