NAME:_____ **PUID: :**_____

SOLUTIONS: ECE 656 Exam 1: Fall 2013

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This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three equally weighted questions. To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 50 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

30 points possible, 10 per question

1) 2 points per part – 10 points total

2) 10 points

3a) 8 points
3b) 2 points

Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) Consider a 1D semiconductor nanowire where electrons can move in the *x*-direction. Assume that the cross-section in the y-z plane is a square with sides of length, *t* and that the lower left corner is at *y* = 0, *z* = 0. What is the wavefunction of the **first** subband? (Assume infinite confining potential all around the write).

a) $\psi(\vec{r}) = \sin(\pi z/t)e^{ik_x x} \times e^{ik_y y}$

b) $\psi(\vec{r}) = \cos(\pi y/t)\sin(\pi z/t)e^{ik_x x}$

c) $\psi(\vec{r}) = \sin(\pi y/t)\cos(\pi z/t)e^{ik_x x}$

d) $\psi(\vec{r}) = \cos(\pi y/t)\cos(\pi z/t)e^{ik_x x}$

e) $\psi(\vec{r}) = \sin(\pi y/t)\sin(\pi z/t)e^{ik_x x}$

1.2) Assume a 3D semiconductor with a conduction band **effective** DOS of $N_C = 10^{19}$ cm^{-3}. What is the electron density in the conduction band if $E_F = E_C$? (DO NOT assume non-degenerate carrier statistics.)

a) $n_0 > 10^{19}$ cm^{-3}

b) $n_0 = 10^{19}$ cm^{-3}.

c) $n_0 < 10^{19}$ **cm^{-3}**

d) $n_0 > 10^{19}$ cm^{-3} for $T > 300$ K and $n_0 < 10^{19}$ cm^{-3} for $T < 300$ K.

e) $n_0 > 10^{19}$ cm^{-3} times 2 for spin.

1.3) Consider a 1D semiconductor of length, L_x with $D_{1D}(E) = 2/(\pi\hbar\upsilon)$. What is $L_x \int_{E_{Cbot}}^{E_{Ctop}} D_{1D}(E)\,dE$?

a) Infinity.
b) The 1D **effective** density-of-states.
c) Zero.
d) The number of states in the band.
e) The Fermi energy.

1.4) What is the quantity, $-d(\chi/q)/dx$, where χ is the electron affinity, called?

a) The electric field.
b) The quasi-electric field for electrons.
c) The quasi-electric field for holes.
d) The quasi-magnetic field.

e) The polarization.

Problem 1) continued:

1.5) Which of the following is generally true of the characteristic times? (Scattering time, τ, momentum relaxation time, τ_m, and energy relaxation time, τ_E.)

a) $\tau > \tau_m > \tau_E$.

b) $\tau > \tau_m < \tau_E$.

c) $\tau < \tau_m > \tau_E$.

d) $\tau < \tau_m < \tau_E$**.**

e) $\tau \approx \tau_m \approx \tau_E$.

2) Work out the following integral: $I = \int_{E_C}^{\infty} \left(E - E_C\right)^3 \left(-\partial f_0 / \partial E\right) dE$.

Draw a box around your answer.

Solution:

$$-\frac{\partial f_0}{\partial E} = +\frac{\partial f_0}{\partial E_F}$$

(This allows us to move the derivative outside the integral, since the integral is over energy.)

$$I = \int_{E_C}^{\infty} \left(E - E_C\right)^3 \left(-\partial f_0 / \partial E\right) dE = \frac{\partial}{\partial E_F} \int_{E_C}^{\infty} \left(E - E_C\right)^3 f_0\left(E\right) dE$$

$$I = \frac{\partial}{\partial E_F} \int_{E_C}^{\infty} \frac{\left(E - E_C\right)^3}{1 + e^{\left(E - E_F\right)/k_B T}} dE$$

$$I = \frac{\partial}{\partial E_F} \int_{E_C}^{\infty} \frac{\left(E - E_C\right)^3}{1 + e^{\left(E - E_C + E_C - E_F\right)/k_B T}} dE$$

(In the last step, we subtracted and then added E_C.) Now define:

$$\eta_F = \frac{E_F - E_C}{k_B T} \qquad \eta = \frac{E - E_C}{k_B T} \qquad d\eta = \frac{dE}{k_B T} \qquad dE = k_B T d\eta$$

with this change of variables, we find:

$$I = \frac{\partial}{\partial E_F}\int_0^\infty \eta^3 \left(k_B T\right)^3 \frac{1}{1+e^{\eta-\eta_F}} k_B T\, d\eta$$

$$I = \left(k_B T\right)^4 \frac{\partial}{\partial E_F}\int_0^\infty \frac{\eta^3}{1+e^{\eta-\eta_F}} d\eta = \left(k_B T\right)^3 \frac{\partial}{\partial\left(E_F/k_B T\right)}\int_0^\infty \frac{\eta^3}{1+e^{\eta-\eta_F}} d\eta$$

$$= \left(k_B T\right)^3 \frac{\partial}{\partial \eta_F}\int_0^\infty \frac{\eta^3}{1+e^{\eta-\eta_F}} d\eta$$

(Note that the lower limit is now zero.)

$$I = \left(k_B T\right)^3 \frac{\partial}{\partial \eta_F}\int_0^\infty \frac{\eta^3}{1+e^{\eta-\eta_F}} d\eta \qquad (*)$$

The integral looks almost like a Fermi-Dirac integral of order 3 (except for a constant.

$$\mathcal{F}_3\left(\eta_F\right) = \frac{1}{\Gamma(4)}\int_0^\infty \frac{\eta^3}{1+e^{\eta-\eta_f}} d\eta = \frac{1}{3!}\int_0^\infty \frac{\eta^3}{1+e^{\eta-\eta_f}} d\eta = \frac{1}{6}\int_0^\infty \frac{\eta^3}{1+e^{\eta-\eta_f}} d\eta$$

$$\int_0^\infty \frac{\eta^3}{1+e^{\eta-\eta_f}} d\eta = 6\mathcal{F}_3\left(\eta_F\right) \qquad (**)$$

Use (**) in (*) to find:

$$I = \left(k_B T\right)^3 \frac{\partial}{\partial \eta_F}\left[\int_0^\infty \frac{\eta^3}{1+e^{\eta-\eta_F}} d\eta\right] = 6\left(k_B T\right)^3 \frac{\partial}{\partial \eta_F}\mathcal{F}_3\left(\eta_F\right) = 6\left(k_B T\right)^3 \mathcal{F}_2\left(\eta_F\right)$$

$$\boxed{I = 6\left(k_B T\right)^3 \mathcal{F}_2\left(\eta_F\right)}$$

3) A carbon nanotube can be considered as a 1D conductor. A metallic carbon nanotube has a linear dispersion (like graphene, but in 1D):

$$E\left(k_x\right) = \pm \hbar \upsilon_F k_x$$

Work out the density of states in energy (per Joule-m) for this metallic carbon nanotube. **Draw a box around your answer.**

3a) Assume $E > 0$

3b) Assume $E < 0$

Solution:

3a) $E > 0$

$$N\left(k_x\right)dk_x = g_v \frac{1}{\left(2\pi/L\right)} \times 2 \times 2dk_x = g_v \frac{2L}{\pi} dk_x$$

One factor of two is for spin degeneracy and the other is to account for both $+k_x$ and $-k_x$.

$$D\left(E\right)dE = \frac{1}{L} N\left(k_x\right)dk_x = g_v \frac{2}{\pi} dk_x$$

$$D\left(E\right) = g_v \frac{2}{\pi} \frac{dk_x}{dE} \qquad (*)$$

Now use the dispersion:

$$E\left(k_x\right) = \pm \hbar \upsilon_F k_x$$

Assume $E > 0$

$$\frac{dE\left(k_x\right)}{dk_x} = \hbar \upsilon_F$$

Equation (*) becomes:

$$\boxed{D\left(E\right) = \frac{2g_v}{\pi \hbar \upsilon_F} \quad \left(E > 0\right)}$$

We get a constant *DOS* for $E > 0$.

3) $E < 0$:

Since the bandstructure is symmetrical about E = 0, we also expect a constant *DOS* for $E < 0$.

$$\boxed{D(E) = \frac{2g_v}{\pi\hbar\upsilon_F} \quad (E < 0)}$$

If we consider the mapping from k-space to energy space for valence bands, where the DOS increases downward, then we see that

$$D(E)dE = \frac{1}{L}N(k_x)dk_x = g_v\frac{2}{\pi}dk_x$$

becomes

$$D(E)(-dE) = \frac{1}{L}N(k_x)dk_x$$

For valence bands.

SCRATCH PAPER

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