

## Week 15 Summary: High-Field Transport

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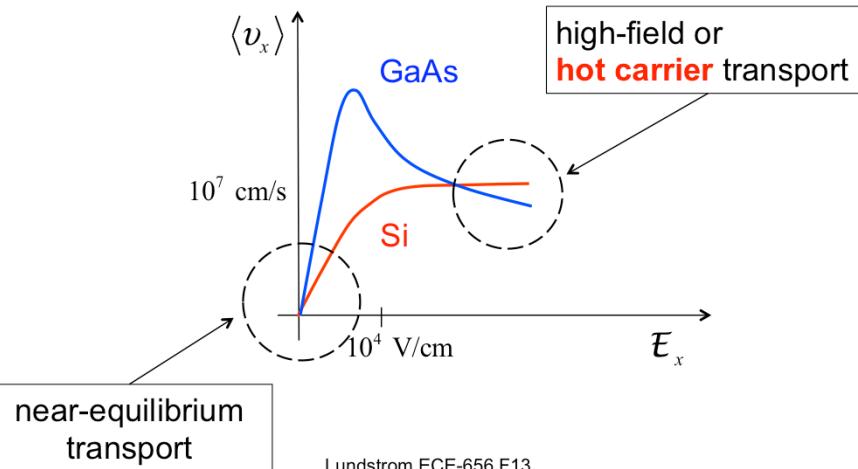


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## velocity vs. field characteristics

(**bulk** semiconductors assumed)



## current equation

$$J_{nx} = nq\mu_n \mathcal{E}_x + 2\mu_n \frac{dW_{xx}}{dx} \quad W_{xx} = \frac{1}{\Omega} \sum_k \frac{1}{2} m^* v_x^2 = n u_{xx}$$

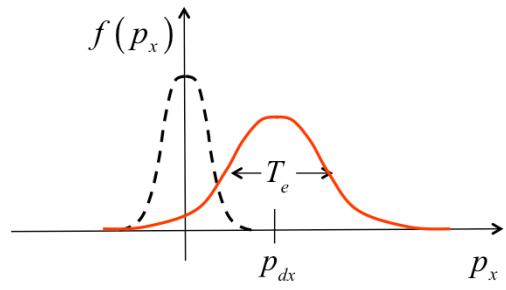
This is an “exact” steady-state current equation, but....

$$\mu_n [f(\vec{r}, \vec{p}, t)] \quad u_{xx} [f(\vec{r}, \vec{p}, t)] = \left\langle \frac{1}{2} p_z v_z \right\rangle$$

## electron kinetic energy

$$W_{xx} = \frac{1}{\Omega} \sum_{\vec{k}} \frac{1}{2} m^* v_x^2 = n u_{xx}$$

$$u_{xx} = \frac{1}{2} m^* v_{dx}^2 + \frac{3}{2} k_B T_e$$



## aside: neglect of the drift energy

$$u = \frac{1}{2} m^* v_d^2 + \frac{3}{2} k_B T_e \approx \frac{3}{2} k_B T_e$$

$$\frac{m^* v_d^2 / 2}{3k_B T_e / 2} = \frac{\langle \tau_m \rangle}{\langle \tau_E \rangle} \ll 1$$

The drift energy is small when the energy relaxation time is much larger than the momentum relaxation time (which is the typical case).

## current equation: bulk semiconductor

$$J_{nx} = nq\mu_n \mathcal{E}_x + 2\mu_n \frac{d(nu_{xx})}{dx}$$

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{2u_{xx}}{q}$$

near equilibrium:  $u_{xx} \approx \frac{k_B T_L}{2}$

high-fields:  $u_{xx} \approx \frac{k_B T_e}{2}$   
 $T_e > T_L$

## field-dependent mobility

$$J_{nx} = nq\mu_n \mathcal{E}_x + qD_n \frac{dn}{dx}$$

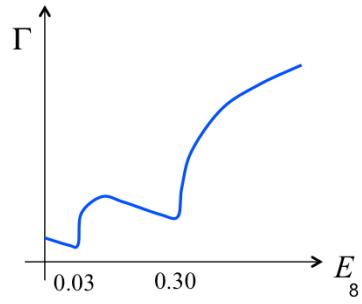
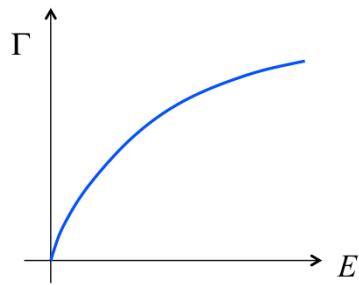
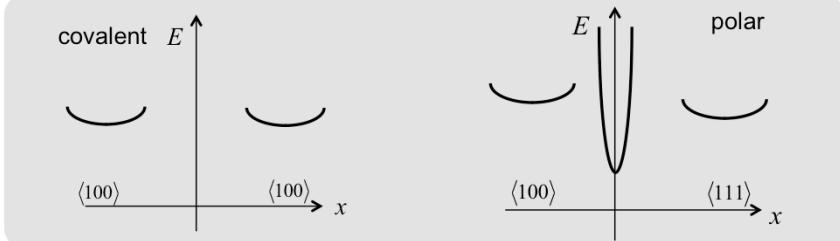
In general:  $\mu_n[f(\vec{r}, \vec{p}, t)] = D_n[f(\vec{r}, \vec{p}, t)]$

In a bulk semiconductor,  $f$  is determined by  $\mathcal{E}$ , so there is a one-to-one mapping between  $\mathcal{E}$  and  $f$ .

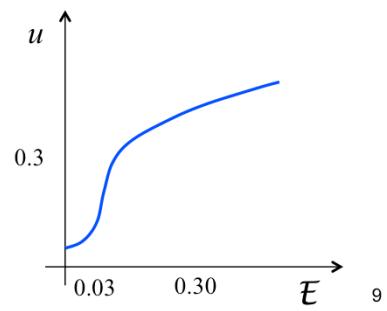
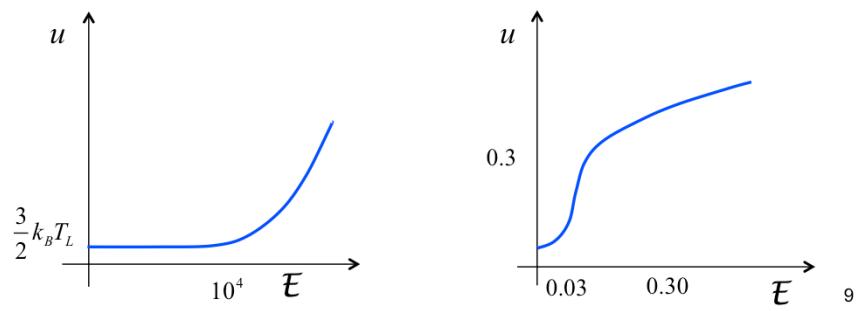
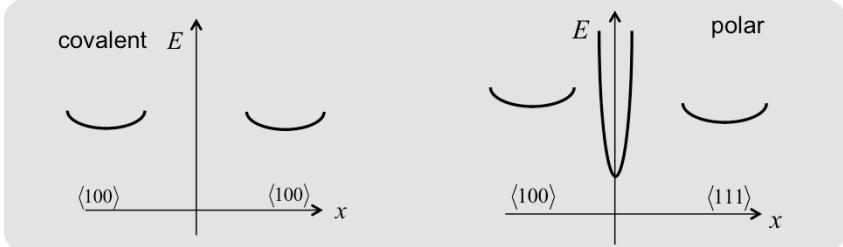
$\mu_n(\mathcal{E})$   $D_n(\mathcal{E})$  Electric **field dependent** mobility and diffusion coefficient.

$$J_{nx} = nq\mu_n(\mathcal{E}) \mathcal{E}_x + qD_n(\mathcal{E}) \frac{dn}{dx}$$

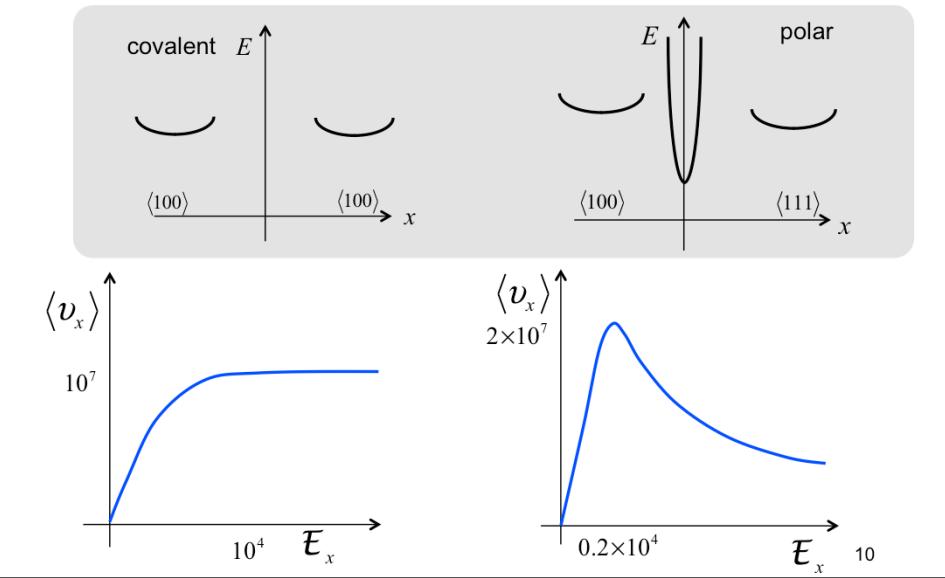
## covalent vs. polar semiconductors



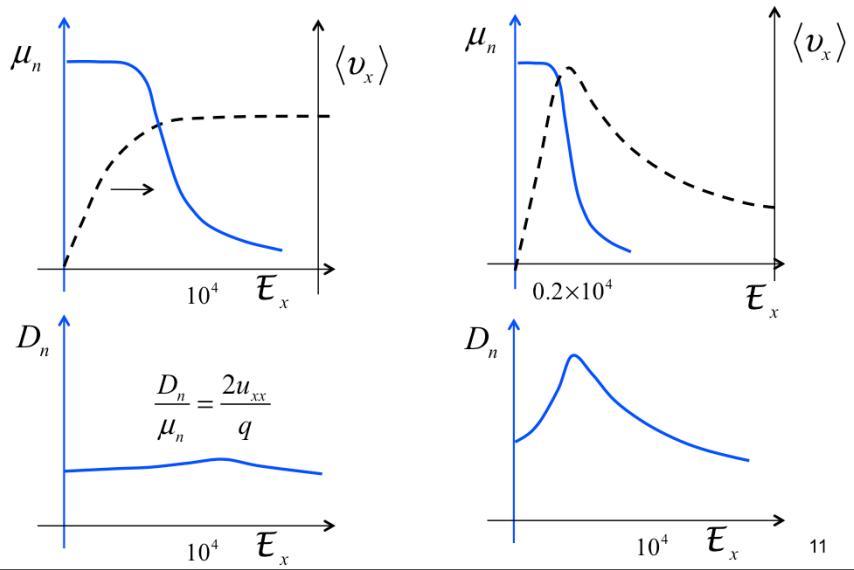
## average energy vs. electric field



## average velocity vs. electric field



## mobility and diffusion coefficient



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## saturation velocity

$$v_{SAT} \approx \sqrt{\frac{\hbar\omega_0}{m^*}}$$

Si:  $\hbar\omega_0 = 0.063$  eV  $v_{SAT} \approx 1.0 \times 10^7$  cm/s

Ge:  $\hbar\omega_0 = 0.037$  eV  $v_{SAT} \approx 0.6 \times 10^7$  cm/s

SiC:  $\hbar\omega_0 = 0.12$  eV  $v_{SAT} \approx 1.5 \times 10^7$  cm/s

## electron temperature approach

$$f(\vec{p}) = e^{-|\vec{p} - m^* \vec{v}_d|^2 / 2m^* k_B T_e}$$

2 unknowns:  $v_{dx}, T_e$  ...need 2 equations

1) Momentum balance:

$$J_{nx} = nq\mu_n \mathcal{E}_x \rightarrow v_{dx} = -\mu_n \mathcal{E}_x$$

2) Energy balance:

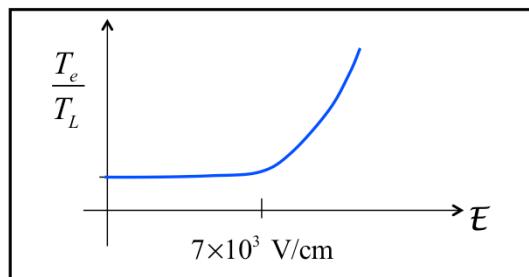
$$J_{nx} \mathcal{E}_x = nq\mu_n \mathcal{E}_x^2 = \frac{n(u - u_0)}{\langle \tau_E \rangle}$$

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## result (for silicon)

$$\frac{T_e}{T_L} = 1 + \frac{q\mu_{n0}}{C} \mathcal{E}_x^2 = 1 + (\mathcal{E}/\mathcal{E}_c)^2 \quad \mathcal{E}_c \approx 7 \times 10^3 \text{ V/cm}$$



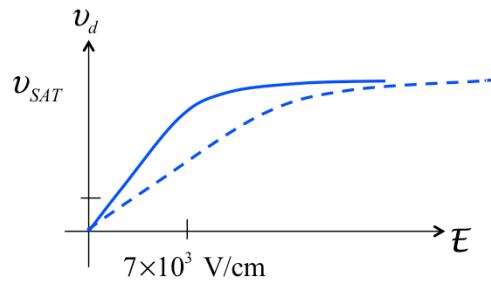
$$\mu_n(T_e) = \mu_0 \sqrt{T_L/T_e} \quad (\text{ADP}) \quad \mu_n(T_e) = \mu_0 (T_e/T_L)^{3/2} \quad (\text{II})$$

## velocity vs. field characteristic

$$\mu_n(T_e) = \frac{\mu_{n0}}{\sqrt{1+(\mathcal{E}/\mathcal{E}_c)^2}}$$

$$v_d = \mu_n(T_e)\mathcal{E} = \frac{\mu_{n0}\mathcal{E}}{\sqrt{1+(\mathcal{E}/\mathcal{E}_c)^2}}$$

$$v_{SAT} = \mu_{n0}\mathcal{E}_c = 1 \times 10^7 \text{ cm/s}$$



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## <111> Silicon: low-field

$$\mathcal{E}_z = -100 \text{ V/cm}$$

$$\langle v_z \rangle = 8.1 \times 10^4 \text{ cm/s}$$

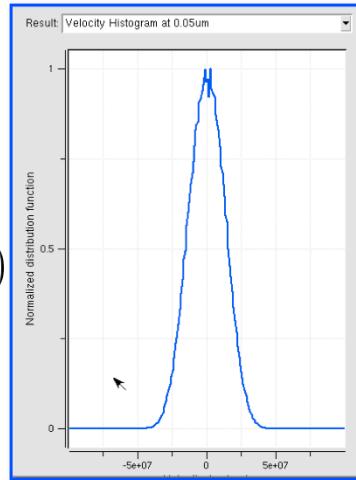
$$\mu_n(\mathcal{E}_z) = 810 \text{ cm}^2/\text{V-s}$$

$$u = 0.04 \text{ eV} \quad (1.5k_B T_L / q = 0.039 \text{ eV})$$

$$u_{zz} = 0.0135 \text{ eV} \quad (u_{zz} / u = 0.34)$$

$$u_{drift} \sim 10^{-7} \text{ eV} \quad (u_{drift} / u \sim 10^{-5})$$

$$n(x, y, z)/n = 0.33 / 0.335 / 0.335$$



(simulations performed with DEMONs on [www.nanoHUB.org](http://www.nanoHUB.org)) 16

## <111> Silicon: high-field

$$\mathcal{E}_z = -10^5 \text{ V/cm}$$

$$\langle v_z \rangle = 1.04 \times 10^7 \text{ cm/s}$$

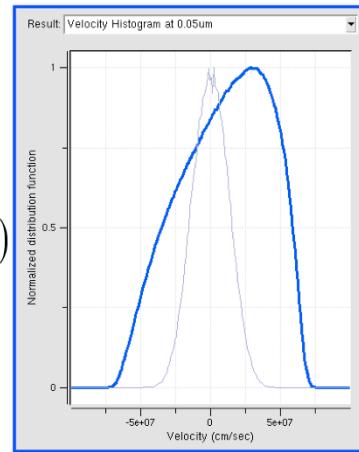
$$\mu_n(\mathcal{E}_z) = 104 \text{ cm}^2/\text{V-s}$$

$$u = 0.364 \text{ eV} \quad (1.5k_B T_L / q = 0.039 \text{ eV})$$

$$u_{zz} = 0.145 \text{ eV} \quad (u_{zz} / u = 0.40)$$

$$u_{drift} = 0.008 \text{ eV} \quad (u_{drift} / u = 0.02)$$

$$n(x, y, z)/n = 0.336 / 0.331 / 0.333$$

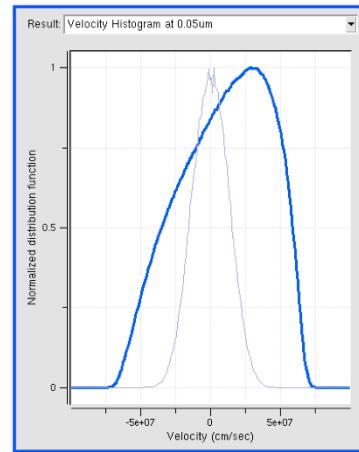
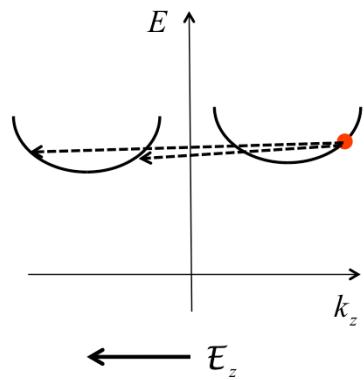


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## <111> Silicon: high-field



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## <100> Silicon: high-field

$$\mathcal{E}_z = -10^5 \text{ V/cm}$$

$$\langle v_z \rangle = 0.98 \times 10^7 \text{ cm/s}$$

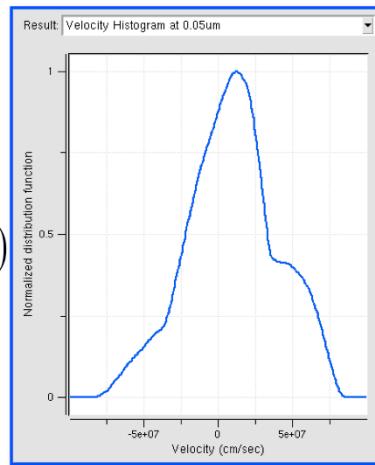
$$\mu_n(\mathcal{E}_z) = 98 \text{ cm}^2/\text{V-s}$$

$$u = 0.346 \text{ eV} \quad (1.5k_B T_L / q = 0.039 \text{ eV})$$

$$u_{zz} = 0.138 \text{ eV} \quad (u_{zz} / u = 0.40)$$

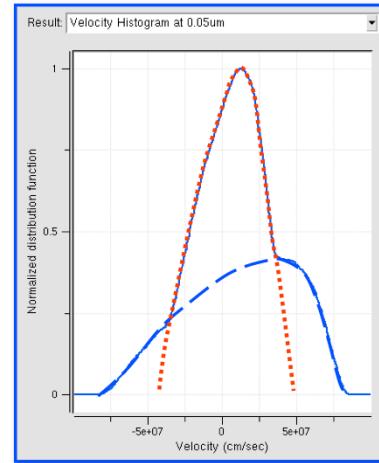
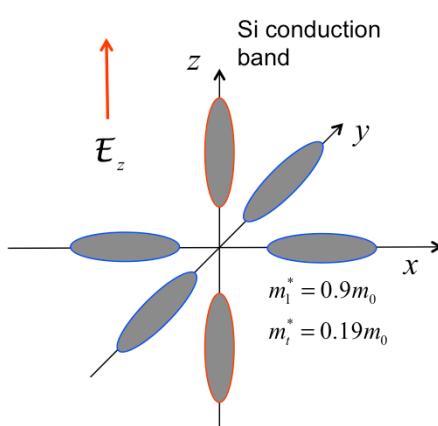
$$u_{drift} = 0.007 \text{ eV} \quad (u_{drift} / u = 0.02)$$

$$n(x, y, z)/n = 0.306 / 0.309 / 0.385$$



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## <100> Silicon: high-field



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## summary

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- 1) High-field transport leads to field-dependent mobilities and diffusion coefficients (when the field varies slowly in space and time).
- 2) Balance equations provide a useful way to interpret detailed (e.g. Monte Carlo) simulations.
- 3) The electron temperature approach provides a qualitative (and sometimes quantitative) way to view high-field (**hot carrier**) transport.

## non-local transport

Rapidly varying electric fields lead to “off-equilibrium”, “non-local” or “non-stationary” transport effects that cannot be described with (local) field-dependent field dependent transport parameters.

