ECE 656: Fundamentals of Carrier Transport Fall 2013

Week 16 Summary: Non-local Transport

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11/29/13

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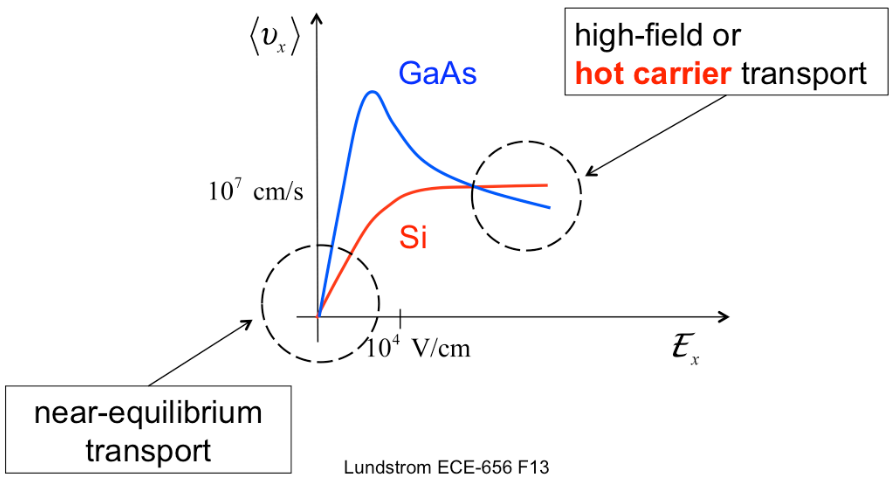
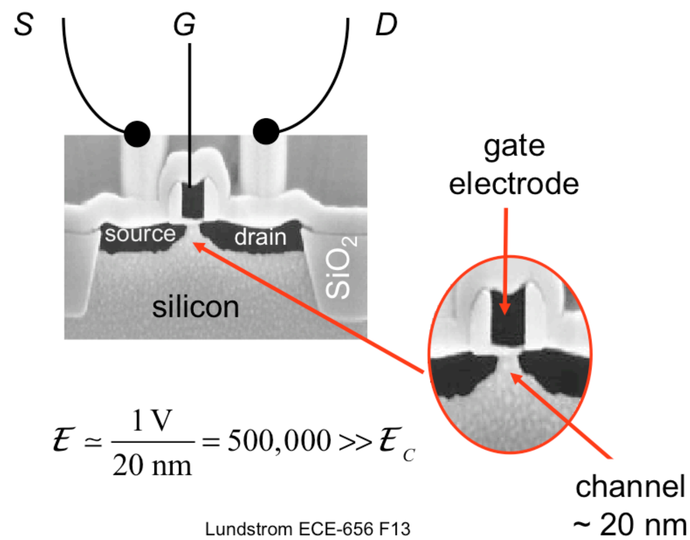
velocity vs. field characteristics

(***bulk*** semiconductors assumed)



nanoscale MOSFETs 2009

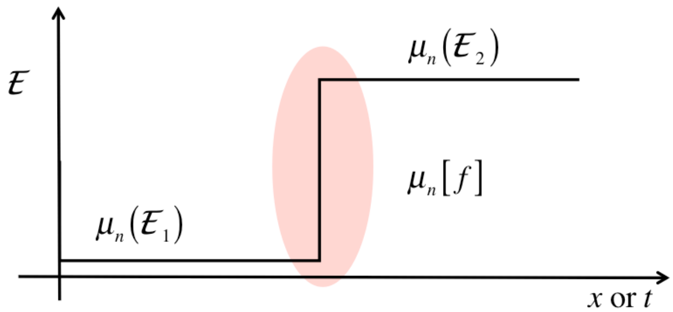


$$\mathcal{E} \simeq \frac{1\text{ V}}{20\text{ nm}} = 500{,}000 >> \mathcal{E}_C$$

this is what it's all about…… making transistors smaller and smaller and keeping Moore's Law going

non-local transport

Rapidly varying electric fields lead to "off-equilibrium", "non-local" or "non-stationary" transport effects that cannot be described with (local) field-dependent field dependent transport parameters.



analysis techniques

1) Field-dependent mobility:

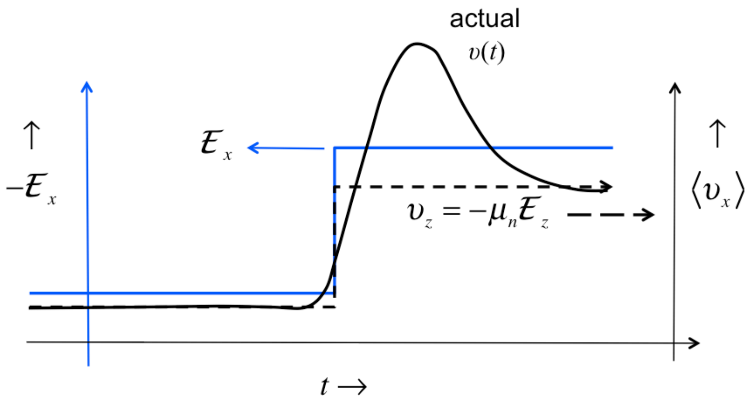
$$J_{nx} = nq\mu_n(\mathcal{E})\mathcal{E}_x + qD_n(\mathcal{E})\frac{dn}{dx}$$

2) energy transport:

$$J_{nx} = nq\mu_n(u)\mathcal{E}_x + 2\mu_n(u)\frac{d(nu)}{dx}$$

3) Monte Carlo simulation:

rapidly varying electric fields



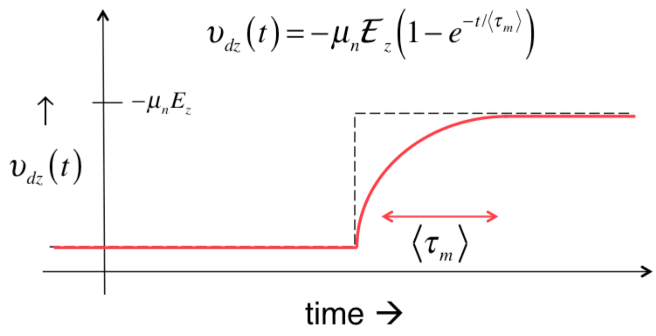
velocity overshoot

$p_{dx} = \langle p_x \rangle$ Let's find an equation for the ave. *x*-directed momentum.

$$\frac{dp_{dx}}{dt} = -q\mathcal{E}_x - \frac{p_{dx}}{\langle \tau_m \rangle}$$ (ignores diffusion)

$$\upsilon_{dx}(t) = -\mu_n \mathcal{E}_x \left(1 - e^{-t/\langle \tau_m \rangle}\right)$$

velocity overshoot



But, μ_n is not constant: $\mu_n(T_e)$

velocity overshoot

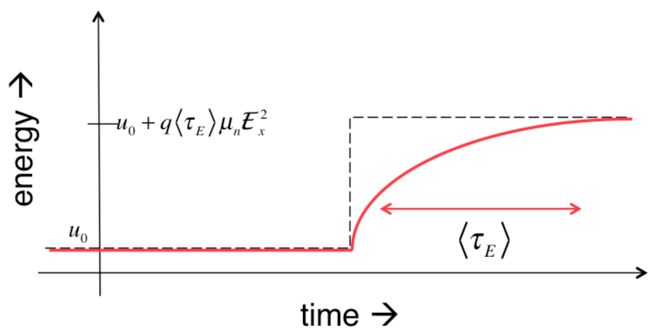
$u = \langle E - E_C \rangle$ Let's find an equation for the ave. kinetic energy

$$\frac{du}{dt} = -q\upsilon_{dx}\mathcal{E}_x - \frac{(u - u_0)}{\langle \tau_E \rangle}$$ (ignores diffusion)

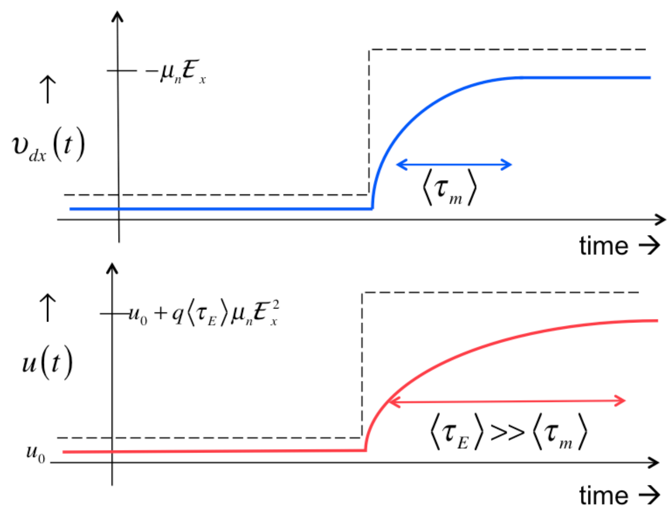
$$u(t) = u_0 + q\langle \tau_E \rangle \mu_n \mathcal{E}_x^2 \left(1 - e^{-t/\langle \tau_E \rangle}\right)$$

velocity overshoot

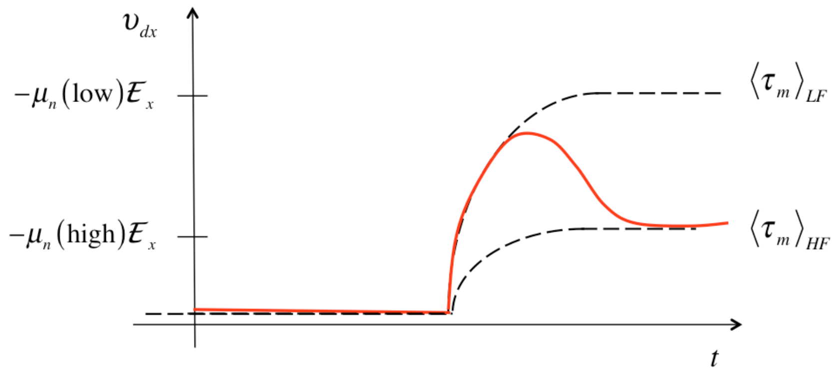
$$u(t) = u_0 + q\langle\tau_E\rangle\mu_n\mathcal{E}_x^2\left(1 - e^{-t/\langle\tau_E\rangle}\right)$$



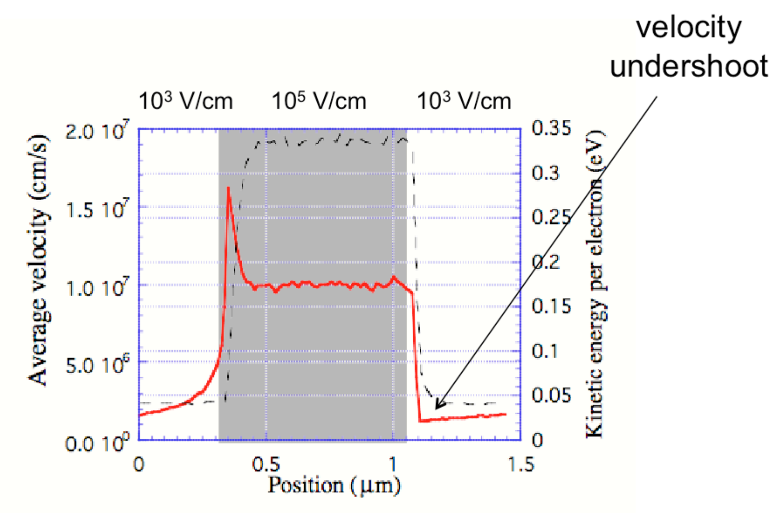
velocity overshoot



velocity overshoot



s.s. velocity overshoot in silicon



temporal velocity overshoot

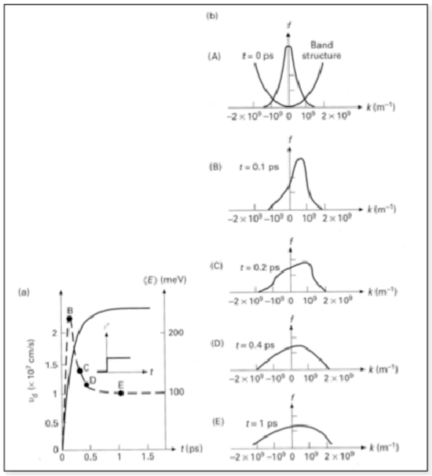


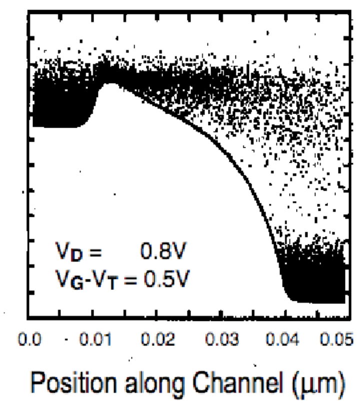
Fig. 8.9 Evolution of the distribution function during a velocity overshoot transient.

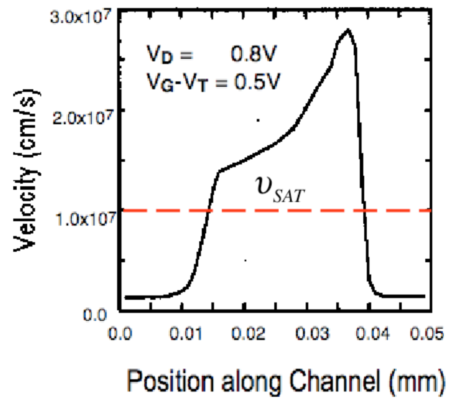
The average drift velocity and energy are shown in (a), and the evolution of the corresponding distribution function is shown in (b).

The results were obtained by Monte Carlo simulation of electron transport in silicon by E. Constant [8.10].

p. 335 of FCT

off-equilibrium nanoscale MOSFETs





Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

outline

1) Velocity overshoot

2) Ballistic BTE

BTE in equilibrium

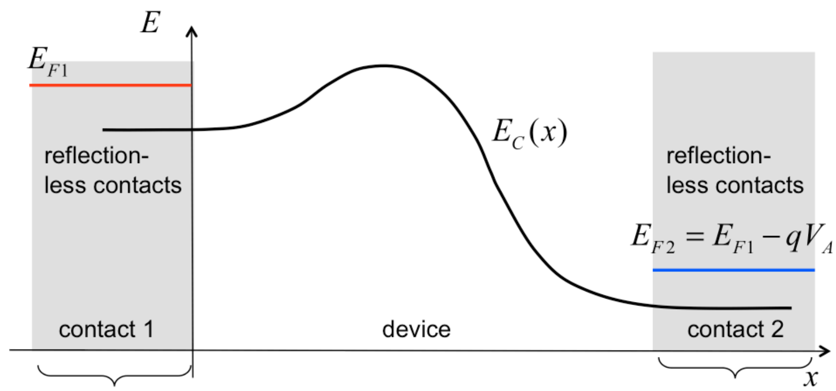
$$\frac{\partial f}{\partial t} + \vec{\upsilon} \bullet \nabla_r f + \vec{F}_e \bullet \nabla_p f = \hat{C} f$$

$\hat{C} f = 0$ in two cases:

-equilibrium

-ballistic transport

generic ballistic device



thermal equilibrium reservoir

ballistic

thermal equilibrium reservoir

$$f(E) = f_0(E_{F1})$$

$$f(E) \neq f_0(E)$$

$$f(E) = f_0(E_{F2})$$

solution for a ballistic device

Steady-state ballistic BTE:

$$\upsilon_x \bullet \frac{\partial f(x, p_x)}{\partial x} - q\mathcal{E}_x \frac{\partial f(x, p_x)}{\partial p_x} = 0$$

Solution:

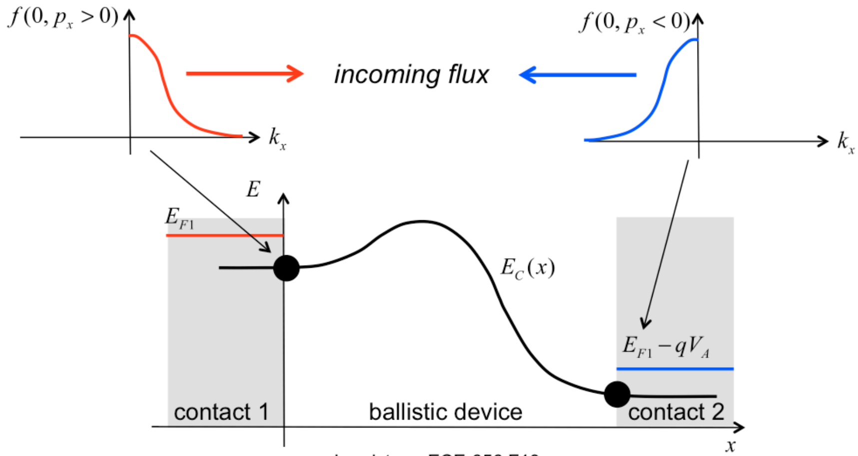
$$f(x, p_x) = g\left(E\right) = g\left[E_C(x) + E(k_x)\right]$$

Boundary conditions:

First-order equation in space --> **one** boundary condition, but we have two contacts!

boundary conditions

Solution: Apply one-half of the boundary condition to each contact.

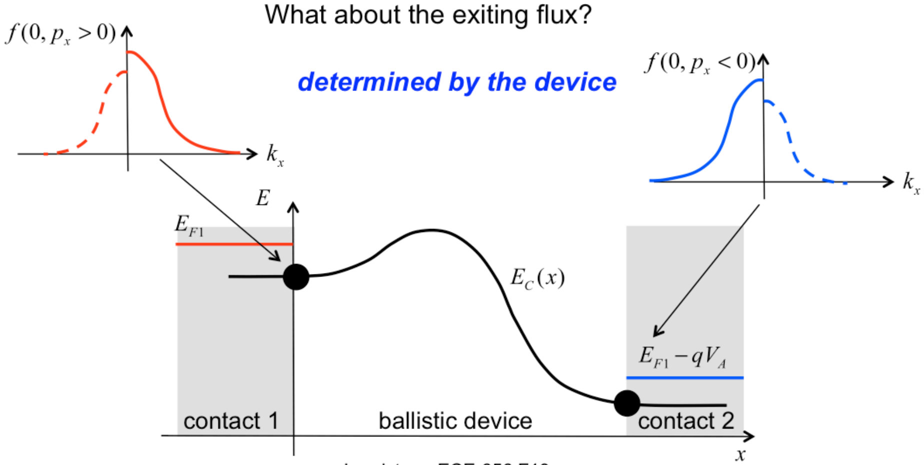


boundary conditions for the BTE

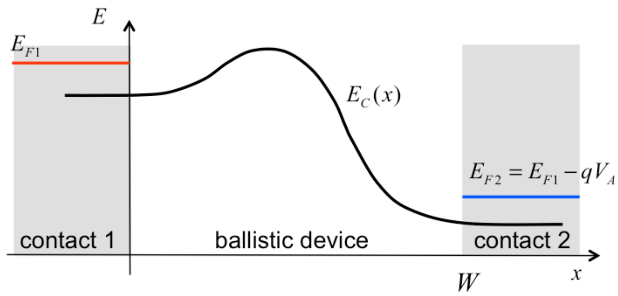
Solution: Specify incoming flux.

What about the exiting flux?

determined by the device



solution to the s.s. ballistic BTE

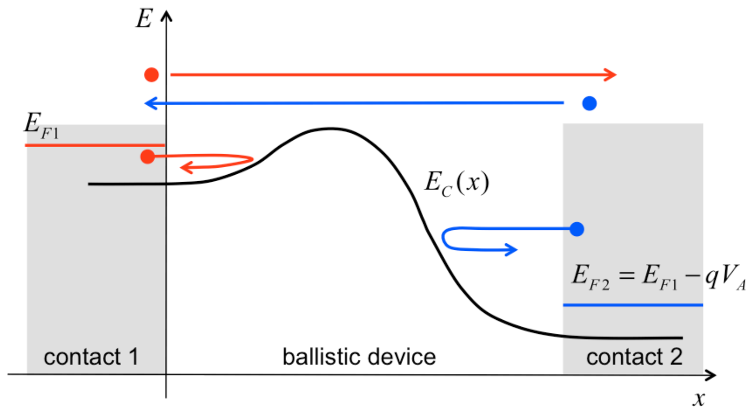


$$f(x,p_x) = g\left[E_C(x) + E(k_x)\right]$$

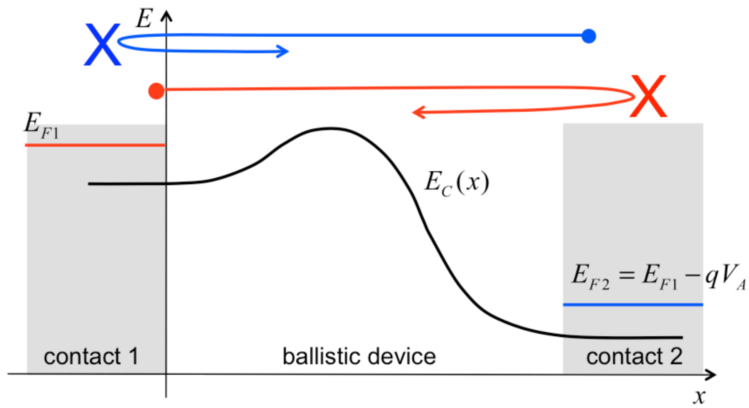
$$f(x,p_x) = \frac{1}{1+e^{(E-E_F)/k_B T_L}}$$

E_{F1} or E_{F2}?

follow trajectories in phase space



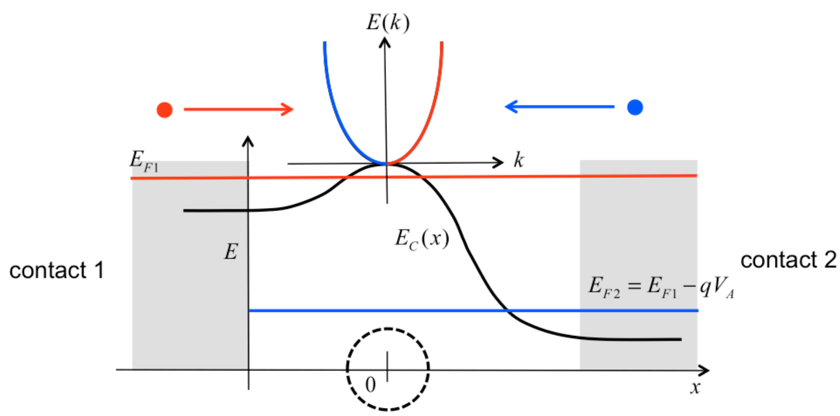
importance of reflection-less contacts



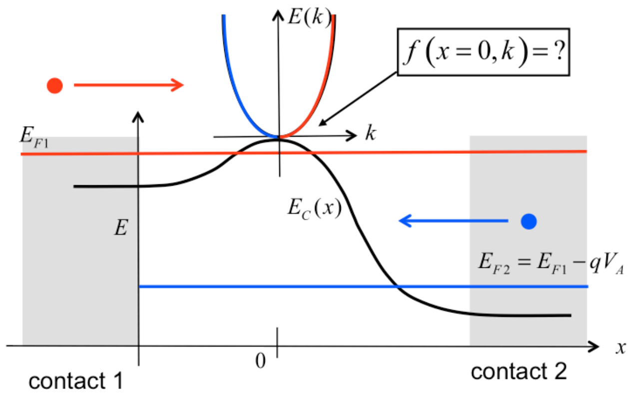
"reflection-less"
or
"absorbing" contacts

"reflection-less"
or
"absorbing" contacts

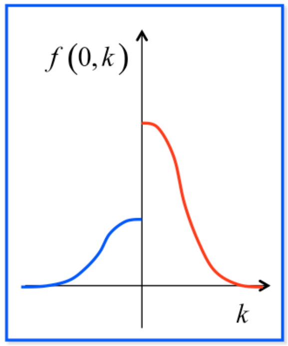
example



example

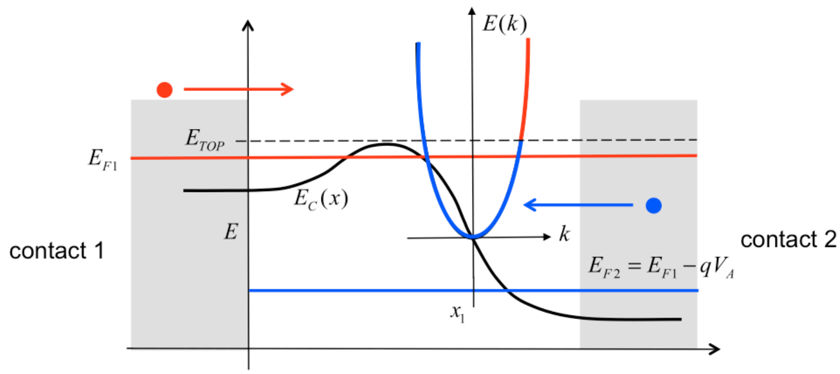


$E(k)$
$f\left(x=0,k\right)=?$
k
E_{F1}
E
$E_C(x)$
$E_{F2}=E_{F1}-qV_A$
x
0
contact 1
contact 2



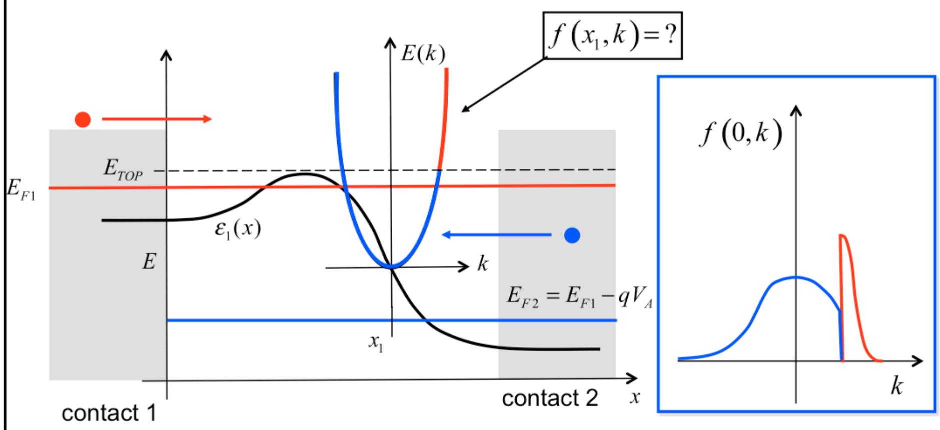
$f\left(0,k\right)$
k

another example



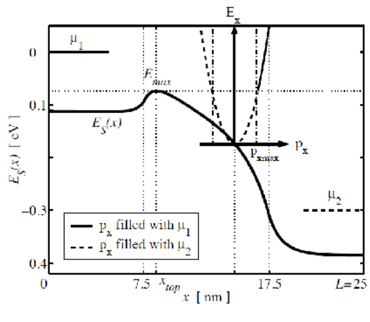
"local density of states"

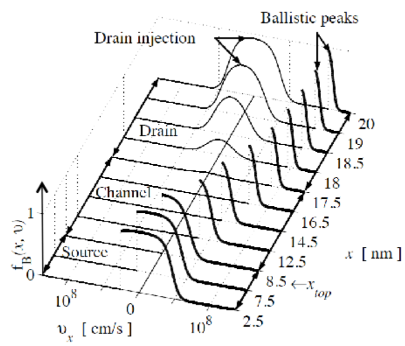
distribution function



J.-H. Rhew, Zhibin Ren, and Mark Lundstrom, "A Numerical Study of Ballistic Transport in a Nanoscale MOSFET," Solid-State Electronics, **46**,1899, 2002.

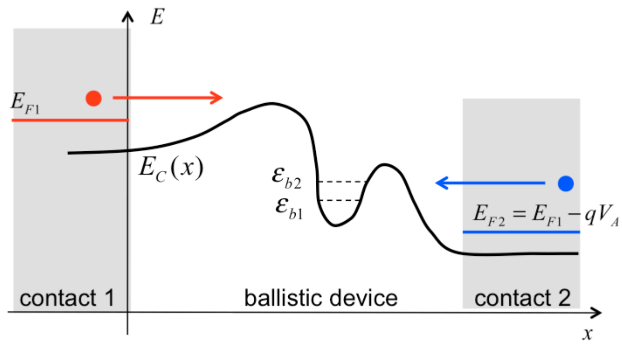
distribution function





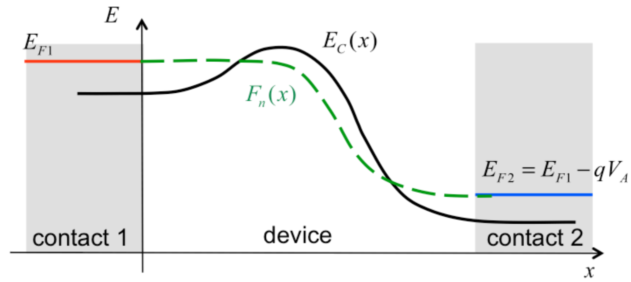
J.-H. Rhew, Zhibin Ren, and Mark Lundstrom, "A Numerical Study of Ballistic Transport in a Nanoscale MOSFET," *Solid-State Electronics*, **46**,1899, 2002.

bound states



Bound states can occur.
They may be difficult (or impossible to fill from the contacts).
In practice, they could be filled by scattering.

diffusive transport



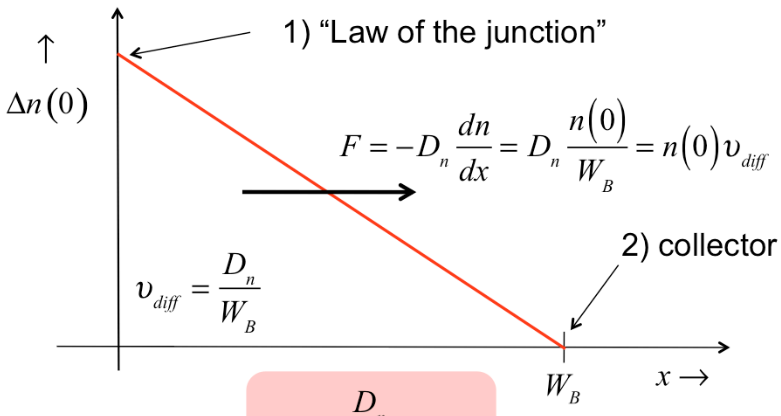
$$f(x,E) = \frac{1}{1 + e^{\left[E - F_n(x)\right]/k_B T_e(x)}} \qquad D_{1D}(x,E) = \frac{1}{\pi\hbar}\sqrt{2m^* / \left(E - E_C(x)\right)}$$

outline

1) Velocity overshoot

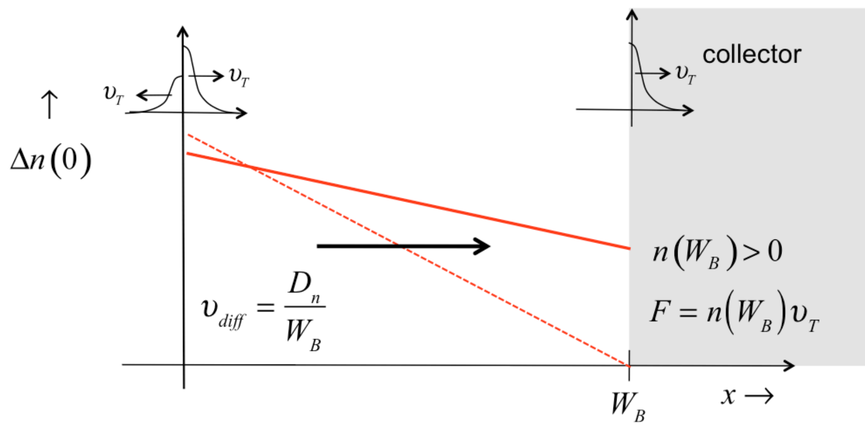
2) Ballistic BTE

Diffusion across a thin base: Fick's Law



$$\upsilon_{diff} = \frac{D_n}{W_B} << \upsilon_T$$

importance of boundary conditions



Fick's Law always holds – no matter how small the base!