measurement of conductivity / resistivity

1) Commonly-used to characterize electronic materials.

2) Results can be clouded by several effects – e.g. contacts, thermoelectric effects, etc.

3) Measurements in the absence of a magnetic field are often combined with those in the presence of a B-field.

This lecture is a brief introduction to the measurement and characterization of near-equilibrium transport.
resistivity / conductivity measurements

\[ J_{nx} = \sigma_n \frac{d(F_n/q)}{dx} \]  
(diffusive transport assumed)

For uniform carrier concentrations:

\[ J_{nx} = \sigma_n E_x \quad E_x = \rho_n J_{nx} \]

We generally measure resistivity (or conductivity) because for diffusive samples, these parameters depend on material properties and not on the length of the resistor or its width or cross-sectional area.

Landauer conductance and conductivity

\[ G = \frac{2q^2}{h} \int M(E) T(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]

\[ \sigma = \frac{G}{A/L} = \frac{2q^2}{h} \int M_{30}(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]

(diffusive)

For ballistic or quasi-ballistic transport, replace the mfp with the "apparent" mfp:

\[ \frac{1}{\lambda_{app}(E)} = \frac{1}{\lambda(E)} + \frac{1}{L} \]
conductivity and mobility

1) Conductivity depends on $E_F$.
2) $E_F$ depends on carrier density.
3) So it is common to characterize the conductivity at a given carrier density.
4) Mobility is often the quantity that is quoted.

So we need techniques to measure two quantities:
1) conductivity  2) carrier density

$$\sigma = \frac{G}{A/L} = \frac{2q^2}{h} \left[ M_{3D}(E) \lambda(E) \left( -\frac{\partial f_n}{\partial E} \right) dE \right]$$

$\sigma_n = nq\mu_n$

2D: conductivity and sheet conductance

$$A = Wt$$

$$G = \sigma_n \frac{A}{L}$$

$$G = \sigma_n \left( \frac{Wt}{L} \right) = \sigma_n \left( \frac{W}{L} \right)$$

$$G = \sigma_s \left( \frac{W}{L} \right)$$

$$\sigma_s = n_s q \mu_n \left( \frac{1}{\Omega} \right)$$

“sheet conductance”
2D electrons vs. 3D electrons

A = Wt

\[ A = Wt \]

\[ n \text{-type semiconductor} \]

Top view

\[ W \]

\[ L \]

\[ t \]

3D electrons:

\[ G = \sigma \frac{A}{L} = \sigma \frac{Wt}{L} \rightarrow \sigma_s = \frac{G}{W/L} = \frac{2q^2}{h} \int t M_{3D}(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]

2D electrons:

\[ G = \sigma_s \frac{W}{L} \rightarrow \sigma_s = \frac{G}{W/L} = \frac{2q^2}{h} \int M_{2D}(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]

---

**mobility**

1) Measure the conductivity: \( \sigma_s \)

2) Measure the sheet carrier density: \( n_s \)

3) Deduce the mobility from: \( \sigma_s = n_s q \mu_n \)

4) Relate the mobility to material parameters:

\[ \sigma_s = \frac{2q^2}{h} \int M_{2D}(E) \lambda(E) \left( -\frac{\partial f_0}{\partial E} \right) \equiv n_s q \mu_n \]
There are three near-equilibrium transport coefficients: conductivity, Seebeck (and Peltier) coefficient, and the electronic thermal conductivity. We can measure all three, but in this brief lecture, we will just discuss the conductivity.

Conductivity depends on the location of the Fermi level, which can be set by controlling the carrier density.

So we need to discuss how to measure the conductivity (or resistivity) and the carrier density. Let’s discuss the resistivity first.

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**outline**

1. Introduction
2. Resistivity / conductivity measurements
3. Hall effect measurements
4. The van der Pauw method
5. Summary

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2-probe measurements

\[ V_{21} = I \left( 2R_c + R_{CH} \right) \]

\[ R_{CH} = \rho_s \frac{L}{W} \]

\[ R_{CH} \neq \frac{V_{21}}{I} \]

"transmission line measurements"

transmission line measurements (TLM)

contact resistance (vertical flow)
contact resistance (vertical flow)

\[ R_c = \frac{\rho_c}{A_c} = \frac{\rho_c}{A_c} \Omega \]

\[ 10^{-8} < \rho_c < 10^{-6} \Omega \cdot \text{cm}^2 \]

“interfacial contact resistivity”

\[ A_c = 0.10 \mu m \times 1.0 \mu m \]
\[ \rho_c = 10^{-7} \Omega \cdot \text{cm}^2 \]
\[ R_c = 100 \Omega \]

contact resistance (vertical + lateral flow)

\[ L_T \quad \text{“transfer length”} \quad L_T = \sqrt{\frac{\rho_c}{\rho_{SD}}} \text{ cm} \]

\[ A_c = WL_c \]
\[ (W \text{ into page}) \]

\[ A_c(\text{eff}) < WL_c \]
contact resistance

\[ L_T = \sqrt{\rho_c / \rho_{SD}} \text{ cm} \]

\[ R_C = \sqrt{\frac{\rho_c \rho_{SD}}{W}} \coth \left( \frac{L_C}{L_T} \right) \]

i) \( L_C << L_T \) : \( R_C = \frac{\rho_c}{L_C W} \)

ii) \( L_C >> L_T \) : \( R_C = \frac{\rho_c}{L_T W} \)

1) Slope gives sheet resistance, intercept gives contact resistance

2) Determine specific contact resistivity and transfer length:

\[ R_C = \sqrt{\frac{\rho_c \rho_{SD}}{W}} \coth \left( \frac{L_C}{L_T} \right) \quad L_T = \sqrt{\rho_c / \rho_{SD}} \text{ cm} \]
four probe measurements

1) force a current through probes 1 and 4
2) with a high impedance voltmeter, measure the voltage between probes 2 and 3

\[ R = \frac{V}{I} = f(\rho_s) \] (no series resistance)

Hall bar geometry

pattern created with photolithography

thin film isolated from substrate

\[ V_{21} = I \times \rho_s \frac{L}{W} \] (high impedance voltmeter)

Contacts 0 and 5: “current probes”
Contacts 1 and 2 (3 and 4): “voltage probes”
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Hall effect

The Hall effect was discovered by Edwin Hall in 1879 and is widely used to characterize electronic materials. It also finds use in magnetic field sensors.
Hall effect: analysis

\[ \mathbf{J}_n = n q \mu_n \mathbf{E} - (\sigma_n \mu_n r_H) \mathbf{E} \times \mathbf{B} \]

- **Hall concentration**

**Example**

\[ I = I_x = 1 \mu A \]

\[ B_z = 2,000 \text{ Gauss} \]

\[ (1 \text{ Tesla} = 10^4 \text{ Gauss}) \]

\[ L = 100 \mu m \]

\[ W = 50 \mu m \]

\[ B = 0: \]

\[ V_{24} = 0.4 \text{ mV} \]

\[ B \neq 0: \]

\[ V_{24} = 13 \mu V \]
example: resistivity

\[ B_z = 2,000 \text{ Gauss} \]
\[ 1 \text{ Tesla} = 10^4 \text{ Gauss} \]
\[ L = 100 \mu \text{m} \]
\[ W = 50 \mu \text{m} \]
\[ B = 0: \]
\[ V_{21} = 0.4 \text{ mV} \]
\[ B = 0.2T: \]
\[ V_{24} = 13 \mu \text{V} \]

\[ I = I_x = 1 \mu \text{A} \]

- \[ R_{xx} = \frac{V_{21}}{I} = 400 \Omega \]
- \[ R_{xx} = \rho_s \frac{L}{W} \rightarrow \rho_s = 200 \Omega/\square \]

example: sheet carrier density

\[ B_z = 2,000 \text{ Gauss} \]
\[ 1 \text{ Tesla} = 10^4 \text{ Gauss} \]
\[ L = 100 \mu \text{m} \]
\[ W = 50 \mu \text{m} \]
\[ B = 0: \]
\[ V_{21} = 0.4 \text{ mV} \]
\[ B = 0.2T: \]
\[ V_{24} = 13 \mu \text{V} \]

\[ I = I_x = 1 \mu \text{A} \]

- \[ n_H = \frac{n_s}{r_n} \]
- \[ n_H = \frac{I B_z}{q V_{21}} = \frac{I_s B_z}{q V_{24}} \]
- \[ n_H = 9.6 \times 10^{12} \text{ cm}^{-2} \]
example: mobility

\[ B_z = 2,000 \text{ Gauss} \]
\[ (1 \text{ Tesla} = 10^4 \text{ Gauss}) \]
\[ L = 100 \mu m \]
\[ W = 50 \mu m \]

\[ B = 0: \]
\[ V_{21} = 0.4 \text{ mV} \]
\[ B = 0.2T: \]
\[ V_{24} = 13 \mu V \]

mobility:

\[ \sigma_x = \frac{1}{\rho_x} = n_x q \mu_n = \left( \frac{n_x}{r_{H1}} \right) q (r_{H2} \mu_n) \]
\[ \mu_{H} = r_{H1} \mu_n = 3125 \text{ cm}^2/\text{V-s} \]
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van der Pauw sample

2D film
arbitrarily shaped
homogeneous, isotropic
(no holes)

Top view

Four small contacts
along the perimeter
van der Pauw approach

**Resistivity**

1) force a current in $M$ and out $N$
2) measure $V_{PO}$
3) $R_{MN, OP} = V_{PO} / I$ related to $\rho_S$

**Hall effect**

1) force a current in $M$ and out $O$
2) measure $V_{PN}$
3) $R_{MO, NP} = V_{PN} / I$ related to $V_H$

van der Pauw approach: Hall effect

**Hall effect**

\[
\begin{align*}
J_x &= \sigma \frac{E}{\rho} \cdot \left( \sigma \mu_n r_{\rho} \right) E_y B_z \\
J_y &= \sigma \frac{E}{\rho} \cdot \left( \sigma \mu_n r_{\rho} \right) E_x B_z \\
\vec{E}_x &= \rho_n J_x + \left( \rho_n \mu H B_z \right) J_y \\
\vec{E}_y &= -\left( \rho_n \mu H B_z \right) J_x + \rho_n J_y \\
V_{PN} \left( B_z \right) &= -\oint_{N} \vec{E} \cdot dl = -\oint_{N} \left[ \vec{E}_x dx + \vec{E}_y dy \right] \\
V_H &= \frac{1}{2} \left[ V_{PN}(+B_z) - V_{PN}(-B_z) \right]
\end{align*}
\]
van der Pauw approach: Hall effect

**Hall effect**

\[ \vec{J}_H = nq \mu E - (\sigma n \mu r_H) \vec{E} \times \vec{B} \]

So we can do Hall effect measurements on such samples.

For the missing steps, see Lundstrom, *Fundamentals of Carrier Transport*, 2nd Ed., Sec. 4.7.1.

van der Pauw approach: resistivity

**Resistivity**

\[ R_{MN,op} = \frac{V_{po}}{I} \]
van der Pauw approach: resistivity

\[ J_r = \frac{I}{\pi r} = \sigma_s \mathcal{E}_r \]

\[ \mathcal{E}_r = \frac{I \rho_s}{\pi r} \]

\[ V(r) - V(r_0) = -\frac{I \rho_s}{\pi} \ln \left( \frac{r}{r_0} \right) \]

\[ V(r) - V(r_0) = -\frac{I \rho_s}{\pi} \ln \left( \frac{r}{r_0} \right) \]

\[ V(P) = -\frac{I \rho_s}{\pi} \ln \left( \frac{a+b+c}{r_0} \right) \]

\[ V(O) = -\frac{I \rho_s}{\pi} \ln \left( \frac{a+b}{r_0} \right) \]

\[ V_{PO} = -\frac{I \rho_s}{\pi} \ln \left( \frac{a+b+c}{a+b} \right) \]

but there is also a contribution from contact \( N \)

\[ V'_{PO} = +\frac{I \rho_s}{\pi} \ln \left( \frac{b+c}{b} \right) \]
van der Pauw approach: resistivity

Given two measurements of resistance, this equation can be solved for the sheet resistance.

\[
R_{MN,OP} = \frac{V_{PO} + V'_{PO}}{I} = \frac{\rho_s}{\pi} \ln \left( \frac{(a+b)(b+c)}{b(a+b+c)} \right)
\]

\[
R_{NO,PM} = \frac{\rho_s}{\pi} \ln \left( \frac{(a+b)(b+c)}{ac} \right)
\]

\[
\frac{\pi}{e} \frac{\rho_{MN,OP}}{\rho_s} + e \frac{\pi}{\rho_s} \frac{\rho_{NO,PM}}{\rho_{NO,PM}} = 1
\]

The same equation applies for an arbitrarily shaped sample!
van der Pauw technique: regular sample

Force $I$ through two contacts, measure $V$ between the other two contacts.

\[ \frac{\pi}{\rho_s} \left( \frac{e}{\rho_s} R_{MN,OP} + e\rho_s R_{NO,PM} \right) = 1 \]

\[ R_{MN,OP} = R_{NQ,PM} = \frac{V}{I} \]

\[ \rho_s = \frac{\pi}{\ln 2} \frac{V}{I} \]

van der Pauw technique: summary

1) measure $n_H$

\[ V_H = \frac{1}{2} [V_{PN}(+B_z) - V_{PN}(-B_z)] = \frac{r_H B_z I}{q n_H} = \frac{B_z I}{q n_H} \]
van der Pauw technique: summary

\[ B = 0 \]

2) measure \( \rho_S \)

3) determine \( \mu_H \)

\[
\begin{align*}
\frac{1}{e\rho_S} &+ \frac{1}{e\rho_D} = 1 & \sigma_S = n_S q \mu_n = \frac{n_S q r_H \mu_n}{r_H} = n_H q \mu_H
\end{align*}
\]

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summary

1) Hall bar or van der Pauw geometries allow measurement of both resistivity and Hall concentration from which the Hall mobility can be deduced.

2) Temperature-dependent measurements (to be discussed in the next lecture) provide information about the dominant scattering mechanisms.

3) Care must be taken to exclude thermoelectric effects (also to be discussed in the next lecture).

for more about low-field measurements


questions

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