

## Week 12 Summary:

# Near-equilibrium Measurements

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## basic equations

$$\mathcal{E}_x = \rho_n J_x + S_n \frac{dT}{dx}$$

$$J_x^q = \pi_n J_x - \kappa_n \frac{dT}{dx}$$

Four transport coefficients:

- 1) resistivity ( $\Omega\text{-cm}$ ) =  $1/\text{conductivity (S/cm)}$
- 2) Seebeck coefficient (V/K)
- 3) Peltier coefficient (W/A)
- 4) Electronic heat conductivity (W/m-K)

Note: These equations describe electric and heat currents due to electrons – in the diffusive limit and in 3D.

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## near-equilibrium measurements

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### **Goal:**

To measure the near-equilibrium transport coefficients.

(electronic parameters)

## approaches

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- 2-probe measurements
- 4-probe measurements
- TLM measurements
- Hall bar measurements
- Van der Pauw measurements

## contact resistance

$$R_C = \frac{\rho_i t}{A_C} = \frac{\rho_c}{A_C} \Omega$$

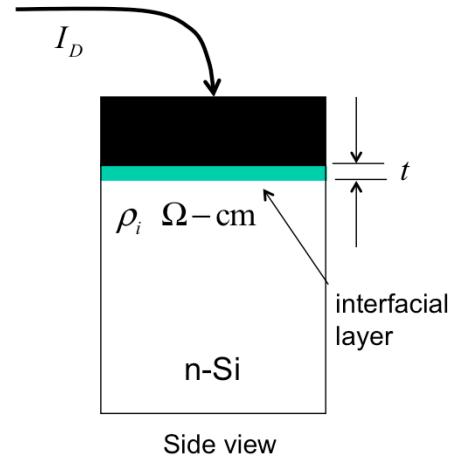
$$10^{-8} < \rho_c < 10^{-6} \Omega\text{-cm}^2$$

“interfacial contact resistivity”

$$A_C = 0.10 \mu\text{m} \times 1.0 \mu\text{m}$$

$$\rho_c = 10^{-7} \Omega\text{-cm}^2$$

$$R_C = 100 \Omega$$

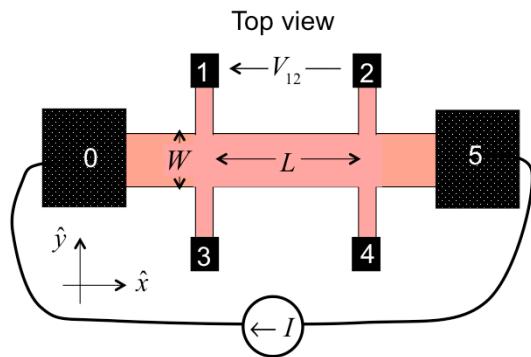


## contact resistance

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What is the fundamental, lower limit for  $\rho_c$  ?

## Hall bar



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## Hall bar analysis

$$\vec{J}_n = \sigma_s \vec{E} - (\sigma_s \mu_n r_H) \vec{E} \times \vec{B}$$

$$\vec{E} = \rho_s \vec{J}_n + (\rho_s \mu_n r_H) \vec{J}_n \times \vec{B}$$

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## indicial notation

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$$\vec{\mathcal{E}} = \rho_s \vec{J}_n + (\rho_n \mu_n r_H) \vec{J}_n \times \vec{B}$$

$$\mathcal{E}_i = \sum_{j=1}^{j=3} \rho_{ij} J_j \equiv \rho_{ij} \mathcal{E}_j \quad \text{“summation convention”}$$

$$\rho_{ij} = \rho_s \delta_{ij}$$

## indicial notation

$$\vec{\mathcal{E}} = \rho_S \vec{J}_n + (\rho_S \mu_n r_H) \vec{J}_n \times \vec{B}$$

$$\vec{C} = \vec{E} \times \vec{B}$$
$$\vec{C} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \mathcal{E}_x & \mathcal{E}_y & \mathcal{E}_z \\ B_x & B_y & B_z \end{bmatrix} = \hat{x}(\mathcal{E}_y B_z - \mathcal{E}_z B_y) + \dots$$

$$C_i = \epsilon_{ijk} \mathcal{E}_j B_k$$

$$\epsilon_{ijk} = +1 (i, j, k \text{ cyclic})$$

$$= -1 (i, j, k \text{ anti-cyclic})$$

$$= 0 (\text{otherwise})$$

$$C_x = \epsilon_{xjk} \mathcal{E}_j B_k$$

$$= \epsilon_{xyz} \mathcal{E}_y B_z + \epsilon_{xzy} \mathcal{E}_z B_y$$

$$= \mathcal{E}_y B_z - \mathcal{E}_z B_y$$

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## indicial notation

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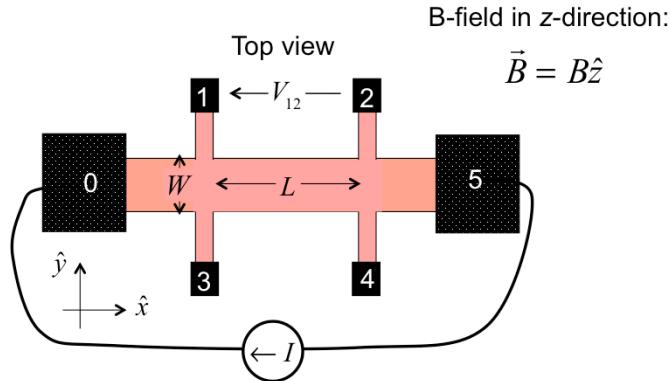
$$\vec{E} = \rho_s \vec{J}_n + (\rho_s \mu_n r_H) \vec{J}_n \times \vec{B}$$

$$E_i = \rho_s J_i + (\rho_s \mu_n r_H) \epsilon_{ijk} J_j B_k$$

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## Hall effect analysis



$$\mathcal{E}_i = \rho_s J_i + (\rho_s \mu_n r_H) \epsilon_{ijk} J_j B_k$$

## Hall effect analysis

$$\mathcal{E}_i = \rho_s J_i + (\rho_s \mu_n r_H) \epsilon_{ijk} J_j B_k$$

$$\mathcal{E}_y = \rho_s J_y + (\rho_s \mu_n r_H) \epsilon_{yxz} J_x B_z$$

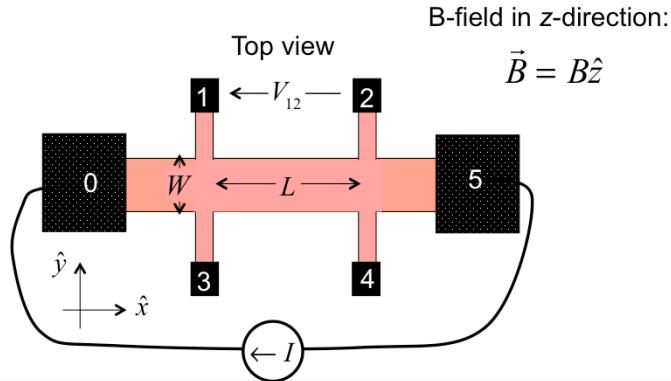
$$\mathcal{E}_y = \rho_s J_y + (\rho_s \mu_n r_H) \epsilon_{yxz} J_x B_z$$

B-field in z-direction:  
 $\vec{B} = B \hat{z}$

$$\mathcal{E}_y = -(\rho_s \mu_n r_H) J_x B_z$$

$$\frac{\mathcal{E}_y}{J_x B_z} = -(\rho_s \mu_n r_H) = \frac{r_H}{(-q) n_s}$$

## Hall effect analysis

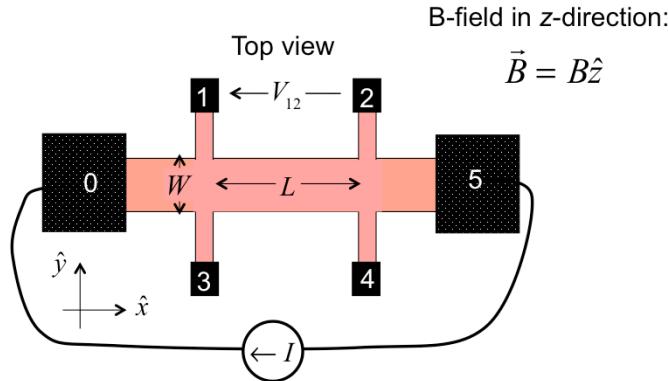


$$\frac{\mathcal{E}_y}{J_x B_z} = -(\rho_s \mu_n r_H) = \frac{r_H}{(-q)n_S} \quad R_{xy} = \frac{V_{1,3}}{I} - \frac{-\mathcal{E}_y W}{W J_x B_z} = \frac{r_H}{q n_S} = \frac{1}{q n_H}$$

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## Hall effect analysis



$$\mathcal{E}_i = \rho_s J_i + (\rho_s \mu_n r_H) \epsilon_{ijk} J_j B_k$$

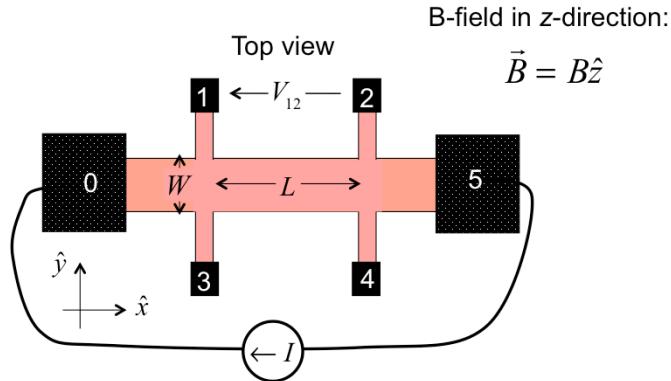
$$\mathcal{E}_x = \rho_s J_x + (\rho_s \mu_n r_H) \epsilon_{xxz} J_x B_z = \rho_s J_x$$

$$R_{xx} = \frac{V_{12}}{I} = \mathcal{E}_x L = \rho_s \frac{L}{W}$$

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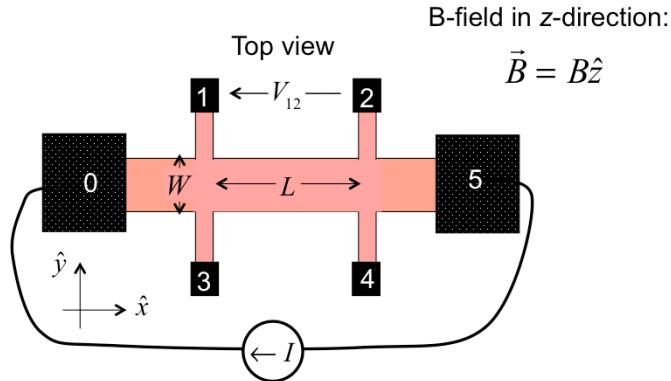
## Hall effect analysis



$$R_{xx} = \frac{V_{1,2}}{I} = \rho_s \frac{L}{W}$$

$$R_{xy} = \frac{V_{1,3}}{I} - \frac{-\mathcal{E}_y W}{W J_x B_z} = \frac{r_H}{q n_s} = \frac{1}{q n_H}$$

## what can go wrong?



## B-dependent transport tensors

$$\mathcal{E}_i = \rho_{ij}(\vec{B}) J_j + S_{ij}(\vec{B}) \partial_j T$$

$$J_i^Q = \pi_{ij}(\vec{B}) J_j - \kappa_{ij}^e(\vec{B}) \partial_j T$$

Magnetic fields introduce off-diagonal elements, which lead to effects like the Hall effect.

$$\rho_{ij} = \rho_0 + \rho_1 \epsilon_{ijk} B_k$$

$$\rho_{ij} = \rho_0 + (\rho_s \mu_n r_H) \epsilon_{ijk} B_k$$

## form of the tensors

$$\rho_{ij}(\vec{B}) = \rho_0 + \rho_0 \mu_H \epsilon_{ijk} B_k + \dots$$

$$\mathcal{E}_i = \rho_{ij}(\vec{B}) J_j + S_{ij}(\vec{B}) \partial_j T$$

$$J_i^Q = \pi_{ij}(\vec{B}) J_j - \kappa_{ij}^e(\vec{B}) \partial_j T$$

$$S_{ij}(\vec{B}) = S_0 + S_1 \epsilon_{ijk} B_k + \dots$$

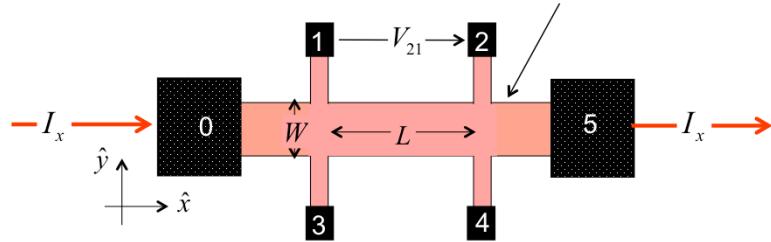
$$\pi_{ij}(\vec{B}) = \pi_0 + \pi_1 \epsilon_{ijk} B_k + \dots$$

$$\kappa_{ij}^e(\vec{B}) = \kappa_0 + \kappa_1 \epsilon_{ijk} B_k + \dots$$

See Smith, Janek, and Adler, Chapter 9, for expressions for the B-field dependent transport tensors.

## Hall effect (errors)

Top view



We have assumed isothermal conditions to compute the Hall voltage, but we expect Peltier cooling at contact 0 and Peltier heating at contact 1. If the sample is not isothermal, how does the Hall voltage change?

## Nernst effect

Assume that there is a temperature gradient in the  $x$ -direction. How is the electric field (Hall voltage) affected?)

$$\rho_{ij}(\vec{B}) = \rho_0 + \rho_0 \mu_H \epsilon_{ijk} B_k + \dots$$
$$\mathcal{E}_i = \rho_{ij}(\vec{B}) J_j + S_{ij}(\vec{B}) \partial_j T$$
$$S_{ij}(\vec{B}) = S_0 + S_1 \epsilon_{ijk} B_k + \dots$$

$$\mathcal{E}_y = \rho_0 J_y + \rho_0 \mu_H \epsilon_{yz} B_z J_z + S_0 \partial_y T + S_1 \epsilon_{yz} B_z \partial_z T$$

$$\mathcal{E}_y = +\rho_0 \mu_H \epsilon_{yxz} B_z J_x + S_1 \epsilon_{yxz} B_z \partial_x T$$

$$\mathcal{E}_y = -\rho_0 \mu_H B_z J_x - S_1 B_z \partial_x T$$

**Nernst voltage**

Reverse direction of  $B_z$  and  $J_x$  and average results to eliminate.

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## other effects

Righi-Leduc effect:

$$\vec{J} = 0 \quad \vec{B} = B_z \hat{z} \quad \frac{\partial T}{\partial y} = \frac{\kappa_1}{\kappa_0} \frac{\partial T}{\partial x}$$
$$J_y^q = 0$$

Ettingshausen effect:

$$\vec{J} = J_x \hat{x} \quad \vec{B} = B_z \hat{z}$$
$$\frac{\partial T}{\partial x} = 0 \quad J_y^q = 0 \quad \left| \frac{\partial T}{\partial y} \right| > 0$$

(See Lundstrom, FCT, Sec. 4.6.2)

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## approaches

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- 2-probe measurements
- 4-probe measurements
- TLM measurements
- Hall bar measurements
- Van der Pauw measurements