1) The figure below shows the electric field vs. position in an MS diode with the semiconductor doped at $1 \times 10^{16}$ cm$^{-3}$. The semiconductor is silicon at room temperature. It is in equilibrium and the depletion approximation is assumed. You may assume that the electron affinity is 4.0 eV. Answer the following questions.

1a) Is the semiconductor N-type or P-type? Explain how you know.
1b) What is the value of the Schottky barrier height?
1c) If the electron affinity is $\chi = 4.03$ eV, then what is the workfunction?
1d) Is the electrostatic potential of the bulk semiconductor greater or less than the electrostatic potential of the metal?
1e) Draw the energy band diagram of the isolated metal and semiconductor – i.e. before they were joined to form the MS diode. Be sure to show the vacuum level, the workfunction of the semiconductor, and other relevant quantities.

1a) Is the semiconductor N-type or P-type? Explain how you know.

**Solution:**

According to the Poisson equation, 
\[
\frac{dE}{dx} = \rho(x) \frac{1}{K_S \varepsilon_0}.
\]
We see from the figure that 
\[
\frac{dE}{dx} > 0 \quad \text{in the semiconductor. So the depleted charge must be positive, which means this is an N-type semiconductor.}
\]
HW9 Solutions (continued):

1b) What is the value of the Schottky barrier height?

Solution:
According to SDF, the built-in potential is \( V_{bi} = \frac{1}{q} \left[ \Phi_B - (E_c - E_F)_{FB} \right] \), where \( \Phi_B \) is the Schottky barrier height. From the figure, we can also determine the built-in potential from the area under the electric field vs. position:

\[
V_{bi} = \frac{1}{2} E(0)W = 0.5 \times 3.59 \times 10^4 \times 0.23 \times 10^{-4} = 0.41 \text{ V.}
\]

We also know that in the bulk, where the band are flat,

\[
n_0 = 10^{16} = N_c e^{(E_F - E_c)/k_BT} = 3.23 \times 10^{19} e^{(E_F - E_c)/k_BT}
\]

\[
\frac{(E_F - E_c)_{FB}}{q} = 0.026 \ln \left( \frac{10^{16}}{3.23 \times 10^{19}} \right) = -0.21 \text{ eV}
\]

Solving for the SB height:

\[
V_{bi} = \frac{1}{q} \left[ \Phi_B - (E_c - E_F)_{FB} \right] \Rightarrow \frac{\Phi_B}{q} = V_{bi} + (E_c - E_F)_{FB}
\]

\[
\frac{\Phi_B}{q} = V_{bi} + (E_c - E_F)_{FB} = 0.41 + 0.21 = 0.62 \text{ eV}
\]

\[
\frac{\Phi_B}{q} = 0.62 \text{ eV}
\]

1c) If the electron affinity is \( \chi = 4.03 \text{ eV} \), then what is the work function of the metal?

Solution:
By definition: \( \Phi_B = \Phi_M - \chi \Rightarrow \Phi_M = \Phi_B + \chi \)

\[
\frac{\Phi_M}{q} = 0.62 + 4.03 = 4.65 \text{ eV}
\]

\[
\frac{\Phi_M}{q} = 4.65 \text{ eV}
\]
HW9 Solutions (continued):

1d) Is the electrostatic potential of the bulk semiconductor greater or less than the electrostatic potential of the metal?

Solution:

The electric field points in the negative x direction. If would take work to push a positive test charge from \( x = 0 \) to \( x \gg W \), since with would be pushing against the force of the electric field. This means that the bulk semiconductor is more positive than the metal.

Another way to get this answer is to use the definition of electrostatic potential.

\[
E_x(x) = \frac{dV}{dx} 
\int_{0}^{x \gg W} dV = V(x \gg W) - V(0) = -\int_{0}^{x \gg W} E(x) dx
\]

Since the electric field is negative everywhere, the integral will give a positive number, so \( V(x \gg W) > V(0) \).

1e) Draw the energy band diagram of the isolated metal and semiconductor – i.e. before they were joined to form the MS diode. Be sure to show the vacuum level, the workfunction of the semiconductor, and other relevant quantities.

Solution:

See the plot below.
HW9 Solutions (continued):

2) Assume typical Metal-Semiconductor and P+N diodes that are both described by 
\[ J_D = J_0 \left( e^{qV/k_BT} - 1 \right) \]. The only difference between these two diodes is the value of the
saturation current density, \( J_0 \). Compare the saturation current density of typical MS
and P+N diodes.

2a) Assume a Si P+N diode with \( N_D = 10^{17} \) cm\(^{-3} \) and \( N_A \gg N_D \). Assume that the N-region
is long and that the minority hole lifetime is \( \tau_n = 1 \mu\text{s} \). Using typical
parameter values for silicon at room temperature, compute \( J_0 \).

Solution:

\[
J_0 = q \frac{D_p}{n_i} \frac{n_i^2}{L_p N_D}
\]

from Fig. 3.5, p. 80 of SDF, \( \mu_p = 331 \) cm\(^2\)/V-s

\[
D_p = \frac{k_BT}{q} \frac{\mu_p}{0.026 \times 331} = 8.6 \text{ cm}^2/\text{s}
\]

\[
L_p = \sqrt{D_p \tau_p} = \sqrt{8.6 \times 10^{-6} \times 10^{-4}} = 29.3 \times 10^{-4} \text{ cm}
\]

\[
J_0 = q \frac{D_p}{L_p} \frac{n_i^2}{N_D} = 1.6 \times 10^{-19} \times \frac{8.6}{29.3 \times 10^{-4}} \times \frac{10^{20}}{10^{17}} = 0.47 \times 10^{-12} \text{ A/cm}^2
\]

\[ J_0 (\text{PN}) = 0.47 \times 10^{-12} \text{ A/cm}^2 \]

2b) Assume a M-Si diode with \( N_D = 10^{17} \) cm\(^{-3} \) and that the metal is gold (Au) with a
Schottky barrier height of \( \Phi_{Bn} = 0.25 \) eV. Assuming typical values for other Si
parameters silicon at room temperature, compute \( J_0 \).

Solution:

\[
J_0 = \frac{4\pi q m^*_{n} k_B^2}{\hbar^3} T^2 e^{-\Phi_{Bn}/k_BT}
\]

For simplicity, let’s assume that \( m^*_n = m_0 \). (A better value might be the so-called
conductivity effective mass, \( m^*_n = m^*_c = 0.26m_0 \), but assuming \( m^*_n = m_0 \) is close
enough for our purposes here.)
HW9 Solutions (continued):

\[
J_0 = \frac{4\pi q m^* k_B^2 k_B}{h^3} T^2 e^{-\Phi_B/k_B T} = \frac{4\pi q m^*}{h} \left( \frac{k_B}{h} \right)^2 T^2 e^{-0.25/0.026} \\
= 2.76 \times 10^{-15} \times \left( \frac{1.38 \times 10^{-23}}{6.626 \times 10^{-34}} \right)^2 \times (300)^2 \times 6.67 \times 10^{-5} = 7.19 \times 10^6 \text{ A/m}^2
\]

Note that we did the above calculation in MKS (SI) units, now convert so that we can compare to 1a).

\[
J_0 = 7.19 \times 10^6 \text{ A/m}^2 \times \left( \frac{10^{-2} \text{ m}}{\text{cm}} \right) = 7.19 \times 10^2 \text{ A/cm}^2
\]

\[
J_0 (\text{MS}) = 7.19 \times 10^2 \text{ A/cm}^2
\]

Note that \( J_0 (\text{MS}) \gg J_0 (\text{PN}) \)

Discussion:
If we were to determine the barrier height from the work function and electron affinity as discussed in the text, we would have obtained a MUCH different number. For gold, \( \Phi_M = 5.10 \text{ eV} \), and the electron affinity of Si is \( \chi = 4.03 \text{ eV} \). The barrier height is \( \Phi_{Bn} = \Phi_M - \chi = 5.10 - 4.03 = 1.07 \) - more that four times the value used in this problem. One difficulty in using work functions and electron affinities is that we end up subtracting two large numbers that each has some uncertainty. The resulting small value has a lot of error. Another problem is that work functions and electron affinities depend on crystallographic orientations and the condition of the surface. The actual metal-semiconductor may have a different interfacial chemistry and, therefore, different barrier height. One example is that defects at the interface may “pin” the Fermi level to a value quite different from that computed from the work function and the electron affinity. For that reason, the metal-semiconductor barrier height is taken to be the key parameter. It is carefully measured – usually by analyzing the IV characteristics of MS diodes. The work functions and electron affinities are useful, conceptual tools, but not always quantitatively reliable in practice.
HW9 Solutions (continued):

3) Answer the follow questions about a Metal-Silicon MS junctions. Assume a hypothetical metal with a workfunction of $\Phi_M = 4.60$ eV. Assume that the electron affinity of Si is $\chi = 4.03$ eV.

3a) Assume that the Si is N-type with $N_D = 10^{17}$ cm$^{-3}$ and draw an energy band diagram. Indicate the Schottky barrier height on your sketch.

**Solution:**

We begin with the energy band diagrams of the metal and the N-type semiconductor before they are connected.

The precise location of the Fermi level in the semiconductor would be determined by solving:

$$n_0 = N_D = N_C e^{(E_{FS} - E_F)/k_BT}$$

for

$$\frac{E_C - E_{FS}}{q} = \frac{k_BT}{q} \ln \left( \frac{N_C}{N_D} \right) \text{ eV}$$

We see that the Fermi level in the semiconductor is above the Fermi level in the metal. So, when brought together, electrons will flow from the semiconductor to the metal. The energy band diagram will have a constant Fermi level, and the conduction band will bend up near the junction to show electron depletion (the valence band must be parallel to the conduction band). Far away from the junction, deep in the semiconductor, it will look like it did before the two materials were connected.
HW 9 Solutions (continued):

3b) Assume that the Si is P-type with $N_A = 10^{17}$ cm$^{-3}$ and draw an energy band diagram. Indicate the Schottky barrier height on your sketch.

Solution:
We begin with the energy band diagrams of the metal and the P-type semiconductor before they are connected.

In this case, the Fermi levels in the separated metal and P-type semiconductors are close, so let’s figure out exactly which one is higher.

$$p_0 = N_A = N_v e^{(E_v - E_{FS})/k_BT}$$

$$\frac{E_{FS} - E_v}{q} = \frac{k_B T}{q} \ln \left( \frac{N_V}{N_A} \right) \text{ eV}$$

For Si $N_v = 1.83 \times 10^{19}$ cm$^{-3}$, so

$$\frac{E_{FS} - E_v}{q} = \frac{k_B T}{q} \ln \left( \frac{N_V}{N_A} \right) = 0.026 \ln \left( \frac{1.83 \times 10^{19}}{10^{17}} \right) \text{ eV} = 0.135 \text{ eV}$$
HW9 Solutions (continued):

Now let’s find out how far above or below the Fermi level in the semiconductor is from the Fermi level in the metal before the two are connected.

\[
E_{FM} - E_{FS} = -\left[ \Phi_M - \chi_{Si} - E_G + (E_{FS} - E_V) \right] = -\left[ 4.60 - 4.03 - 1.12 + 0.135 \right] = 0.415 \text{ eV}
\]

The Fermi level in the metal is above the Fermi level in the semiconductor, so electrons will flow from the metal to the semiconductor when the two are brought together. We conclude that bands in the semiconductor will bend down to show more electrons near the junction and fewer holes. The amount of the bandbending will be 0.415 eV.

\[ E_0 = \text{field-free vacuum level} \quad E_F = \text{local vacuum level} \]
\[ \Phi_M = 4.60 \text{ eV} \]
\[ \chi_{Si} = 4.03 \text{ eV} \]

3c) Explain why each of the above two examples is, or is not, expected to be a rectifying junction.

Solution:

In the first case, 2a), the semiconductor is depleted near the junction. There is a barrier to electron flow from the semiconductor to the metal that can be changed by an applied bias. A negative bias on the N-type semiconductor lowers the barrier and increases the current exponentially. A positive bias on the N-type semiconductor increase the barrier height, and lowers the current. This MS diode is rectifying.

In the second case, 2b), the semiconductor is also depleted near the junction. There is a barrier to hole flow from the semiconductor to the metal that can be changed by an applied bias. A positive bias on the P-type semiconductor lowers the barrier and increases the current exponentially. A negative bias on the P-type semiconductor increases the barrier height, and lowers the current. This MS diode is also rectifying.
HW9 Solutions (continued):

3d) Compute the Schottky barrier height for each of the two examples.

**Solution:**
For the n-type semiconductor:
\[ \Phi_{Bn} = E_C(0) - E_{FM} = \Phi_M - \chi_{Si} = 4.60 - 4.03 = 0.57 \text{ eV} \]

\[ \Phi_{Bn} = 0.57 \text{ eV} \]

Note from the figure in 2a), the barrier height for an N-type semiconductor is the difference between the conduction band in the semiconductor (at the junction) and the Fermi level in the metal. The positive value means that there is a barrier for electrons in the metal to be injected into the conduction band of the semiconductor.

For the p-type semiconductor:
\[ \Phi_{Bp} = E_V(0) - E_{FM} = -\left[ \Phi_M - \chi_{Si} - E_G \right] = -\left[ 4.60 - 4.03 - 1.12 \right] = 0.55 \text{ eV} \]

\[ \Phi_{Bp} = 0.55 \text{ eV} \]

Note from the figure in 2b), the barrier height for a P-type semiconductor is the difference between the valence band in the semiconductor (at the junction) and the Fermi level in the metal.

The Schottky barrier height is the barrier for majority carriers (in the semiconductor) to flow from the metal to the semiconductor.

Finally, note that even though the metal and semiconductor have not changed, the Schottky barrier height for N-type material is different than for P-type. They happen to be close in this example, but depending on the work function of the metal, they can be quite different.

See the discussion about Schottky barrier heights in prob. 1.
HW9 Solutions (continued):

4) The I-V characteristic of both MS and PN diodes are described by \( J_D = J_0 \left( e^{qV_D/k_BT} - 1 \right) \).

The only difference between these two diodes is the value of the saturation current density, \( J_0 \). For an MS diode, the saturation current density is

\[
J_0 = \frac{4\pi q m^* n_i^2}{h^3} T^2 e^{-\Phi_s/k_BT} = \mathcal{A} T^2 e^{-\Phi_s/k_BT}
\]

Re-write the saturation current for a P+N diode in this form, and use the results to explain why \( J_0 \) is much smaller for PN junctions than for MS junctions.

Solution:

The saturation current density of the P+N diode is

\[
J_0 = q \frac{D_p}{L_p} n_i^2 = q \sqrt{\frac{D_p}{\tau_p} \frac{N_C N_V e^{-E_G/k_BT}}{N_D}}
\]

The term, \( N_D \) is really the majority electron density in the N-type semiconductor:

\[
N_D = n_0 = N_C e^{(E_C - E_F)/k_BT}
\]

so the saturation current density can be written as

\[
J_0 = q \sqrt{\frac{D_p N_C N_V e^{-E_G/k_BT}}{\tau_p N_D}} = q \sqrt{\frac{k_BT \mu_n}{\tau_p}} \frac{N_C N_V e^{-(E_C - E_F)/k_BT}}{N_D}
\]

Recall the definition of the effective density-of-states:

\[
N_V = \frac{1}{4} \left( \frac{2 m_p^* k_B T}{\pi \hbar^2} \right)^{3/2} = \frac{1}{4} \left( \frac{8\pi m_p^* k_B T}{h} \right)^{3/2} = \frac{2\sqrt{2}\pi^{3/2}}{4} \left( \frac{m_p^* k_B T}{h} \right)^{3/2}
\]

so the saturation current density becomes:

\[
J_0 = \sqrt{\frac{q \mu_n}{\tau_p}} \frac{2\sqrt{2}\pi^{3/2}}{4} \left( \frac{m_p^* k_B T}{h} \right)^{3/2} \left( k_B T \right)^{1/2} e^{-\frac{E_F - E_F(\omega)}{k_BT}}
\]

Now recall how the mobility is related to the scattering time, \( \tau_m \):

\[
\mu_p = \frac{q \tau_m}{m_p}
\]

\[
J_0 = \sqrt{\frac{q \tau_m}{m_p \tau_p}} \frac{2\sqrt{2}\pi^{3/2}}{2} \left( \frac{m_p^* k_B T}{h} \right)^{3/2} \left( k_B T \right)^{1/2} e^{-\frac{E_F - E_F(\omega)}{k_BT}}
\]
HW9 Solutions (continued):

Now group some terms:

$$J_0 = \sqrt{\frac{\tau_m}{\tau_p}} \left( \frac{\sqrt{2\pi}^{3/2}}{8\pi} \right) \left( \frac{4\pi q m_p^* k_B^2}{h^3} \right) T^2 e^{-\frac{(E_F - E_V(\infty))}{k_B T}}$$

Finally, we can define some parameters:

The “effective Richardson constant” of the PN junction:

$$\mathcal{A}_{PN} = \sqrt{\frac{\tau_m}{\tau_p}} \left( \frac{\sqrt{2\pi}^{3/2}}{8\pi} \right) \left( \frac{4\pi q m_p^* k_B^2}{h^3} \right)$$

and the “barrier height” of the PN junction

$$\Phi_{PN} = \left( E_F - E_V(\infty) \right)$$

Then we can write the saturation current density of the PN junction as

$$J_0 = \mathcal{A}_{PN} T^2 e^{-\Phi_{PN}/k_B T}$$

All we have done is algebra, but now the saturation current density for a P+ N junction is written in the same form as the saturation current density of a MS diode. Typically, $J_0(MS) >> J_0(PN)$. Why?

Consider first the “barrier heights” of the two devices. Since the lightly doped side of the P+ N junction is N-type, the Fermi level is near the conduction band, the barrier height for the PN diode is close to the bandgap of the semiconductor. For MS diodes, barrier heights are typically much less than the bandgap. This difference in barrier heights makes the $J_0$ of a PN diode orders of magnitude smaller than the $J_0$ of a MS diode (as we saw in problem 1).

Consider next the Richardson constants of the two devices. The Richardson constant for the PN junction is two numerical factors times the Richardson constant of the MS diode. Consider first

$$N_1 = \left( \frac{\sqrt{2\pi}^{3/2}}{8\pi} \right) = 0.3$$

so this factor is on the order of one and does not make a big difference.

Consider next:

$$N_2 = \sqrt{\frac{\tau_m}{\tau_p}}$$

A typical scattering time that controls the mobility is a picosecond (or maybe a tenth of a ps). A typical lifetime for minority carrier recombination is a microsecond – or up to a thousand times longer in very high quality material.
HW9 Solutions (continued):

So, we conclude:

\[ N_2 = \sqrt{\frac{\tau_m}{\tau_p}} < \sqrt{\frac{10^{-12}}{10^{-6}}} = 10^{-3} \]

Physically, this occurs because in a PN junction, when a hole is injected into the semiconductor, current does not flow in the leads until the minority carrier hole recombines, which can be a quite long time. When an electron is injected from a semiconductor into the metal contact, an electron flows out the contact almost immediately (within a dielectric relaxation time). The difference between these two times leads to a large difference in the Richardson constants.

So, the reasons that \( J_0 \) in a typical PN diode is orders of magnitude smaller than \( J_0 \) in a typical MS diode are: 1) the barrier height of a PN junction is much larger than the typical MS barrier height, and 2) the Richardson constant of a PN junction is much smaller that that of a MS diode.

5) Assume a diode described by \( I_D = I_0 \left(e^{qV_A/nk_BT} - 1\right)\). Answer the following questions.

5a) Assume that the diode is forward biased moderately (i.e. \( V_A \) is large enough so that the -1 term can be neglected). Derive a general expression for the increase, \( \Delta V_A \) in \( V_A \) needed to increase the diode current by a factor of 10.

Solution:

\[ I_D = I_0 \left(e^{qV_A/nk_BT} - 1\right) \]
\[ 10I_D = I_0 \left(e^{q(V_A+\Delta V_A)/nk_BT} - 1\right) \approx I_0 e^{qV_A/nk_BT} e^{q\Delta V_A/nk_BT} = I_D e^{q\Delta V_A/nk_BT} \]
\[ e^{q\Delta V_A/nk_BT} = 10 \]
\[ \Delta V_A = \ln(10) \frac{nk_BT}{q} = 2.3 \frac{nk_BT}{q} \]
\[ \Delta V_A = 2.3nk_BT/q \] This is the voltage needed to increase the current by a factor of 10.
HW9 Solutions (continued):

5b) Evaluate $\Delta V_A$ assuming room temperature and $n = 1$. Express your answer in millivolts.

Solution:

$$\Delta V_A = 2.3 nk_BT/q = 2.3 \times 0.026 = 0.06$$

$\Delta V_A = 60 \text{ mV/decade}$

This is an important result. It occurs whenever current is due to injection over a barrier. It is a fundamental limitation for how fast the current can increase with voltage (when the physical process for current is injection over a barrier). The same 60 mV/decade limit applies to MOSFETs and bipolar transistors.