

NAME: _____

PUID: : _____

SOLUTIONS: ECE 656 Exam 3: Fall 2013

October 14, 2013

Mark Lundstrom
Purdue University

This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three equally weighted questions. To receive full credit, you must **show your work** (scratch paper is attached).

The exam is designed to be taken in 50 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

30 points possible, 10 per question

- 1) 2 points per part – 10 points total
 - 2a) 4 points
 - 2b) 3 points
 - 2c) 3 points
- 3) 10 points

Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) Some of the thermoelectric transport coefficients are **proportional** to each other. Two are related fundamentally (independent of details of bandstructure and scattering) and two are related phenomenologically, in a way that depends on details of bandstructure and scattering. Which of the following is correct?

- a) Fundamental: σ and S . Phenomenological: π and κ_e .
- b) Fundamental: σ and π . Phenomenological: S and κ_e .
- c) Fundamental: σ and κ_0 . Phenomenological: κ_0 and κ_e .
- d) Fundamental: π and κ_e . Phenomenological: π and S .
- e) Fundamental: π and S . Phenomenological: σ and κ_e .**

1.2) What are the most general driving forces for current flow?

- a) Gradients in the electrostatic potential and carrier concentration.
- b) Gradients in the electric field and temperature.
- c) Gradients in the electrochemical potential and temperature.**
- d) Gradients in the carrier concentration and temperature.
- e) Gradients in the electron affinity and temperature.

1.3) Consider the expression for the electrical conductivity: $\sigma = \frac{2q^2}{h} \int \lambda(E) \frac{M(E)}{A} \left(-\frac{\partial f_0}{\partial E} \right) dE$

When can we use it?

- a) When the current flows in the conduction band.
- b) When the current flows in the valence band.
- c) When the current flows in both the conduction and valence bands.
- d) When the temperature is constant.
- e) All of the above.**

1.4) How do the lattice thermal conductivity, κ_L , and the electronic thermal conductivity, κ_e compare?

- a) $\kappa_L \gg \kappa_e$ for a metal and for a semiconductor.
- b) $\kappa_L \ll \kappa_e$ for a metal and for a semiconductor.
- c) $\kappa_L \ll \kappa_e$ for a metal and $\kappa_L \approx \kappa_e$ for a semiconductor.
- d) $\kappa_L \ll \kappa_e$ for a metal and $\kappa_L \gg \kappa_e$ for a semiconductor.**
- e) $\kappa_L \approx \kappa_e$ for a metal and a semiconductor.

1.5) If the Peltier coefficient is -0.3 V, contact 1 is at a temperature of 300 K and contact 2 at 310 K, then what open circuit voltage would be measured on contact 2 with respect to contact 1 (i.e. what is $V(2) - V(1)$)?

- a) +0.3 V
- b) -0.3V
- c) +310 millivolts
- d) -10 millivolts
- e) +10 millivolts**

2) Consider a metallic carbon nanotube, which has a linear dispersion $E(k_x) = \pm \hbar v_F k_x$. The density of states in this case is a constant, independent of energy, $D(E) = 2g_v / (\pi \hbar v_F)$, where $g_v = 2$ is the valley degeneracy for a carbon nanotube, and v_F is a velocity.

2a) Determine the number of channels vs. energy, $M(E)$.

Solution:

Begin with:

$$M(E) = \frac{h}{4} \langle v_x^+ \rangle D(E)$$

$$v(k_x) = \frac{1}{\hbar} \frac{\partial E(k_x)}{\partial k_x} = \pm v_F$$

$$\langle v_x^+ \rangle = v_F$$

$$M(E) = \frac{h}{4} \langle v_x^+ \rangle D(E) = \frac{h}{4} v_F \frac{2g_v}{\pi \hbar v_F} = g_v = 2$$

$M(E) = 2$

 (this is valid for $-\infty < E < +\infty$).

- 2b) **Work out** an expression for the differential conductivity, $\sigma'(E)$, and make a **sketch** of $\sigma'(E)$ vs. E . You should assume that the mean-free-path is independent of energy. SHOW YOUR WORK.

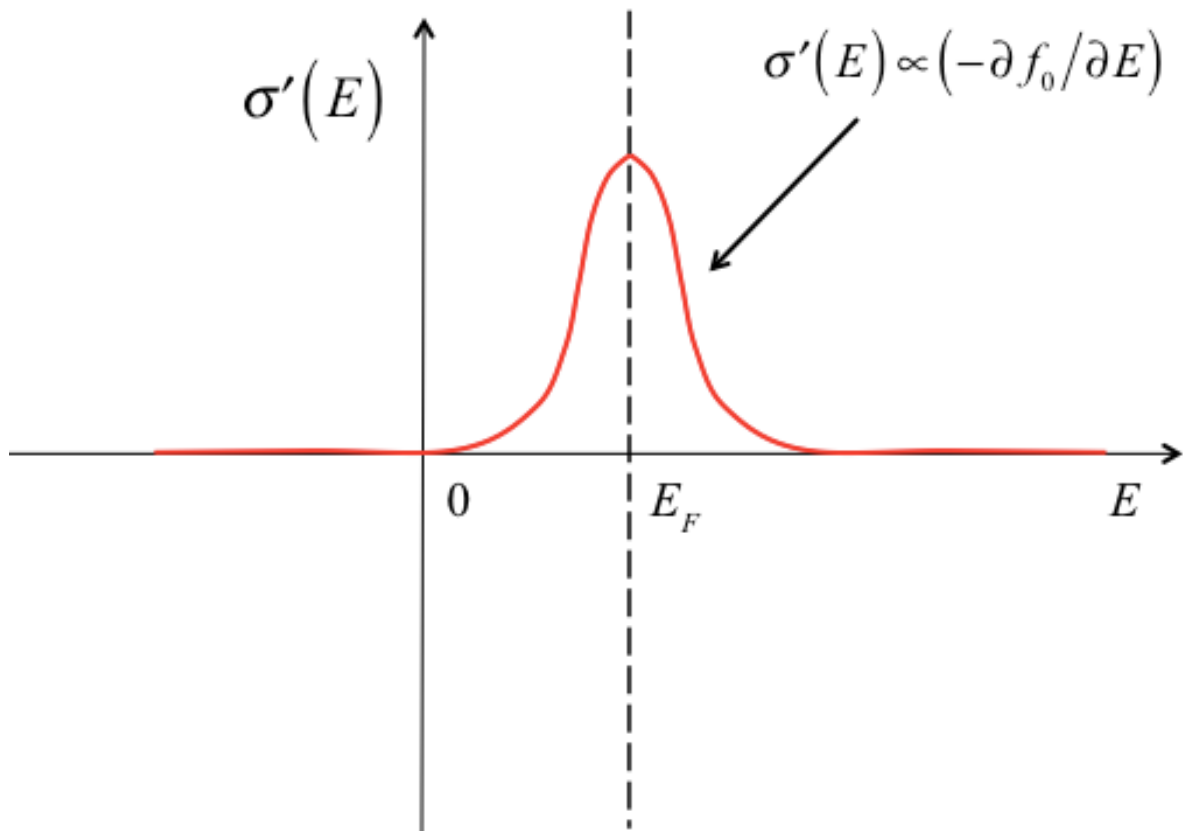
Solution:

$$\sigma'(E) = \frac{2q^2}{h} \lambda(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right)$$

assume a constant MFP and used the answer from part a):

$$\sigma'(E) = \frac{2q^2}{h} \times \lambda_0 \times 2 \times \left(-\frac{\partial f_0}{\partial E} \right) \quad \boxed{\sigma'(E) = \left(\frac{4q^2}{h} \lambda_0 \right) \times \left(-\frac{\partial f_0}{\partial E} \right)}$$

so the differential conductivity vs. energy has the shape of the Fermi window, $(-\partial f_0 / \partial E)$



- 2c) Find an expression for the Seebeck coefficient for an arbitrary location of the Fermi level. **HINT:** You can find the answer without a lot of work, but you must give a **mathematical explanation** of the result.

Solution:

$$S = \left(-\frac{k_B}{q} \right) \times \frac{\int_{-\infty}^{+\infty} \left(\frac{E - E_F}{k_B T} \right) \sigma'(E) dE}{\int_{-\infty}^{+\infty} \sigma'(E) dE}$$

$$S = \left(-\frac{k_B}{q} \right) \times \frac{\int_{-\infty}^{+\infty} \left(\frac{E - E_F}{k_B T} \right) (-\partial f_0 / \partial E) dE}{\int_{-\infty}^{+\infty} (-\partial f_0 / \partial E) dE}$$

The denominator integrates to 1:

$$\text{den} = \int_{-\infty}^{+\infty} \left(-\frac{\partial f_0}{\partial E} \right) dE = - \int_{-\infty}^{+\infty} df_0 = -[f_0(+\infty) - f_0(-\infty)] = -[0 - 1] = 1$$

Now looking at the numerator, we can see that it must be zero, so the answer must be zero, $\boxed{S=0}$ independent of the location of the Fermi level.

The **mathematical explanation** is that the numerator is the product of a function that is anti-symmetric about the Fermi level, $(E - E_F)$, and a function that is symmetric about the Fermi level, $-\partial f_0 / \partial E$, so when we integrate from $-\infty$ to $+\infty$, we must get zero.

Alternatively, we could argue that the Seebeck coefficient is proportional to the average energy of current flow with respect to the Fermi level:

$$S = \left(-\frac{k_B}{q} \right) \times \frac{\int_{-\infty}^{+\infty} \left(\frac{E - E_F}{k_B T} \right) \sigma'(E) dE}{\int_{-\infty}^{+\infty} \sigma'(E) dE} = \left(-\frac{k_B}{q} \right) \times \left(\frac{E_J - E_F}{k_B T} \right)$$

$$E_J = \frac{\int_{-\infty}^{+\infty} E \sigma'(E) dE}{\int_{-\infty}^{+\infty} \sigma'(E) dE}$$

Because of the symmetry of $\sigma'(E)$ as plotted in part b), it is clear that $E_J = E_F$, so $S = 0$ independent of the location of the Fermi level.

3) For this problem you will make use of the two coupled current equations:

$$\bar{E}_x = \rho J_x + S \frac{dT}{dx}$$

$$J_{Qx} = \pi J_x - (\kappa_L + \kappa_e) \frac{dT}{dx}$$

Assume that we have a sample of length, L , and that we force a current J_x through it and measure the voltage across it. The voltage consists of two components, one due to the resistance, V_R and one due to the Seebeck effect, V_S . Assume that we perform this measurement under adiabatic conditions (no heat flow). **What is the ratio of the two voltage components, V_R and V_S , in terms of the thermoelectric transport coefficients?**

Solution:

The measured voltage is:

$$V = \bar{E}_x L = \rho J_x L + S \frac{dT}{dx} L = V_R + V_S \quad (i)$$

$$V_R = \rho J_x L \quad (ii)$$

$$V_S = S \frac{dT}{dx} L \quad (iii)$$

From the second equation under adiabatic conditions,

$$J_{Qx} = \pi J_x - \kappa_e \frac{dT}{dx} = 0 \rightarrow \frac{dT}{dx} = \frac{\pi J_x}{\kappa_e} \quad (iv)$$

Use (iv) in (iii) to find

$$V_S = S \frac{dT}{dx} L = S \frac{\pi J_x}{\kappa_e} L \quad (v)$$

Now (ii) and (v) give:

$$\frac{V_S}{V_R} = \frac{S \frac{\pi J_x L}{\kappa_e}}{\rho J_x L} = \frac{S^2 \sigma T}{\kappa_e} = ZT \quad \boxed{\frac{V_S}{V_R} = ZT}$$

Following the work of Harmon (T.C. Harmon, "Special Techniques for Measurement of Thermoelectric Properties," *J. Applied Physics*, vol. 29, p. 1373, 1958.) several thermoelectric characterization methods based on this analysis have been developed. These techniques are known as "Z-meters".