Lecture 7: Resistance: Ballistic to Diffusive

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\[ R_{1D} = \rho_{1D} L \quad \rho_{1D} = \frac{1}{n_e q \mu_n} \]

\[ R_{2D} = \rho_{2D} \frac{L}{W} \quad \rho_{2D} = \frac{1}{n_s q \mu_n} \]

\[ R_{3D} = \rho_{3D} \frac{L}{A} \quad \rho_{3D} = \frac{1}{n q \mu_n} \]
Landauer picture

1D, 2D, or 3D resistor

ideal contact

ideal contact

I

x

Current is positive when it flows into contact 2 (i.e. positive current flows in the -x direction).

\[ I = GV = \frac{2qa^2}{h} \left( \int T(E)M(E) \left( -\frac{\partial f}{\partial E} \right) dE \right) V \]

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driving “forces” for transport

\[ I = \frac{2qa}{h} \int T(E)M(E)(f_1 - f_2) dE \]

Differences in occupation, \( f \), produce current.

\[ (f_1 - f_2) = f_1 - \left( f_1 + \frac{\partial f}{\partial E} \Delta E_f \right) = -\frac{\partial f}{\partial E} \Delta E_f \]

Assumes \( T_{L1} = T_{L2} \).

but differences in temperature also produce differences in \( f \) and can, therefore, drive current (thermoelectric effects).

\[ \Delta E_f = -q \Delta V = -q \left( V_2 - V_1 \right) \]

\[ \Delta T_L = T_{L2} - T_{L1} \]

\( \leftarrow \) this week

(\( \text{next week} \))

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transport regimes

1) Ballistic: \( \lambda >> L, T \approx 1 \)
2) Diffusive: \( \lambda << L, T << 1 \)
3) Quasi-Ballistic: \( \lambda \approx L, T < 1 \) \( T(E) = \frac{\lambda(E)}{\lambda(E) + L} \)

outline

1) Review
2) 2D ballistic resistors
3) 2D diffusive resistors
4) Discussion
5) Summary
an isothermal 2D resistor

\[ G = \frac{1}{R} = \frac{I}{V} = \frac{2q^2}{h} \int T(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]

the ballistic conductance

\[ G = \frac{2q^2}{h} \int_{-\infty}^{\infty} T(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]

\[ T(E) = 1 \quad \text{(ballistic)} \]

\[ G_{\text{ball}} = \frac{2q^2}{h} M(E_F) \]

\[ f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T_L}} \]

\[ f_0(E) \approx k_B T_L \]

\[ \left( -\frac{\partial f_0}{\partial E} \right) \approx \delta(E_F) \]
$T = 0$ K ballistic conductance

\[ G_{\text{ball}} = \frac{2q^2}{h} M(E_F) \]

"quantized conductance"

\[ R_{\text{ball}} = \frac{1}{M(E_F)} \frac{h}{2q^2} = \frac{12.8 \text{k}\Omega}{M} \]

Resistance is independent of length, $L$.

If the number of modes is small, we can simply count them.

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**quantized conductance**


Conductance is **quantized** in units of $2q^2/h$ - it increases with $W$, but not linearly.

The conductance is finite as $L \rightarrow 0$. 
for wider resistors, $M(E) \sim W$

$$M(E) = g_v W \frac{2m^* (E_F - E_C)}{\pi \hbar}$$

depends on bandstructure!

$$\frac{\hbar^2 k_F^2}{2m^*} = (E_F - E_C)$$

How is $M$ related to the sheet carrier density?

$$M(E) = g_v \frac{W k_F}{\pi}$$

Sheet carrier density at $T_L = 0$ K

$$n_S = \frac{\pi k_F^2}{(2\pi)^2 / A} \times 2 \times g_v \times \frac{1}{A}$$

$$n_S = g_v \frac{k_F^2}{2\pi}$$
**M(E) and \( n_S \)**

\[
M(E_F) = g_v \frac{W k_F}{\pi}
\]

\[
n_S = g_v \frac{k_F^2}{2\pi}
\]

\[
M(E_F) = W \sqrt{\frac{2g_v n_S}{\pi}}
\]

depends only on \( n_S \)

---

**Example: the channel of an n-MOSFET**

\[
n_S = 10^{13} \text{ cm}^{-2}
\]

\[
W = 2L = 100 \text{ nm}
\]

\[
M(E_F) = ?
\]

for a rough estimate, assume \( T_L = 0 \) K

\[
M(E_F) = W \sqrt{\frac{2g_v n_S}{\pi}}
\]
quantum confinement in a Si inversion layer

\[ E_n = \frac{\hbar^2 n^2 \pi^2}{2m^*} \]

\( m_i^* = 0.9m_0 \)

\( m_y^* = 0.19m_0 \)

\[ m^* = ? \]

bandstructure effects on quantum confinement

unprimed ladder: \( m^* = m_t^* \quad g_V = 2 \)

primed ladder: \( m^* = m_t^* \quad g_V = 4 \)

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quasi-2D carriers in the plane

Si conduction band

confinement masses:
unprimed ladder: \( m^* = m_t \quad g_V = 2 \)
primed ladder: \( m^* = m_t^* \quad g_V = 4 \)

\[ D_{2D} = g_V \frac{m_{\text{DOS}}^*}{\pi h^2} \]

example: the channel of an n-MOSFET

\[ n_S = 10^{13} \text{ cm}^{-2} \]
\[ W = 2L = 100 \text{ nm} \]
\[ M(E_F) = W \frac{2g_f n_S}{\pi} \]

Assume \( T_L = 0 \text{ K} \) and that only the lowest subband is occupied.

\[ g_f = 2 \]
\[ M(E_F) = 36 \]
$T = 0 \text{ K ballistic conductance}$

$$G_{\text{ball}} = \frac{2q^2}{h} M(E_F)$$

“quantized conductance”

$$R_{\text{ball}} = \frac{1}{M(E_F)} \frac{h}{2q^2} = \frac{12.8 \text{k}\Omega}{M}$$

Resistance is independent of length, $L$.

If the number of modes is small, we can simply count them.

conductance in 2D ($T > 0\text{K}$)

$$G_{\text{ball}} = \frac{2q^2}{h} \left( \int M(E) \left(-\frac{\partial f_0}{\partial E}\right) dE \right)$$

$$M(E) = W_{E_F} \sqrt{\frac{2m(E - E_F)}{\pi \hbar}}$$

$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_BT}}$$

$$\left(-\frac{\partial f_0}{\partial E}\right) + \left(-\frac{\partial f_0}{\partial E_F}\right)$$
conductance in 2D \((T > 0K)\)

\[
G_{\text{ball}} = \frac{2q^2}{\hbar} \left( \frac{1}{1+e^{\eta_F}} \right) \left( \int M(E) f_0(E) dE \right)
\]

\[
f_0(\eta) = \frac{1}{1 + e^{\eta - \eta_F}}
\]

\[
M(E) = W g_y \frac{2m^* k_B T_L \eta}{\pi \hbar}
\]

\[
dE = k_B T_L d\eta
\]

\[
\frac{\partial}{\partial E_F} = k_B T_L \frac{\partial}{\partial \eta_F}
\]

\[
\frac{\partial}{\partial E_F} = \frac{d}{d\eta_F} \left( \frac{\partial}{\partial E_F} \right)
\]

\[
M(E) = W g_y \frac{2m^* (E - E_C)}{\pi \hbar}
\]

\[
f_0(E) = \frac{1}{1 + e^{E - E_C \eta_F}}
\]

\[
f_0(E) = \frac{1}{1 + e^{E - E_C \eta_F}}
\]

\[
\eta_F = \frac{E - E_C}{k_B T_L}
\]

\[
\eta_F = \frac{E - E_C}{k_B T_L}
\]

\[
f(\eta_F) = \frac{1}{1 + e^{\eta - \eta_F}}
\]

\[
\int f(\eta_F) d\eta_F = \frac{2}{\sqrt{\pi}} \int \frac{\eta^{1/2} d\eta}{1 + e^{\eta - \eta_F}}
\]

\[
\eta = \frac{E - E_C}{k_B T_L}
\]

\[
\eta_F = \frac{E - E_C}{k_B T_L}
\]

\[
\frac{dF_{1/2}}{d\eta_F} \to F_{1/2}
\]
conductance in 2D \((T > 0K)\)

\[ G_{\text{ball}} = \frac{2q^2}{h} W \langle M_{2D} \rangle \]

\(<\!M\!> \) is the no. of modes in the “Fermi window”

\[ \langle M_{2D} \rangle = \frac{\sqrt{2m^* k_B T_L}}{\pi h} \sqrt{\frac{\pi}{2}} F_{-\pi/2} (\eta_F) = M_{2D} (E - E_C = k_B T_L) \frac{\sqrt{\pi}}{2} F_{-\pi/2} (\eta_F) \]

\[ \eta_F = \frac{E_F - E_C}{k_B T_L} \]

\[ n_s = N_{2D} F_0 (\eta_F) = \left( g_y \frac{m^* k_B T_L}{\pi h^2} \right) F_0 (\eta_F) \]

\[ \langle M \rangle = \text{no. of modes in the “Fermi window”} \]

example: the channel of an n-MOSFET

\[ n_s = 10^{13} \text{ cm}^{-2} \]

\[ W = 2L = 100 \text{ nm} \]

\[ M(E_F) = W \sqrt{\frac{2g_y n_s}{\pi}} = 36 \left( T_L = 0K \right) \]

Repeat the calculation for \( T_L = 300 \text{ K} \) and find \(<\!M\!>\).
outline

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2) 2D ballistic resistors
3) **2D diffusive resistors**
4) Discussion
5) Summary

an isothermal 2D resistor

\[ G = \frac{2q^2}{h} \int T(E)M(E) \left( -\frac{\partial f}{\partial E} \right) dE \quad (T_{L1} = T_{L2}) \]
the \( T_L = 0 \) diffusive conductance

\[
G = \frac{2q^2}{h} \int_{-\infty}^{\infty} T(E)M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE
\]

\[
T(E) = \frac{\lambda(E)}{\lambda(E) + L}
\]

\[
f_0(E) = \frac{1}{1 + e^{(E-E_F)/\lambda L}}
\]

\[
\approx k_B T
\]

\[\left( \frac{\partial f_0}{\partial E} \right) = \delta(E_F)\]

ballistic to diffusive

\[
G = \frac{2q^2}{h} \int_{-\infty}^{\infty} T(E)M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE
\]

\[
T = \frac{\lambda(E_F)}{\lambda(E_F) + L}
\]

\[
G = \frac{\lambda(E_F)}{\lambda(E_F) + L} G_{\text{ball}}
\]

\[
R = \left( 1 + \frac{L}{\lambda(E_F)} \right) R_{\text{ball}}
\]
example: nanoscale MOSFETs

(Courtesy, Shuji Ikeda, ATDF, Dec. 2007)

Question: Do we expect this device to be closer to ballistic or to diffusive?

$L = 60 \text{ nm}$

$\mu_n = 260 \text{ cm}^2/\text{V-s}$

$n_s = 6.7 \times 10^{12} \text{ cm}^2$

$R_{CH} = 215 \Omega \cdot \mu\text{m}$
mfp in a Si MOSFET

\[ D_n = \frac{k_B T_L}{q} \mu_n \]

(near-equilibrium, MB statistics)

\[ D_n = \frac{\nu_T \lambda_0}{2} \]

(near-equilibrium, MB statistics, constant mfp)

\[ \lambda_0 = \frac{2 \left( \frac{k_B T_L}{q} \right)}{\nu_T} \mu_n \]

\[ \nu_T = \sqrt{\frac{2 k_B T_L}{\pi m}} = 1.2 \times 10^7 \text{ cm/s} \]

\[ L = 60 \text{ nm} \]
\[ \mu_n = 260 \text{ cm}^2/\text{V-s} \]
\[ n_S = 6.7 \times 10^{12} \text{ cm}^{-2} \]
\[ R_{\text{ch}} = 215 \Omega \cdot \mu\text{m} \]

\[ \lambda_0 \approx 11 \text{ nm} \ll L \text{ diffusive} \]

40 nm channel length III-V FETs

\[ \mu_n = 10,000 \text{ cm}^2/\text{V-s} \]

\[ D_n = \frac{\nu_T \lambda_0}{2} \]

\[ \nu_T = \sqrt{\frac{2 k_B T}{\pi m}} = 2.7 \times 10^7 \text{ cm/s} \]

\[ (m^* = 0.041 m_0) \]

\[ \lambda_0 \approx 200 \text{ nm} \]

\[ \lambda_0 \gg L \text{ ballistic} \]
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key points

1) Conductors display a finite conductance (resistance) - even in the absence of scattering.

2) The resistance is quantized, and the quantum of conductance is $2q^2/h$.

3) The ballistic resistance sets a lower limit to device resistance and is becoming important in practical, room temperature devices.

4) Transport from the ballistic to diffusive limit is easily treated by using the transmission.
2D resistor summary

\[ G = \frac{2q^2}{h} T \langle M \rangle = \frac{2q^2}{h} \frac{\lambda_0}{\lambda_0 + L} \langle M \rangle \]

\[ \langle M \rangle = \frac{\sqrt{\pi}}{2} W M_{2D} \left( E - E_C = k_B T_L \right) F_{-1/2} \left( \eta_F \right) \]

\[ M_{2D} \left( E - E_C = k_B T_L \right) = g_v \frac{\sqrt{2m^* k_B T_L}}{\pi h} \]

\[ \eta_F = \left( E_F - \epsilon_1 \right) / k_B T_L \]

(parabolic energy bands) (constant mfp)

\[ T_L = 0 \text{ K:} \]

\[ \langle M \rangle = M \left( E_F \right) \]

MB statistics:

\[ \langle M \rangle = W \frac{\hbar}{4} \frac{\eta}{k_B T_L} \]

\[ G = \frac{\lambda_0}{\lambda_0 + L} G_{\text{ball}} \]

\[ R = \left( 1 + \frac{\lambda_0}{L} \right) R_{\text{ball}} \]

questions

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3) 2D diffusive resistors
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