

## ECE-656 Key Equations (Weeks 3-5)

### Physical constants:

$$\begin{aligned} \hbar &= 1.055 \times 10^{-34} \text{ [J-s]} & m_0 &= 9.109 \times 10^{-31} \text{ [kg]} \\ k_B &= 1.380 \times 10^{-23} \text{ [J/K]} & q &= 1.602 \times 10^{-19} \text{ [C]} & \epsilon_0 &= 8.854 \times 10^{-14} \text{ [F/cm]} \end{aligned}$$


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### Density of states in k-space:

$$1D: N_k = 2 \times (L/2\pi) = L/\pi \quad 2D: N_k = 2 \times (A/4\pi^2) = A/2\pi^2 \quad 3D: N_k = 2 \times (\Omega/8\pi^2) = \Omega/4\pi^3$$

### Density of states in energy (parabolic bands, per length, area, or volume):

$$D_{1D}(E) = \frac{g_v}{\pi\hbar} \sqrt{\frac{2m^*}{(E - \epsilon_1)}} \quad D_{2D}(E) = g_v \frac{m^*}{\pi\hbar^2} \quad D_{3D}(E) = g_v \frac{m^* \sqrt{2m^*(E - E_C)}}{\pi^2\hbar^3}$$


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### Fermi function and Fermi-Dirac Integrals:

$$\begin{aligned} f_0(E) &= \frac{1}{1 + e^{(E - E_F)/k_B T}} \\ \mathcal{F}_j(\eta_F) &= \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{\eta^j d\eta}{1 + e^{\eta - \eta_F}} & \mathcal{F}_j(\eta_F) &\rightarrow e^{\eta} \quad \eta_F \ll 0 & \frac{d\mathcal{F}_j}{d\eta_F} &= \mathcal{F}_{j-1} \\ \Gamma(n) &= (n-1)! \quad (n \text{ an integer}) & \Gamma(1/2) &= \sqrt{\pi} & \Gamma(p+1) &= p\Gamma(p) \end{aligned}$$


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### Scattering:

$$\begin{aligned} S(\vec{p}, \vec{p}') &= \frac{2\pi}{\hbar} |H_{\vec{p}', \vec{p}}|^2 \delta(E' - E - \Delta E) & H_{\vec{p}', \vec{p}} &= \frac{1}{\Omega} \int_{-\infty}^{+\infty} e^{-i\vec{p}' \cdot \vec{r}/\hbar} U_S(\vec{r}) e^{i\vec{p} \cdot \vec{r}/\hbar} d\vec{r} \\ \frac{1}{\tau(\vec{p})} &= \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') & \frac{1}{\tau_m(\vec{p})} &= \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{\Delta p_z}{p_{z0}} & \frac{1}{\tau_E(\vec{p})} &= \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{\Delta E}{E_0} \\ \text{ADP: } |K_\beta|^2 &= \beta^2 D_A^2 & \text{ODP: } |K_\beta|^2 &= D_0^2 & \text{PZ: } |K_\beta|^2 &= (q e_{PZ} / \kappa_S \epsilon_0)^2 & \text{POP: } |K_\beta|^2 &= \frac{\rho q^2 \omega_0^2}{\beta^2 \kappa_0 \epsilon_0} \left( \frac{\kappa_0}{\kappa_\infty} - 1 \right) \\ S(\vec{p}, \vec{p}') &= \frac{\pi}{\Omega \rho \omega} |K_\beta|^2 \left( N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \delta_{\vec{p}', \vec{p} \pm \hbar \beta} \delta(E' - E \mp \hbar \omega) \\ \delta_{\vec{p}', \vec{p} \pm \hbar \beta} \delta(E' - E \mp \hbar \omega) &\rightarrow \frac{1}{\hbar v \beta} \delta \left( \pm \cos \theta + \frac{\hbar \beta}{2p} \mp \frac{\omega_\beta}{v \beta} \right) \\ \frac{1}{\tau} &= \frac{1}{\tau_{abs}} + \frac{1}{\tau_{ems}} = \frac{2\pi}{\hbar} \left( \frac{D_A^2 k_B T}{c_l} \right) \frac{D_{3D}(E)}{2} \quad (\text{ADP}) & L_D &= \sqrt{\frac{\kappa_S \epsilon_0 k_B T_L}{q^2 n_0}} \\ \frac{1}{\tau} &= \frac{1}{\tau_m} = \frac{2\pi}{\hbar} \left( \frac{\hbar D_0^2}{2\rho \omega_0} \right) \left( N_0 + \frac{1}{2} \mp \frac{1}{2} \right) \frac{D_{3D}(E \pm \hbar \omega_0)}{2} & N_0 &= \frac{1}{e^{\hbar \omega_0 / k_B T_L} - 1} \quad (\text{ODP}) \end{aligned}$$