NAME:________________________________ **PUID: :**________________________________

ECE 656 Exam 1: Fall 2013
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(Revised 9/11/13)

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three equally weighted questions. To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 50 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

30 points possible, 10 per question

1) 2 points per part – 10 points total

2) 10 points

3a) 8 points
3b) 2 points

Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) Consider a 1D semiconductor nanowire where electrons can move in the x-direction. Assume that the cross-section in the y-z plane is a square with sides of length, t and that the lower left corner is at y = 0, z = 0. What is the wavefunction of the **first** subband? (Assume infinite confining potential all around the write).

a) $\psi(\vec{r}) = \sin(\pi z/t) e^{ik_x x} \times e^{ik_y y}$

b) $\psi(\vec{r}) = \cos(\pi y/t)\sin(\pi z/t) e^{ik_x x}$

c) $\psi(\vec{r}) = \sin(\pi y/t)\cos(\pi z/t) e^{ik_x x}$

d) $\psi(\vec{r}) = \cos(\pi y/t)\cos(\pi z/t) e^{ik_x x}$

e) $\psi(\vec{r}) = \sin(\pi y/t)\sin(\pi z/t) e^{ik_x x}$

1.2) Assume a 3D semiconductor with a conduction band **effective** DOS of $N_C = 10^{19}$ cm^{-3}. What is the electron density in the conduction band if $E_F = E_C$? (DO NOT assume non-degenerate carrier statistics.)

a) $n_0 > 10^{19}$ cm^{-3}

b) $n_0 = 10^{19}$ cm^{-3}.

c) $n_0 < 10^{19}$ cm^{-3}

d) $n_0 > 10^{19}$ cm^{-3} for T > 300 K and $n_0 < 10^{19}$ cm^{-3} for T < 300 K.

e) $n_0 > 10^{19}$ cm^{-3} times 2 for spin.

1.3) Consider a 1D semiconductor of length, L_x with $D_{1D}(E) = 2/(\pi\hbar\upsilon)$. What is

$$L_x \int_{E_{Cbot}}^{E_{Ctop}} D_{1D}(E)\,dE\,?$$

a) Infinity.

b) The 1D **effective** density-of-states.

c) Zero.

d) The number of states in the band.

e) The Fermi energy.

1.4) What is the quantity, $-d(\chi/q)/dx$, where χ is the electron affinity, called?

a) The electric field.

b) The quasi-electric field for electrons.

c) The quasi-electric field for holes.

d) The quasi-magnetic field.

e) The polarization.

Problem 1) continued:

1.5) Which of the following is generally true of the characteristic times? (Scattering time, τ, momentum relaxation time, τ_m, and energy relaxation time, τ_E.)

a) $\tau > \tau_m > \tau_E$.

b) $\tau > \tau_m < \tau_E$.

c) $\tau < \tau_m > \tau_E$.

d) $\tau < \tau_m < \tau_E$.

e) $\tau \approx \tau_m \approx \tau_E$.

2) Work out the following integral: $I = \int_{E_C}^{\infty} \left(E - E_C\right)^3 \left(-\partial f_0 / \partial E\right) dE$.

Draw a box around your answer.

3) A carbon nanotube can be considered as a 1D conductor. A metallic carbon nanotube has a linear dispersion (like graphene, but in 1D):

$$E\left(k_x\right) = \pm\hbar\upsilon_F k_x$$

Work out the density of states in energy (per Joule-m) for this metallic carbon nanotube. **Draw a box around your answers.**

3a) Assume $E > 0$

3b) Assume $E < 0$

SCRATCH PAPER

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ECE-656 Key Equations (Weeks 1-3)

Physical constants:

$\hbar = 1.055 \times 10^{-34}$ [J-s] $\quad m_0 = 9.109 \times 10^{-31}$ [kg]

$k_B = 1.380 \times 10^{-23}$ [J/K] $\quad q = 1.602 \times 10^{-19}$ [C] $\quad \varepsilon_0 = 8.854 \times 10^{-14}$ [F/cm]

Density of states in k-space:

1D: $N_k = 2 \times (L/2\pi) = L/\pi$ $\quad$ 2D: $N_k = 2 \times (A/4\pi^2) = A/2\pi^2$ $\quad$ 3D: $N_k = 2 \times (\Omega/8\pi^2) = \Omega/4\pi^3$

Density of states in energy (parabolic bands, per length, area, or volume):

$$D_{1D}(E) = \frac{g_v}{\pi\hbar}\sqrt{\frac{2m^*}{(E-\varepsilon_1)}} \qquad D_{2D}(E) = g_V \frac{m^*}{\pi\hbar^2} \qquad D_{3D}(E) = g_v \frac{m^*\sqrt{2m^*(E-E_C)}}{\pi^2\hbar^3}$$

Fermi function and Fermi-Dirac Integrals:

$$f_0(E) = \frac{1}{1+e^{(E-E_F)/k_BT}}$$

$$\mathcal{F}_j(\eta_F) = \frac{1}{\Gamma(j+1)}\int_0^\infty \frac{\eta^j d\eta}{1+e^{\eta-\eta_F}} \qquad \mathcal{F}_j(\eta_F) \to e^{\eta} \quad \eta_F << 0 \qquad \frac{d\mathcal{F}_j}{d\eta_F} = \mathcal{F}_{j-1}$$

$\Gamma(n) = (n-1)!$ (n an integer) $\qquad \Gamma(1/2) = \sqrt{\pi} \qquad \Gamma(p+1) = p\Gamma(p)$

Mark Lundstrom 8/29/2013