Ideal Diode Equation II
+ Intro to Solar Cells

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outline

1) Review
2) Ideal diode equation (long base)
3) Ideal diode equation (short base)
4) Discussion
5) A Primer Solar Cells
e-band diagram under bias

\[ V = 0 \]

\[ F_n - F_p = qV_A \]

\[ F_n \rightarrow q(V_{bi} - V_A) \uparrow \]

\[ F_p \]

\[ E_C \]

\[ V_A > 0 \]

\[ x \]

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recombination and current

minority carriers injected across junction

\[ F_n \rightarrow \quad qV_A \quad \rightarrow \quad F_p \]

\[ -V_A + \]

\[ I_D \]

Every time a minority electron recombines on the p-side, one electron flows in the external current.
excess electrons on the p-side of junction

\[ n_{0N} \approx N_D \quad \text{and} \quad n_p >> n_{0P} \]

\[ q(V_n - V_A) \]

What is \( \Delta n(x) \) on the p-side?

Answer: Solve the MCDE.

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solving the MCDE

\[ \Delta n(0) = \frac{n_i^2}{N_A} \left( e^{q(V_a - V_b)T} - 1 \right) \]

\[ \Delta n(x \rightarrow \infty) = 0 \]

\[ \Delta n(x) \rightarrow 0 \]

\[ \frac{d^2 \Delta n}{dx^2} \left|_{x=0} \right. = 0 \quad L_n = \frac{1}{\sqrt{D_n \tau_n}} \]

**What is \( \Delta n(x) \) on the p-side?**

**Answer:** Solve the MCDE.
total recombination on p-side of junction

$$\Delta n(x) = \Delta n(0) e^{-x/L_n}$$

How many electrons recombine per second?

Answer:

$$R_{TOT} = A \left( -D_n \frac{d\Delta n}{dx} \right)_0 = A \frac{D_n}{L_n} \Delta n(0)$$

current (long region)

$$\Delta n(0) = \frac{n_i^2}{N_A} \left( e^{\nu_i/\beta_{kT}} - 1 \right)$$

$$\Delta n(x) = \Delta n(0) e^{-x/L_n}$$

$$W_p >> L_n$$

$$I_D = qR_{TOT} = qA \frac{D_n}{L_n} \Delta n(0)$$

$$I_D = qA \frac{D_n}{L_n} \frac{n_i^2}{N_A} \left( e^{\nu_i/\beta_{kT}} - 1 \right)$$

$$\Delta n(x) \to 0$$

$$x$$
diode current (long)

\[ I_n = \frac{qA\Delta n(0)D_n}{L_n} = qA \left( \frac{D_n n_i^2}{L_n N_A} \right) \left( e^{\frac{qV_A kT}{kT}} - 1 \right) \]

\[ V = 0 \quad \quad \quad V_A > 0 \]

\[ I_p = \frac{qA\Delta p(0)D_p}{L_p} = qA \left( \frac{D_p n_i^2}{L_p N_D} \right) \left( e^{\frac{qV_A kT}{kT}} - 1 \right) \]

\[ I_D(V_A) = I_p(V_A) + I_n(V_A) \]

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diode current

“ideal diode equation”

“Shockley diode equation”

\[ I_D = I_0 \left( e^{\frac{qV_A kT}{kT}} - 1 \right) \]

\[ I_0 = qA \left( \frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right) \]
1) Review

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3) Ideal diode equation (short case)

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We have assumed that $W_p \gg L_n$ (long diode).

What happens if $W_p \ll L_n$ (short diode)?
solving the MCDE

\[ \Delta n(0) = \frac{n_i^2}{N_A} \left( e^{q(V_n - V_A) / kT} - 1 \right) \]

\[ \Delta n(x = W_p) = 0 \]

\[ \frac{d^2 \Delta n}{dx^2} = 0 \quad L_n = \sqrt{D_n \tau_n} \gg W_p \]

“short” p-side

\[ \Delta n(x) = \frac{n_i^2}{N_A} \left( e^{q(V_n - V_A) / kT} - 1 \right) \]

\[ I_n = -qA D_n \frac{d \Delta n(x)}{dx} \bigg|_{x=0} \]

\[ I_n = qA \frac{D_n}{W_p} \Delta n(0) \]
diode current (short)

\[ I_n = qA \left( \frac{D_n}{W_P N_A} \right) \left( e^{\frac{qV_A}{kT}} - 1 \right) \]

\[ V = 0 \quad V_A > 0 \]

\[ I_p = qA \left( \frac{D_p}{W_N N_D} \right) \left( e^{\frac{qV_A}{kT}} - 1 \right) \]

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Ideal diode equation

\[ I_D = I_0 \left( e^{\frac{qV_A}{k_B T}} - 1 \right) \]

Long:
\[ I_0 = qA \left( \frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D} \right) \]

Short:
\[ I_0 = qA \left( \frac{D_n n_i^2}{W_p N_A} + \frac{D_p n_i^2}{W_n N_D} \right) \]

Diode current

\[ I_D = I_0 \left( e^{\frac{qV_A}{k_B T}} - 1 \right) \]

0.6 – 0.7 V
How large of a voltage can we apply?

\[ I_D = I_0 \left( e^{\frac{qV}{kT}} - 1 \right) \]

Reverse bias: breakdown

Forward bias: ?

e-band diagram under bias

\[ V_A < V_{bi} \]

\[ V_A > 0 \]
Question: one-sided?

Is this a one-sided diode? Yes: \( N_D >> N_A \)

We only need to consider the electrons injected into the P-region.
question (long or short)

\[ I_0 = qA \left( \frac{D_n n_i^2}{L_n N_A} \right) \]  

\[ I_0 = qA \left( \frac{D_n n_i^2}{W_p N_A} \right) \]

\[ \tau_n = 10 \mu s \]
\[ L_n = \sqrt{D_n \tau_n} \]
\[ L_n = 144 \mu m \]

example

\[ N_A = 10^{17} \text{ cm}^{-3} \]
\[ \mu_n = 800 \text{ cm}^2/\text{V-s} \]
\[ D_n = \frac{k_B T}{\mu_n q} \]
\[ D_n = 21 \text{ cm}^2/\text{s} \]

\[ W_p = 50 \mu m \]
\[ L_n > W_p \]

\[ I_0 = qA \left( \frac{D_n n_i^2}{L_n N_A} \right) \]

\[ I_0 = qA \left( \frac{D_n n_i^2}{W_p N_A} \right) \]

\[ \tau_n = 10 \mu s \]
\[ L_n = \sqrt{D_n \tau_n} \]
\[ L_n = 144 \mu m \]

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What is the diode current for a forward bias of 0.6 V?

\[
I_D = I_0 \left( e^{\frac{V_F}{kT}} - 1 \right)
\]

\[
I_0 = qA \left( \frac{D_p}{W_P} \frac{n_i^2}{N_A} \right)
\]

\[
A = 1 \text{ cm}^2
\]

\[
I_0 = 1.6 \times 10^{-19} \times 1 \times \left( \frac{21}{50 \times 10^{-4}} \times \frac{10^{20}}{10^{17}} \right)
\]

\[
= 0.7 \times 10^{-12} \text{ A}
\]

\[
I_D = 0.7 \times 10^{-12} \left( e^{0.6/0.026} - 1 \right)
\]

\[
I_D = 7 \text{ mA}
\]
Solar cells

modern Si solar cell

Chapin, Pearson, Fuller, 1954

http://www.bell-labs.com/org/physicalsciences/timeline/span10.html#

solar cells today

SunPower http://us.sunpower.com
Every time a minority electron **recombines** on the p-side, one electron flows in the external current.

Every time a minority electron is generated and collected by the PN junction, one electron flows in the external current.
light and dark current

\[ J_D = J_0 \left( e^{\frac{V_A}{k_B T}} - 1 \right) \]

\[ J_L < 0 \]

solar cell operation

1) Light generates e-h pairs

\[ E_F \]
solar cell operation

2) PN junction collects e-h pairs

3) Current flows through load

$I_L < 0$

forward bias across PN junction develops

4) Forward bias reduces current

5) IV characteristic is a superposition

$I_{TOT} = I_D \left( e^{qV_D/kT} - 1 \right) - I_{SC}$

diode (dark) current

light-generated current

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IV characteristic

IV = \frac{P_{out}}{P_{in}} = \frac{I_{SC} V_{OC} FF}{P_{in}}

\eta = \frac{P_{out}}{P_{in}} = \frac{I_{SC} V_{OC} FF}{P_{in}}

1) Short circuit current
2) Open-circuit voltage
3) Fill factor