NAME:________________________________ **PUID: :**________________________________

SOLUTIONS: ECE 656 Exam 2: Fall 2013

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(Revised 10/24/13)

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three equally weighted questions. To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 50 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

30 points possible, 10 per question

1) 2 points per part – 10 points total

2a) 5 points
2b) 5 points

3a) 4 points
3b) 4 points
3c) 2 points

Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) What are the important scattering mechanisms for electrons in undoped Si at room temperature?

a) ADP and PZ intravalley scattering
b) ADP intravalley scattering and intervalley phonon scattering.
c) ADP intravalley and POP inter valley scattering.
d) ADP intravalley, alloy and neutral defect scattering.
e) ADP intravalley and plasmon scattering.

1.2) What is the most important scattering mechanism in undoped GaAs at room temperature?

a) IV scattering
b) ODP intravalley scattering
c) ADP intravelley scattering
d) POP intravalley scattering
e) PZ intravalley scattering

1.3) How does the energy relaxation **time** generally compare to the other characteristic times?

a) $\tau_E \approx \tau_m, \tau_E \approx \tau$
b) $\tau_E \approx \tau_m, \tau_E > \tau$
c) $\tau_E > \tau_m, \tau_E \approx \tau$
d) $\tau_E > \tau_m, \tau_E > \tau$
e) $\tau_E < \tau_m, \tau_E \approx \tau$.

1.4) How does the ADP momentum relaxation **time** vary with temperature?

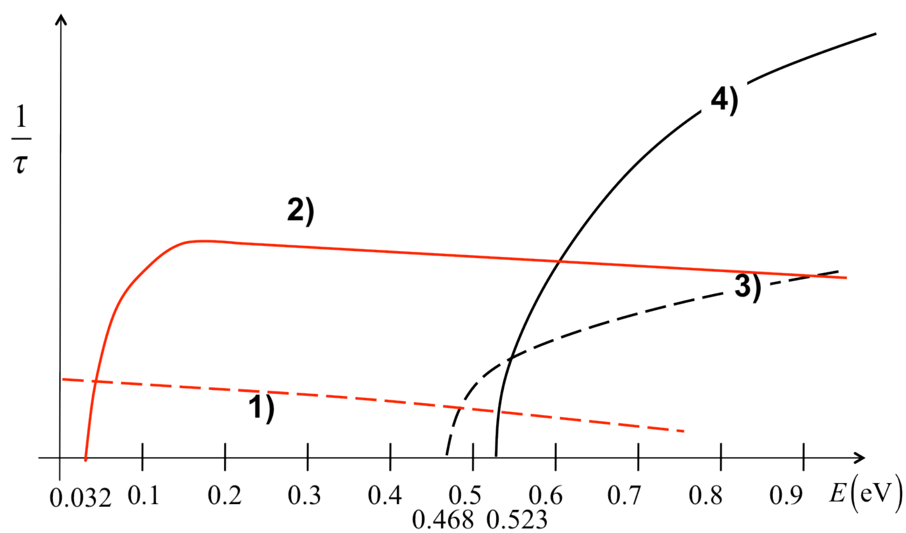
a) Approximately independent of temperature.
b) Increases as temperature increases.
c) Decreases as temperature increases.
d) Displays a maximum at the Debye temperature.
e) Displays a minimum at the Debye temperature.

1.5) How does the II momentum relaxation **time** vary with temperature?

a) Approximately independent of temperature.
b) Increases as temperature increases.
c) Decreases as temperature increases.
d) Displays a maximum at the Debye temperature.
e) Displays a minimum at the Debye temperature

2) The material, $In_{0.53}Ga_{0.47}As$ is an important semiconductor because it is lattice matched to InP. It is a direct bandgap material with a bandgap of $E_{G\Gamma} = 0.75$ eV and an effective mass of $m_n^*/m_0 = 0.041$. It contains heavy mass, upper valleys located at an energy of $\Delta E_{\Gamma-L} = 0.55$ eV above the Γ valley minimum. It has an optical phonon energy of 32 meV. The following two questions concern electron scattering in undoped $In_{0.53}Ga_{0.47}As$ at room temperature.

2a) Sketch the scattering rate vs. energy for **electrons in the Γ valley** from E = 0 (bottom of the Γ valley) to E = 0.9 eV. Your sketch should label **each** of the main phonon scattering processes, the critical energies, and the relative magnitudes of the processes. (DO NOT add all of the processes to get the total scattering rate, just sketch each one separately.) SHOW YOUR WORK.



$$N_0 = \frac{1}{e^{\hbar\omega_o/k_B T_L} - 1} = 0.41 \quad N_0 + 1 = 1.41$$

1) POP ABS, $\Gamma \to \Gamma$, $1/\tau \propto N_0$

2) POP EMS, $\Gamma \to \Gamma$, $1/\tau \propto (N_0 + 1)$

3) IV ABS, $\Gamma \to L$, $1/\tau \propto N_0 \times \sqrt{(E-0.5) + \hbar\omega_0}$

4) IV EMS, $\Gamma \to L$, $1/\tau \propto (N_0 + 1) \times \sqrt{(E-0.5) - \hbar\omega_0}$

Processes 3) and 4) are proportional to the density of final states.

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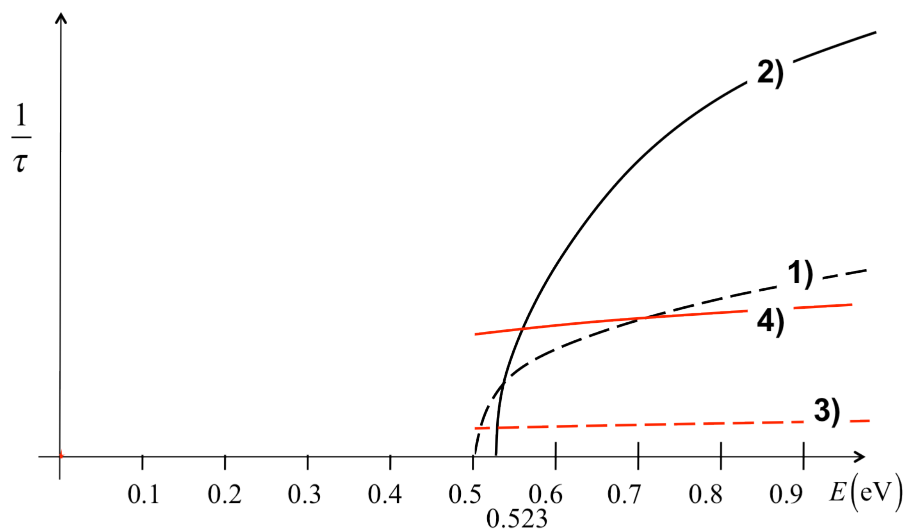
2b) Sketch the **intervalley** scattering rate vs. energy for **electrons in the L valley** from E = 0 (bottom of the Γ valley) to E = 0.9 eV. Your sketch should label **each** of the main phonon scattering processes, the critical energies, and the relative magnitudes of the processes. (DO NOT add all of the processes to get the total scattering rate, just sketch each one separately.) SHOW YOUR WORK.

Note: There is also intravalley scattering withing the L-valley, but you are not being asked to plot this.



1) IV phonon ABS scattering, $\mathrm{L} \to \mathrm{L}$, $1/\tau \propto 3 \times N_0 \times \sqrt{(E-0.5)+\hbar\omega_0}$ (the three is the number of final valleys)
2) IV phonon EMS, $\mathrm{L} \to \mathrm{L}$, $1/\tau \propto 3 \times (N_0+1) \times \sqrt{(E-0.5)-\hbar\omega_0}$
3) IV phonon ABS scattering, $\mathrm{L} \to \Gamma$, $1/\tau \propto 1 \times N_0 \times \sqrt{E+\hbar\omega_0}$ (the one is for the number of final valleys)
4) IV phonon EMS, scattering $\mathrm{L} \to \Gamma$, $1/\tau \propto 1 \times (N_0+1) \times \sqrt{E-\hbar\omega_0}$

All rates are proportional to the density of final states.

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3) This problem involves ODP **intravalley** scattering of electrons in graphene. Recall that the dispersion of graphene is

$$E(k) = \pm\hbar\upsilon_F\sqrt{k_x^2 + k_y^2} = \pm\hbar\upsilon_F k\,,$$

and the density-of-states is

$$D(E) = \frac{2E}{\pi\hbar^2\upsilon_F^2} \quad E > 0\,.$$

(Note that the factor of 2 in the above expression comes from the valley degeneracy of 2 for graphene.) The transition rate is given by FGR as:

$$S(\vec{p}_\parallel, \vec{p}_\parallel') = \frac{2\pi}{\hbar}\left|H_{\vec{p}_\parallel', \vec{p}_\parallel}\right|^2 \delta(E' - E \mp \hbar\omega_0)\,,$$

where $\vec{p}_\parallel$ refers to an electron in the plane of the graphene sheet (the *x*-*y* plane). The ODP scattering potential is

$$U_S = D_0 u_\beta$$

The lattice vibration is written as

$$u_\beta(\vec{\rho}) = A_\beta e^{\pm i\vec{\beta}_\parallel \bullet \vec{\rho}}\,,$$

where $\vec{\rho}$ is a vector in the *x*-*y* plane and $\vec{\beta}_\parallel$ is a phonon wavevector in the x-y plane. Following the procedure in the text, we write the amplitude of the phonon wavevector as

$$\left|A_\beta\right|^2 = \frac{\hbar}{2\rho_m A\,\omega_0}\left(N_0 + \frac{1}{2} \mp \frac{1}{2}\right),$$

where ρ_m is the mass density in Kg/m^2.

Answer the following three questions. **Draw a box around your answers.**

For this problem, you should just assume simple plane waves; do not worry about the two-component wavefunction.

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3a) Derive an expression for the transition rate, $S\left(\vec{p}_{\parallel},\vec{p}_{\parallel}'\right)$ due to intravalley phonon **emission.** Assume that electrons are in the graphene conduction band, with $E >> 0$. You must show your work.

Solution:

$$S\left(\vec{p}_{\parallel},\vec{p}_{\parallel}'\right)=\frac{2\pi}{\hbar}\left|H_{\vec{p}_{\parallel}',\vec{p}_{\parallel}}\right|^{2}\delta\left(E'-E\mp\hbar\omega_{0}\right)$$

$$H_{\vec{p}_{\parallel}',\vec{p}_{\parallel}}=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{A}}e^{-i\vec{p}_{\parallel}'\cdot\vec{\rho}/\hbar}U_{S}(\vec{r})\frac{1}{\sqrt{A}}e^{i\vec{p}_{\parallel}\cdot\vec{\rho}/\hbar}d\vec{\rho}=\frac{1}{A}\int_{-\infty}^{+\infty}e^{-i\vec{p}_{\parallel}'\cdot\vec{\rho}/\hbar}\left(D_{0}A_{\beta}e^{\pm i\vec{\beta}_{\parallel}\cdot\vec{\rho}/\hbar}\right)e^{i\vec{p}_{\parallel}\cdot\vec{\rho}/\hbar}d\vec{\rho}$$

$$H_{\vec{p}_{\parallel}',\vec{p}_{\parallel}}=D_{0}A_{\beta}\frac{1}{A}\int_{-\infty}^{+\infty}e^{i\left(\vec{p}_{\parallel}-\vec{p}_{\parallel}'\pm\hbar\vec{\beta}_{\parallel}\right)\cdot\vec{\rho}/\hbar}\,d\vec{\rho}=D_{0}A_{\beta}\delta_{\vec{p}_{\parallel},\vec{p}_{\parallel}'\pm\hbar\vec{\beta}_{\parallel}}$$

$$S\left(\vec{p}_{\parallel},\vec{p}_{\parallel}'\right)=\frac{2\pi}{\hbar}\left|H_{\vec{p}_{\parallel}',\vec{p}_{\parallel}}\right|^{2}\delta\left(E'-E\mp\hbar\omega_{0}\right)\rightarrow\frac{2\pi}{\hbar}D_{0}^{2}\left|A_{\beta}\right|^{2}\delta_{\vec{p}_{\parallel},\vec{p}_{\parallel}'\pm\hbar\vec{\beta}_{\parallel}}\delta\left(E'-E\mp\hbar\omega_{0}\right)$$

Use:

$$\left|A_{\beta}\right|^{2}=\frac{\hbar}{2\rho_{m}A\,\omega_{0}}\left(N_{0}+\frac{1}{2}\mp\frac{1}{2}\right)$$

to find

$$S\left(\vec{p}_{\parallel},\vec{p}_{\parallel}'\right)=\frac{2\pi}{\hbar}D_{0}^{2}\frac{\hbar}{2\rho_{m}A\,\omega_{0}}\left(N_{0}+\frac{1}{2}\mp\frac{1}{2}\right)\delta_{\vec{p}_{\parallel},\vec{p}_{\parallel}'\pm\hbar\vec{\beta}_{\parallel}}\delta\left(E'-E\mp\hbar\omega_{0}\right)$$

For phonon emission, we find

$$\boxed{S\left(\vec{p}_{\parallel},\vec{p}_{\parallel}'\right)=\frac{\pi D_{0}^{2}}{\rho_{m}\omega_{0}}\left(N_{0}+1\right)\frac{1}{A}\delta_{\vec{p}_{\parallel},\vec{p}_{\parallel}'\pm\hbar\vec{\beta}_{\parallel}}\delta\left(E'-E+\hbar\omega_{0}\right)}$$

3b) Using your result from part 3a) derive an expression for the **intravalley** scattering rate. HINT: You **do not** need to explicitly consider momentum conservation for this problem. You must show your work.

Solution:

$$\frac{1}{\tau\left(\vec{p}_{\parallel}\right)} == \sum_{\vec{p}'_{\parallel},\uparrow} S\left(\vec{p}_{\parallel}, \vec{p}'_{\parallel}\right)$$

Since there is no wavevector dependence in the transition rate, we don't need to worry about momentum conservation.

$$\frac{1}{\tau\left(\vec{p}_{\parallel}\right)} = \frac{\pi D_0^2}{\rho_m \omega_0}\left(N_0 + 1\right)\frac{1}{A}\sum_{\vec{p}'_{\parallel},\uparrow} \delta\left(E' - E + \hbar\omega_0\right)$$

We only sum over one-half of the spin states, and only one of the two valleys, since we are dealing with intravalley scattering, so this sum is recognized as one-fourth of the graphene density of states as given earlier in the problem.

$$\frac{1}{\tau\left(\vec{p}_{\parallel}\right)} = \frac{\pi D_0^2}{\rho_m \omega_0}\left(N_0 + 1\right)\frac{D_{2D}\left(E + \hbar\omega_0\right)}{4} = \frac{D_0^2}{2\rho_m \omega_0 \hbar^2 \upsilon_F^2}\left(N_0 + 1\right)\left(E + \hbar\omega_0\right)$$

$$\boxed{\frac{1}{\tau\left(E\right)} = \frac{D_0^2}{2\rho_m \omega_0 \hbar^2 \upsilon_F^2}\left(N_0 + 1\right)\left(E - \hbar\omega_0\right)}$$

This result agrees with the paper:

J. Chauhan and J. Guo, *Applied Physics Letters*, Vol. 98, 023120, 2009.

continued on next page

3c) Using your results from above, derive the energy relaxation time assuming energetic carriers for which phonon emission dominates. You must show your work.

Solution:

$$\frac{1}{\tau_E(E)} = \sum_{\vec{p}',\uparrow} S\left(\vec{p}_\parallel, \vec{p}_\parallel'\right)\frac{\Delta E}{E} = \frac{\hbar\omega_0}{E}\sum_{\vec{p}',\uparrow} S\left(\vec{p}_\parallel, \vec{p}_\parallel'\right) = \frac{\hbar\omega_0}{E}\frac{1}{\tau}$$

$$\boxed{\frac{1}{\tau_E(E)} = \frac{D_0^2}{2\rho_m \hbar \upsilon_F^2}\left(N_0+1\right)\left(1-\frac{\hbar\omega_0}{E}\right)}$$

SCRATCH PAPER

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ECE-656 Key Equations (Weeks 3-5)

Physical constants:

$\hbar = 1.055\times10^{-34}$ [J-s] $\quad m_0 = 9.109\times10^{-31}$ [kg]

$k_B = 1.380\times10^{-23}$ [J/K] $\quad q = 1.602\times10^{-19}$ [C] $\quad \varepsilon_0 = 8.854\times10^{-14}$ [F/cm]

Density of states in k-space:

1D: $N_k = 2\times\left(L/2\pi\right) = L/\pi$ 2D: $N_k = 2\times\left(A/4\pi^2\right) = A/2\pi^2$ 3D: $N_k = 2\times\left(\Omega/8\pi^2\right) = \Omega/4\pi^3$

Density of states in energy (parabolic bands, per length, area, or volume):

$$D_{1D}(E) = \frac{g_v}{\pi\hbar}\sqrt{\frac{2m^*}{(E-\varepsilon_1)}} \qquad D_{2D}(E) = g_V\frac{m^*}{\pi\hbar^2} \qquad D_{3D}(E) = g_v\frac{m^*\sqrt{2m^*(E-E_C)}}{\pi^2\hbar^3}$$

Fermi function and Fermi-Dirac Integrals:

$$f_0(E) = \frac{1}{1+e^{(E-E_F)/k_BT}}$$

$$\mathcal{F}_j(\eta_F) = \frac{1}{\Gamma(j+1)}\int_0^\infty \frac{\eta^j d\eta}{1+e^{\eta-\eta_F}} \qquad \mathcal{F}_j(\eta_F) \to e^{\eta} \quad \eta_F << 0 \qquad \frac{d\mathcal{F}_j}{d\eta_F} = \mathcal{F}_{j-1}$$

$\Gamma(n) = (n-1)!$ (n an integer) $\qquad \Gamma(1/2) = \sqrt{\pi} \qquad \Gamma(p+1) = p\Gamma(p)$

Scattering:

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|H_{\vec{p}\,',\vec{p}}\right|^2 \delta(E'-E-\Delta E) \qquad H_{\vec{p}\,',\vec{p}} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\vec{p}\,'\bullet r/\hbar}U_S(\vec{r})e^{i\vec{p}\bullet\vec{r}/\hbar}d\vec{r}$$

$$\frac{1}{\tau(\vec{p})} = \sum_{\vec{p}\,',\uparrow} S(\vec{p},\vec{p}\,') \qquad \frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}\,',\uparrow} S(\vec{p},\vec{p}\,')\frac{\Delta p_z}{p_{z0}} \qquad \frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}\,',\uparrow} S(\vec{p},\vec{p}\,')\frac{\Delta E}{E_0}$$

ADP: $\left|K_\beta\right|^2 = \beta^2 D_A^2$ ODP: $\left|K_\beta\right|^2 = D_0^2$ PZ: $\left|K_\beta\right|^2 = \left(qe_{PZ}/\kappa_S\varepsilon_0\right)^2$ POP: $\left|K_\beta\right|^2 = \dfrac{\rho q^2\omega_0^2}{\beta^2\kappa_0\varepsilon_0}\left(\dfrac{\kappa_0}{\kappa_\infty}-1\right)$

$$S(\vec{p},\vec{p}\,') = \frac{\pi}{\Omega\rho\,\omega}\left|K_\beta\right|^2\left(N_\omega + \frac{1}{2}\mp\frac{1}{2}\right)\delta_{\vec{p}\,',\vec{p}\pm\hbar\vec{\beta}}\,\delta(E'-E\mp\hbar\omega)$$

$$\delta_{\vec{p}\,',\vec{p}\pm\hbar\vec{\beta}}\,\delta(E'-E\mp\hbar\omega_\beta) \to \frac{1}{\hbar\upsilon\beta}\delta\left(\pm\cos\theta + \frac{\hbar\beta}{2p}\mp\frac{\omega_\beta}{\upsilon\beta}\right)$$

$$\frac{1}{\tau} = \frac{1}{\tau_{abs}} + \frac{1}{\tau_{ems}} = \frac{2\pi}{\hbar}\left(\frac{D_A^2 k_B T}{c_l}\right)\frac{D_{3D}(E)}{2} \text{ (ADP)} \qquad L_D = \sqrt{\frac{\kappa_S\varepsilon_0 k_B T_L}{q^2 n_0}}$$

$$\frac{1}{\tau} = \frac{1}{\tau_m} = \frac{2\pi}{\hbar}\left(\frac{\hbar D_O^2}{2\rho\omega_0}\right)\left(N_0 + \frac{1}{2}\mp\frac{1}{2}\right)\frac{D_{3D}(E\pm\hbar\omega_0)}{2} \qquad N_0 = \frac{1}{e^{\hbar\omega_o/k_BT_L}-1} \text{ (ODP)}$$

Mark Lundstrom 8/29/2013