

Week 13 Summary: Balance Equations

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ECE 656

Transport physics:

- Near-equilibrium (coupled flows)
- Hot carrier or high-field
- Non-local and ballistic transport in devices

Analytical and numerical techniques:

- Landauer approach
- Boltzmann Transport Equation
- Moments of the BTE
- Monte Carlo simulation
- (Quantum transport)

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Boltzmann Transport Equation (BTE)

$$\frac{\partial f}{\partial t} + \vec{v} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C}f$$

in general, very difficult to solve

moments of the BTE

- physical quantities are moments of $f(\vec{r}, \vec{p}, t)$

$$n_\phi(\vec{r}, t) = \sum_{\vec{p}} \phi(\vec{p}) f(\vec{r}, \vec{p}, t)$$

$$\text{e.g. } \vec{J}_n(\vec{r}, t) = \sum_{\vec{p}} (-q) \vec{v} f(\vec{r}, \vec{p}, t)$$

more moments

$$\phi(\vec{p}) = 1: \quad n_\phi(\vec{r}, t) = n(\vec{r}, t) \quad \text{electron density}$$

$$\phi(\vec{p}) = (-q)\vec{v}(\vec{p}): \quad n_\phi(\vec{r}, t) = \vec{J}_n(\vec{r}, t) \quad \text{current density}$$

$$\phi(\vec{p}) = E(\vec{p}): \quad n_\phi(\vec{r}, t) = W(\vec{r}, t) \quad \text{kinetic energy density}$$

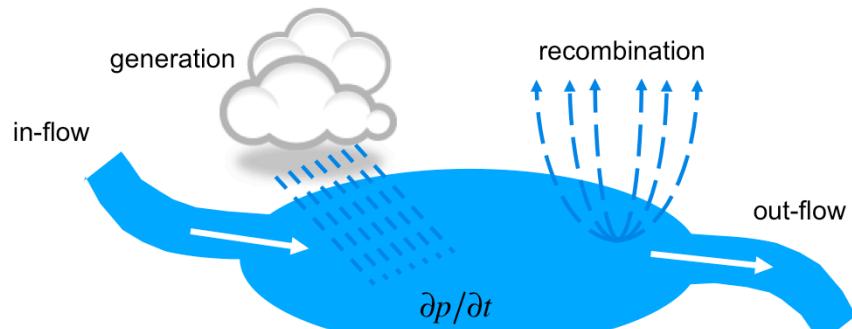
$$\phi(\vec{p}) = \vec{v}(\vec{p})E(\vec{p}): \quad n_\phi(\vec{r}, t) = \vec{F}_E(\vec{r}, t) \quad \text{energy current density}$$

Can we bypass solving the BTE and solve directly for the physical quantities of interest?

the continuity equation for holes

A familiar balance equation:

$$\frac{\partial p}{\partial t} = -\nabla \cdot \frac{\vec{J}_p}{q} + G_p - R_p$$

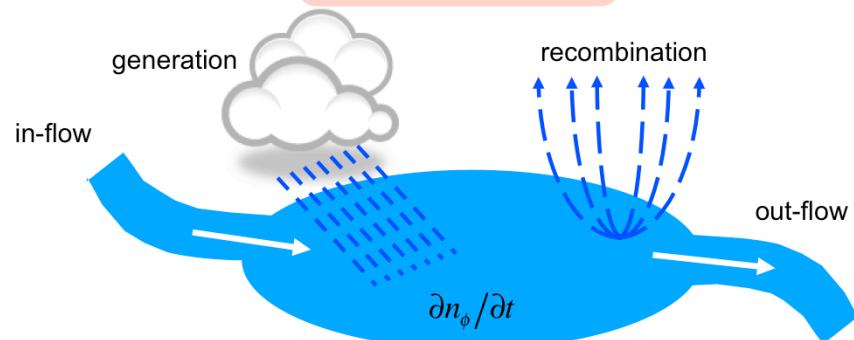


$$\text{in-flow} - \text{out-flow} = - \text{divergence of flux}$$

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the continuity equation for n_ϕ

$$\frac{\partial n_\phi}{\partial t} = -\nabla \cdot \vec{F}_\phi + G_\phi - R_\phi$$



in-flow - out-flow = - divergence of flux

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putting it all together (1D)

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} - \frac{q\mathcal{E}_x}{\hbar} \frac{\partial f}{\partial k_x} = \hat{C}f$$

$$\frac{\partial n_\phi}{\partial t} = -\nabla \cdot \vec{F}_\phi + G_\phi - R_\phi$$

$$n_\phi(x,t) = \frac{1}{L} \sum_p \phi(p) f(x,p,t) \quad G_\phi = -q\mathcal{E}_x \left\{ \frac{1}{L} \sum_{p_x} \frac{\partial \phi}{\partial p_x} f \right\}$$

$$F_\phi \equiv \frac{1}{L} \sum_{p_x} \phi(p_x) v_x f(x,p_x,t) \quad R_\phi \equiv \frac{n_\phi - n_\phi^0}{\langle \tau_\phi \rangle}$$

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“generation” of n_ϕ

$$G_\phi \equiv -q\mathcal{E}_x \left\{ \frac{1}{L} \sum_{p_x} \frac{\partial \phi}{\partial p_x} f \right\} \quad \text{why?}$$

$$G_\phi \equiv \left\{ \frac{1}{L} \sum_{p_x} \frac{\partial \phi}{\partial t} f \right\} = \left\{ \frac{1}{L} \sum_{p_x} \frac{\partial \phi}{\partial p_x} \frac{dp_x}{dt} f \right\}$$

$$\frac{dp_x}{dt} = -q\mathcal{E}_x \quad \text{equation of motion in momentum (k) space}$$

$$G_\phi = -q\mathcal{E}_x \left\{ \frac{1}{L} \sum_{p_x} \frac{\partial \phi}{\partial p_x} f \right\}$$

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relaxation time approximation

Microscopic RTA

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} - \frac{q\mathcal{E}_x}{\hbar} \frac{\partial f}{\partial k_x} = \hat{C}f$$

$$\hat{C}f(\vec{p}) = -\frac{f(\vec{p}) - f_s(\vec{p})}{\tau_m(\vec{p})}$$

Macroscopic RTA

$$\frac{\partial n_\phi}{\partial t} = -\nabla \cdot \vec{F}_\phi + G_\phi - R_\phi$$

$$R_\phi \equiv \frac{n_\phi - n_\phi^0}{\langle \tau_\phi \rangle}$$

balance equation prescription

$$\sum_{\vec{p}} \phi(\vec{p}) \left\{ \frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} - \frac{q \mathcal{E}_x}{\hbar} \frac{\partial f}{\partial k_x} = \hat{C}f \right\}$$

$$\frac{\partial n_\phi}{\partial t} = -\nabla \cdot \vec{F}_\phi + G_\phi - R_\phi$$

$$n_\phi(\vec{r}, t) = \frac{1}{L^d} \sum_{\vec{p}} \phi(\vec{p}) f(\vec{r}, \vec{p}, t) \quad G_\phi = -q \vec{\mathcal{E}} \cdot \left\{ \frac{1}{L^d} \sum_{\vec{p}} \vec{\nabla}_{\vec{p}} \phi f \right\}$$

$$\vec{F}_\phi \equiv \frac{1}{L^d} \sum_{\vec{p}} \phi(\vec{p}) \vec{v} f(\vec{r}, \vec{p}, t) \quad R_\phi \equiv \frac{n_\phi - n_\phi^0}{\langle \tau_\phi \rangle}$$

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balance equations hierarchy

$$\phi(\vec{p}) = 1 \quad \text{electron continuity equation}$$

$$\phi(\vec{p}) = \vec{p} \quad \text{momentum balance equation}$$

$$\phi(\vec{p}) = E(\vec{p}) \quad \text{energy balance equation}$$

$$\phi(\vec{p}) = v(\vec{p})E(\vec{p}) \quad \text{energy flux balance equation}$$

Terminating the hierarchy

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