

**SOLUTIONS: ECE 305 Homework: Week 3**

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- 1) Assume Silicon (bandgap 1.12 eV) at room temperature (300 K) with the Fermi level located **exactly** in the middle of the bandgap. Answer the following questions.
- a) What is the probability that a state located at the bottom of the conduction band is **filled**?
- b) What is the probability that a state located at the top of the valence band is **empty**?

**Solution:**

1a) Begin with the Fermi function:  $f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}$

For our problem:  $E = E_C$

$$f(E_C) = \frac{1}{1 + e^{(E_C - E_F)/k_B T}} \approx e^{-(E_C - E_F)/k_B T}$$

The approximation is extremely good, because the bottom of the conduction band is very far above the Fermi level. The semiconductor is said to be **nondegenerate**.

$$E_F = \frac{E_C + E_V}{2} \quad (\text{The Fermi level is exactly in the middle of the bandgap.})$$

$$E_C - E_F = E_C - \frac{E_C + E_V}{2} = \frac{E_C - E_V}{2} = \frac{E_G}{2}$$

so the probability is

$$f(E_C) \approx e^{-(E_C - E_F)/k_B T} = e^{-E_G/2k_B T}$$

$$E_G = 1.12 \text{ eV} \quad k_B T = 0.026 \text{ eV} \quad E_G/2k_B T = 21.5$$

$$\boxed{f(E_C) = e^{-E_G/2k_B T} = e^{-21.5} = 4.43 \times 10^{-10}}$$

What if we had not made the non-degenerate approximation? Then

$$f(E_C) = \frac{1}{1 + e^{(E_C - E_F)/k_B T}} = \frac{1}{1 + e^{E_G/2k_B T}} = \frac{1}{1 + 2.26 \times 10^9} = 4.43 \times 10^{-10}$$

Same answer as with the non-degenerate assumption because this semiconductor is non-degenerate.

**HW3 Solutions (continued):**

1b) Begin again with the Fermi function:  $f(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}$

For this part:  $E = E_V$

$$f(E_V) = \frac{1}{1 + e^{(E_V - E_F)/k_B T}}$$

The Fermi function gives the probability that a state is **full**. We want the probability that the state is empty, which is

$$1 - f(E_V) = 1 - \frac{1}{1 + e^{(E_V - E_F)/k_B T}} = \frac{e^{(E_V - E_F)/k_B T}}{1 + e^{(E_V - E_F)/k_B T}} = \frac{1}{e^{-(E_V - E_F)/k_B T} + 1}$$

$$1 - f(E_V) = \frac{1}{1 + e^{(E_F - E_V)/k_B T}} \approx e^{-(E_F - E_V)/k_B T}$$

The approximation is extremely good, because the Fermi level is very far above the top of the valence band. The **nondegenerate** approximation can be used for the valence band too.

$$E_F = \frac{E_C + E_V}{2} \quad (\text{The Fermi level is exactly in the middle of the bandgap.})$$

$$E_F - E_V = \frac{E_C + E_V}{2} - E_V = \frac{E_C - E_V}{2} = \frac{E_G}{2}$$

so the probability is

$$1 - f(E_V) = e^{-(E_F - E_V)/k_B T} = e^{-E_G/2k_B T}$$

$$1 - f(E_V) = e^{-E_G/2k_B T} = 4.43 \times 10^{-10}$$

As expected, when the Fermi level is exactly in the middle of the bandgap, the probability that a state at the conduction band edge is occupied is exactly the same as the probability that a state at the valence band edge is empty.

- 2) For Si at room temperature, calculate the following quantities. (Numerical answers required, and don't forget to include units with your answers).
- The density of states in the conduction band,  $g_c(E)$ , at an energy 26 meV above  $E_C$ .
  - The density of states in the valence band,  $g_v(E)$ , at an energy 26 meV below  $E_V$ .
  - The effective density of conduction band states,  $N_C$ .
  - The effective density of valence band states,  $N_V$ .
  - Compute the ratio of the effective density of conduction band states to the atomic density of Si.

**HW3 Solutions (continued):****Solution:**

2a) Begin with the DOS expression (SDF, eqn.(2.6a), p. 41)

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^* (E - E_C)}}{\pi^2 \hbar^3} \quad E > E_C$$

$$g_c(E) = \frac{m_0 \sqrt{2m_0} \left(\frac{m_n^*}{m_0}\right)^{3/2}}{\pi^2 \hbar^3} \sqrt{E - E_C}$$

$$g_c(E) = \frac{9.11 \times 10^{-31} \sqrt{2 \times 9.11 \times 10^{-31}} \left(\frac{m_n^*}{m_0}\right)^{3/2}}{3.14^2 \times (1.06 \times 10^{-34})^3} \sqrt{E - E_C} \quad (\text{everything in MKS (SI) units})$$

$$g_c(E) = 1.05 \times 10^{56} \left(\frac{m_n^*}{m_0}\right)^{3/2} \sqrt{E - E_C}$$

According to SDF, p. 34,  $m_n^*/m_0 = 1.18$

$$g_c(E) = 1.05 \times 10^{56} (1.18)^{3/2} \sqrt{E - E_C} = 1.35 \times 10^{56} \sqrt{E - E_C} \text{ (J-m}^3\text{)}^{-1}$$

Note the units. This is the number of states per unit energy (in Joules) per unit volume (in cubic meters).

We are asked for the density of states 26 meV (milli electron volts) above the bottom of the conduction band. In Joules:

$$E - E_C = 0.026 \times 1.60 \times 10^{-19} = 4.16 \times 10^{-21} \text{ J}$$

$$g_c(0.026 \text{ eV} + E_C) = 1.35 \times 10^{56} \sqrt{4.16 \times 10^{-21}} \frac{1}{\text{J-m}^3} = 8.71 \times 10^{45} \frac{1}{\text{J-m}^3}$$

The answer is correct, but the units are inconvenient. Let's express the answer per eV per cubic cm, which is the more typical way of doing things for semiconductors.

$$g_c(0.026 \text{ eV} + E_C) = 8.71 \times 10^{45} \frac{1}{\text{J}} \times \frac{1.6 \times 10^{-19} \text{ J}}{\text{eV}} \frac{1}{\text{m}^3} \times \left(\frac{10^{-2} \text{ m}}{\text{cm}}\right)^3$$

$$\boxed{g_c(0.026 \text{ eV} + E_C) = 1.39 \times 10^{21} \frac{1}{\text{eV-cm}^3}}$$

**HW3 Solutions (continued):**

Note: The total number of states in a range of energy of  $k_B T = 0.026$  eV is approximately

$$g_C(0.026 \text{ eV} + E_C) \times 0.026 = 3.62 \times 10^{19} \frac{1}{\text{cm}^3},$$

which is a reasonable number, as we shall see below. This answer is approximate, because to get the total number of states in the bottom  $k_B T$  of the conduction band, we should do the integral

$$N_{TOT} = \int_{E_C}^{E_C + k_B T} g_C(E) dE .$$

**A note about units.** Calculations should be done in the MKS system of units (also called SI). All of the fundamental constants (e.g. Plank's constant, Boltzmann's constant, rest mass of an electron, etc.) are in these units. After we have an answer, it is often convenient to convert to different units. For example, in semiconductor work, we like to quote carrier and doping densities per cubic cm, not per cubic meter, which is the proper MKS unit.

**Be careful about units!**

2b) Begin with the DOS expression (SDF, eqn.(2.6b), p. 41)

$$g_V(E) = \frac{m_p^* \sqrt{2m_p^* (E_V - E)}}{\pi^2 \hbar^3} \quad E < E_V$$

$$g_V(E) = \frac{m_0 \sqrt{2m_0} \left(\frac{m_p^*}{m_0}\right)^{3/2}}{\pi^2 \hbar^3} \sqrt{E_V - E}$$

$$g_V(E) = \frac{9.11 \times 10^{-31} \sqrt{2 \times 9.11 \times 10^{-31}} \left(\frac{m_p^*}{m_0}\right)^{3/2}}{3.14^2 \times (1.06 \times 10^{-34})^3} \sqrt{E_V - E} \quad (\text{everything in MKS (SI) units})$$

$$g_V(E) = 1.05 \times 10^{56} \left(\frac{m_p^*}{m_0}\right)^{3/2} \sqrt{E_V - E}$$

According to SDF, p. 34,  $m_p^*/m_0 = 0.81$

**HW3 Solutions (continued):**

$$g_v(E) = 1.05 \times 10^{56} (0.81)^{3/2} \sqrt{E_v - E} = 7.65 \times 10^{55} \sqrt{E_v - E} (\text{J-m}^3)^{-1}$$

Note that the DOS in the valence band is a little smaller than in the conduction band because the hole effective mass is smaller than the electron effective mass.

We are asked for the density of states 26 meV (milli-electron volts) below the top of the valence band. In Joules

$$E_v - E = 0.026 \times 1.60 \times 10^{-19} = 4.16 \times 10^{-21} \text{ J}$$

$$g_c(E_v - 0.026 \text{ eV}) = 7.65 \times 10^{55} \sqrt{4.16 \times 10^{-21}} \frac{1}{\text{J-m}^3} = 4.94 \times 10^{45} \frac{1}{\text{J-m}^3}$$

The answer is correct, but the units are inconvenient. Let express the answer per eV per cubic cm, which is the more typical way of doing things for semiconductors.

$$g_v(E_v - 0.026 \text{ eV}) = 4.94 \times 10^{45} \frac{1}{\text{J}} \times \frac{1.6 \times 10^{-19} \text{ J}}{\text{eV}} \frac{1}{\text{m}^3} \times \left( \frac{10^{-2} \text{ m}}{\text{cm}} \right)^3$$

$$\boxed{g_v(E_v - 0.026 \text{ eV}) = 7.90 \times 10^{20} \frac{1}{\text{eV-cm}^3}}$$

Note: The total number of states in a range of energy of  $k_B T = 0.026 \text{ eV}$  is approximately

$$g_v(E_v - 0.026 \text{ eV}) \times 0.026 = 2.06 \times 10^{19} \frac{1}{\text{cm}^3}$$

which is a reasonable number, as we shall see below.

- 2c) This is a problem about the “effective density of states” which is different from the density-of-states. The effective density of states, is roughly the total number of states in an energy range of  $k_B T$  near the bottom of the conduction band or top of the valence band. It is derived by integration as discussed in the text. The result is given in SDF on p. 51

$$N_c = 2.510 \times 10^{19} \left( \frac{m_n^*}{m_0} \right)^{3/2} \frac{1}{\text{cm}^3}$$

Using  $m_n^* = 1.18 m_0$ , we find

**HW3 Solutions (continued):**

$$N_C = 2.510 \times 10^{19} (1.18)^{3/2} \text{ cm}^{-3} = 3.22 \times 10^{19} \text{ cm}^{-3}$$

$$\boxed{N_C = 3.22 \times 10^{19} \text{ cm}^{-3}}$$

Note that this is close to the value we estimated in part 2a):

$$g_c (0.026 \text{ eV} + E_C) \times 0.026 = 3.62 \times 10^{19} \frac{1}{\text{cm}^3}$$

So we can interpret the conduction band effective DOS as the number of states within about on  $k_B T$  of the bottom of the conduction band.

2d) Repeat 2c) but for the valence band.

$$N_V = 2.510 \times 10^{19} \left( \frac{m_p^*}{m_0} \right)^{3/2} \frac{1}{\text{cm}^3}$$

Using  $m_n^* = 0.81 m_0$ , we find

$$N_V = 2.510 \times 10^{19} (0.81)^{3/2} \text{ cm}^{-3} = 1.83 \times 10^{19} \text{ cm}^{-3}$$

$$\boxed{N_V = 1.83 \times 10^{19} \text{ cm}^{-3}}$$

Note that this is close to the value we estimated in part 2b):

$$g_v (E_V - 0.026 \text{ eV}) \times 0.026 = 2.06 \times 10^{19} \frac{1}{\text{cm}^3}$$

2e) Recall the atomic density of Si is  $N_{Si} = 5.00 \times 10^{22} \text{ cm}^{-3}$  so the ratio

$$\frac{N_C}{N_{Si}} = \frac{3.22 \times 10^{19}}{5.00 \times 10^{22}} = 6.44 \times 10^{-4}$$

$$\boxed{\frac{N_C}{N_{Si}} = 0.06 \%}$$

The number of states near the bottom of the conduction band that can be occupied is a small fraction of the density of silicon atoms.

**HW3 Solutions (continued):**

3) Consider a region of Si at room temperature. For each of the following cases, calculate the equilibrium electron and hole concentrations ( $n$  and  $p$ ). Assume that the dopants are fully ionized.

- a) Intrinsic material ( $N_D = N_A = 0$ )
- b)  $N_D = 1.00 \times 10^{13} \text{ cm}^{-3}$   $N_A = 0$
- c)  $N_D = 1.00 \times 10^{17} \text{ cm}^{-3}$   $N_A = 0$
- d)  $N_D = 0$   $N_A = 1.00 \times 10^{17} \text{ cm}^{-3}$
- e)  $N_D = 1.00 \times 10^{17} \text{ cm}^{-3}$   $N_A = 3.00 \times 10^{17} \text{ cm}^{-3}$

**Solution:**

3a) For intrinsic material, we know that  $n = p = n_i$ . According to SDF, p. 54,

$$n_i = 1.00 \times 10^{10} \text{ cm}^{-3} \text{ so}$$

$$\boxed{n = p = n_i = 1.00 \times 10^{10} \text{ cm}^{-3}}$$

3b) First, let's ask what to expect. The donor density is 1000 times the intrinsic density. All of the donor electrons will go in the conduction band, overwhelming the number of intrinsic carriers that were there. So we expect

$$n \approx N_D = 1.00 \times 10^{13} \text{ cm}^{-3}$$

We would determine the hole density from  $np = n_i^2$ , so

$$p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{13}} = 1.00 \times 10^7 \text{ cm}^{-3}$$

Note that doping the semiconductor n-type means that we now have more electrons in the conduction band, and we have fewer holes in the valence band than for the intrinsic semiconductor.

How would we do this problem more accurately?

Begin by assuming that the semiconductor is neutral:

$$p - n + N_D^+ - N_A^- = 0$$

Assume that the dopants are fully ionized and use  $p = n_i^2/n$

$$\frac{n_i^2}{n} - n + N_D - N_A = 0$$

**HW3 Solutions (continued):**

Solve the quadratic equation for  $n$

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

Plug in numbers:

$$\begin{aligned} n &= \frac{10^{13} - 0}{2} + \sqrt{\left(\frac{10^{13}}{2}\right)^2 + 10^{20}} = 0.5 \times 10^{13} + \sqrt{0.25 \times 10^{26} + 10^{20}} \\ &= 0.5 \times 10^{13} + \sqrt{0.25 \times 10^{26} + 0.000001 \times 10^{26}} \end{aligned}$$

$$n = 0.5 \times 10^{13} + 0.5 \times 10^{13} = 1.0 \times 10^{13}$$

$$\boxed{n = 1.00 \times 10^{13} \text{ cm}^{-3}}$$

Just as expected. We find the hole density from

$$p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{13}} = 1.00 \times 10^7 \text{ cm}^{-3}$$

$$\boxed{p = 1.00 \times 10^7 \text{ cm}^{-3}}$$

- 3c) First, let's ask what to expect. The donor density is  $10^7$  times the intrinsic density. All of the donor electrons will go in the conduction band, overwhelming the number of intrinsic carriers that were there. So we expect

$$n \approx N_D = 1.00 \times 10^{17} \text{ cm}^{-3}$$

We would determine the hole density from  $np = n_i^2$ , so

$$p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{17}} = 1.00 \times 10^3 \text{ cm}^{-3}$$

Note that doping the semiconductor n-type means that we now have more electrons and we have fewer holes than the intrinsic semiconductor.

Now do this problem more accurately.

Use the quadratic equation for  $n$ :



**HW3 Solutions (continued):**

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

Plug in numbers:

$$\begin{aligned} n &= \frac{10^{17} - 0}{2} + \sqrt{\left(\frac{10^{17}}{2}\right)^2 + 10^{20}} = 0.5 \times 10^{17} + \sqrt{0.25 \times 10^{34} + 10^{20}} \\ &= 0.5 \times 10^{17} + \sqrt{0.25 \times 10^{34} + 0} \end{aligned}$$

$$n = 0.50 \times 10^{17} + 0.50 \times 10^{17} = 1.00 \times 10^{17}$$

$$\boxed{n = 1.00 \times 10^{17} \text{ cm}^{-3}}$$

Just as expected. Next, we find the hole density from

$$p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{17}} = 1.00 \times 10^3 \text{ cm}^{-3}$$

$$\boxed{p = 1.00 \times 10^3 \text{ cm}^{-3}}$$

- 3d) Again, let's ask what to expect. The acceptor density is  $10^7$  times the intrinsic density. All of the acceptor holes will go in the valence band, overwhelming the number of intrinsic carriers that were there. So we expect

$$p \approx N_A = 1.00 \times 10^{17} \text{ cm}^{-3}$$

We would determine the electron density from  $np = n_i^2$ , so

$$n = \frac{n_i^2}{p} \approx \frac{n_i^2}{N_A} = \frac{10^{20}}{10^{17}} = 1.00 \times 10^3 \text{ cm}^{-3}$$

Note that doping the semiconductor p-type means that we now have more holes in the valence band, and we have fewer electrons in the conduction band than the intrinsic semiconductor.

Now do this problem more accurately.

Use the quadratic equation for  $p$ :

**HW3 Solutions (continued):**

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

Plug in numbers:

$$p = \frac{10^{17} - 0}{2} + \sqrt{\left(\frac{10^{17}}{2}\right)^2 + 10^{20}} = 0.5 \times 10^{17} + \sqrt{0.25 \times 10^{34} + 10^{20}}$$

$$= 0.5 \times 10^{17} + \sqrt{0.25 \times 10^{34} + 0}$$

$$p = 0.50 \times 10^{17} + 0.50 \times 10^{17} = 1.00 \times 10^{17}$$

$$p = 1.00 \times 10^{17} \text{ cm}^{-3}$$

Just as expected. We find the electron density from

$$n = \frac{n_i^2}{p} \approx \frac{n_i^2}{N_A} = \frac{10^{20}}{10^{17}} = 1.00 \times 10^3 \text{ cm}^{-3}$$

$$n = 1.00 \times 10^3 \text{ cm}^{-3}$$

- 3e) Since the net p-type doping is  $N_A - N_D = 3 \times 10^{17} - 1 \times 10^{17} = 2 \times 10^{17}$  is much greater than the intrinsic density, we expect:  $p = 2 \times 10^{17}$  and  $n = n_i^2 / p = 0.5 \times 10^3 \text{ cm}^{-3}$ . This is correct, as we can see by solving the problem properly.

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2} = \frac{3 \times 10^{17} - 1 \times 10^{17}}{2} + \sqrt{\left(\frac{2 \times 10^{17}}{2}\right)^2 + 10^{20}}$$

$$p = 1 \times 10^{17} + \sqrt{2 \times 10^{34} + 10^{20}} = 2 \times 10^{17}$$

$$p = 2 \times 10^{17} \text{ cm}^{-3}$$

$$n = \frac{n_i^2}{p} \approx \frac{n_i^2}{2 \times 10^{17}} = \frac{10^{20}}{2 \times 10^{17}} = 5.00 \times 10^2 \text{ cm}^{-3}$$

$$n = 5.00 \times 10^2 \text{ cm}^{-3}$$

**HW3 Solutions (continued):**

One might conclude from these examples, that there is no need to use space-charge neutrality and the quadratic equation. That would be wrong, as seen in the next problem. It only happened in this problem because in each case (except for case a)), the net doping was much larger than the intrinsic carrier density.

- 4) Assuming silicon with completely ionized dopants, compute  $n$  and  $p$  for the following case.

$$N_D = 5.00 \times 10^{16} \text{ cm}^{-3}$$

$$N_A = 0$$

$$T = 700 \text{ K}$$

**Solution:**

We expect the intrinsic carrier density to be MUCH larger at higher temperatures. Using the information on p. 57 of SDF, we have

$$n_i(700 \text{ K}) = 2.865 \times 10^{16} \text{ cm}^{-3}$$

This is close to the doping density, so we need to use the quadratic equation.

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$n = 2.5 \times 10^{16} + \sqrt{(2.5 \times 10^{16})^2 + (2.865 \times 10^{16})^2}$$

$$n = 2.5 \times 10^{16} + \sqrt{6.25 \times 10^{32} + 8.21 \times 10^{32}} = 6.3 \times 10^{16}$$

$$\boxed{n = 6.3 \times 10^{16} \text{ cm}^{-3}}$$

$$p = \frac{n_i^2}{n} = \frac{(2.865 \times 10^{16})^2}{6.3 \times 10^{16}} = \frac{8.21 \times 10^{32}}{6.3 \times 10^{16}} = 1.30 \times 10^{16} \text{ cm}^{-3}$$

$$\boxed{p = 1.30 \times 10^{16} \text{ cm}^{-3}}$$

**HW3 Solutions (continued):**

5) This problem asks you to compute the location of the Fermi level.

- 5a) For each of the cases (a-e) in Prob. 3, calculate the Fermi level position, with respect to the **intrinsic level** ( $E_F - E_i$ ). Note that you need to consider sign.

**Solution:**

In problem 3), we computed  $n$  and  $p$ . For this problem, we need an expression that relates  $n$  and  $p$  to the Fermi level and intrinsic level. The expressions are on p. 53 of SDF:

$$n = n_i e^{(E_F - E_i)/k_B T}$$

$$p = n_i e^{(E_i - E_F)/k_B T}$$

Solving for  $(E_F - E_i)$  from the first equation, we have

$$(E_F - E_i) = k_B T \ln\left(\frac{n}{n_i}\right)$$

or in electron volts

$$\frac{(E_F - E_i)}{q} = \frac{k_B T}{q} \ln\left(\frac{n}{n_i}\right) = 0.026 \ln\left(\frac{n}{10^{10}}\right)$$

For the case of 3a)  $n = p = n_i = 1.00 \times 10^{10} \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln(1) = 0$$

$$\boxed{\frac{(E_F - E_i)}{q} = 0} \text{ or we could say: } (E_F - E_i) = 0 \text{ eV}$$

For the case of 3b)  $n = 1.00 \times 10^{13} \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln\left(\frac{10^{13}}{10^{10}}\right) = 0.180$$

$$\boxed{\frac{(E_F - E_i)}{q} = 0.180}$$

**HW3 Solutions (continued):**

For the case of 3c)  $n = 1.00 \times 10^{17} \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln\left(\frac{10^{17}}{10^{10}}\right) = 0.026 \ln(10^7) = 0.419$$

$$\boxed{\frac{(E_F - E_i)}{q} = 0.419}$$

Note: The larger the n-type doping, the higher the Fermi level is above the intrinsic level.

For the case of 3d)  $n = 1.00 \times 10^3 \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln\left(\frac{10^3}{10^{10}}\right) = 0.026 \ln(10^{-7}) = -0.419$$

$$\boxed{\frac{(E_F - E_i)}{q} = -0.419}$$

Note the sign – the Fermi level is now **below the intrinsic level** because this is a p-type semiconductor.

For the case of 3e)  $n = 5.00 \times 10^2 \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = 0.026 \ln\left(\frac{5 \times 10^2}{10^{10}}\right) = 0.026 \ln(5 \times 10^{-8}) = -0.437$$

$$\boxed{\frac{(E_F - E_i)}{q} = -0.437}$$

Also a p-type semiconductor, so the Fermi level is below the intrinsic level.

**HW3 Solutions (continued):**

- 5b) For each of the cases (a-e) in Prob. 3, calculate the Fermi level position, with respect to the **bottom of the conduction band** ( $E_F - E_C$ ). Note that you need to consider sign.

**Solution:**

This is basically the same problem as 5a) – we just want to know the position of the Fermi level with respect to another energy, bottom of the conduction band instead of the intrinsic level.

To solve this problem, we need an expression that relates  $n$  to the Fermi level and bottom of the conduction band. The expression is on p. 52 of SDF:

$$n = N_C e^{(E_F - E_C)/k_B T}$$

Solving for  $(E_F - E_C)$  we have

$$(E_F - E_C) = k_B T \ln\left(\frac{n}{N_C}\right)$$

or in electron volts

$$\frac{(E_F - E_C)}{q} = \frac{k_B T}{q} \ln\left(\frac{n}{N_C}\right) = 0.026 \ln\left(\frac{n}{N_C}\right)$$

We computed  $N_C$  in prob. 2c:  $N_C = 3.22 \times 10^{19} \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln\left(\frac{n}{3.22 \times 10^{19}}\right)$$

Note that this quantity will be negative for a non-degenerate semiconductor, because the Fermi level is always well below the conduction band.

For case 3a)  $n = p = n_i = 1.00 \times 10^{10} \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln\left(\frac{10^{10}}{3.22 \times 10^{19}}\right) = -0.569$$

$$\boxed{\frac{(E_F - E_C)}{q} = -0.569} \text{ or we could say: } (E_F - E_C) = -0.569 \text{ eV}$$

**HW3 Solutions (continued):**

For the case of 3b)  $n = 1.00 \times 10^{13} \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln \left( \frac{10^{13}}{3.22 \times 10^{19}} \right) = -0.390$$

$$\boxed{\frac{(E_F - E_C)}{q} = -0.390}$$

For the case of 3c)  $n = 1.00 \times 10^{17} \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln \left( \frac{10^{17}}{3.22 \times 10^{19}} \right) = -0.150$$

$$\boxed{\frac{(E_F - E_C)}{q} = -0.150}$$

Note: The larger the n-type doping, the closer the Fermi level is to the conduction band.

For the case of 3d)  $n = 1.00 \times 10^3 \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln \left( \frac{10^3}{3.22 \times 10^{19}} \right) = -0.988$$

$$\boxed{\frac{(E_F - E_C)}{q} = -0.988}$$

Note: This value is getting close to the bandgap of Si. The Fermi level is getting close to the top of the valence band.

For the case of 3e)  $n = 5.00 \times 10^2 \text{ cm}^{-3}$

$$\frac{(E_F - E_C)}{q} = 0.026 \ln \left( \frac{5 \times 10^2}{3.22 \times 10^{19}} \right) = -1.01$$

$$\boxed{\frac{(E_F - E_C)}{q} = -1.01}$$

Also a p-type semiconductor, so the Fermi level is close to the top of the valence band.

- 5c) For the case of problem 4), calculate the Fermi level position, with respect to the **intrinsic level** ( $E_F - E_i$ ). Note that you need to consider sign.

**Solution:**

The electron density was  $n = 6.3 \times 10^{16} \text{ cm}^{-3}$

and the intrinsic carrier concentration was  $n_i(700 \text{ K}) = 2.865 \times 10^{16} \text{ cm}^{-3}$

$$\frac{(E_F - E_i)}{q} = \frac{k_B T}{q} \ln \left( \frac{n}{n_i} \right)$$

**HW3 Solutions (continued):**

$$\text{at } T = 700 \text{ K: } \frac{k_B T}{q} = \frac{1.38 \times 10^{-23} \times 700}{1.6 \times 10^{-19}} = 0.0604 \text{ eV}$$

$$\frac{(E_F - E_i)}{q} = 0.060 \ln \left( \frac{6.3 \times 10^{16}}{2.865 \times 10^{16}} \right) = 0.048$$

$$\boxed{\frac{(E_F - E_i)}{q} = 0.048}$$

The Fermi level is almost in the middle of the gap – very close to the intrinsic level. This happens because  $n = 6.3 \times 10^{16} \text{ cm}^{-3}$  is only a little bigger than  $n_i(700 \text{ K})$ .