ECE 656 Homework 1: Week 1

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1) Working out Fermi-Dirac integrals just takes some practice. For practice, work out the integral

$$I_1 = \int_{-\infty}^{\infty} M(E) f_0(E) dE$$

where

$$f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}}$$

and

$$M(E) = W \frac{\sqrt{2m^*(E-E_C)}}{\pi\hbar} H(E-E_C)$$

where

$H(E-E_C)$ is the unit step function.

2) For more practice, work out the integral in problem 1) assuming non-degenerate carrier statistics.

3) For still more practice, work out this integral:

$$I_2 = \int_{E_C}^{\infty} M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE \, ,$$

where $M(E)$ is as given in problem 1).

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4) It is important to understand when Fermi-Dirac statistics must be used and when non-degenerate (Maxwell-Boltzmann) statistics are good enough. The electron density in 1D is

$$n_L = N_{1D}\mathcal{F}_{-1/2}\left(\eta_F\right) \text{cm}^{-1} \,,$$

where N_{1D} is the 1D effective density of states and $\eta_F = \left(E_F - E_C\right)/k_B T$. In 3D,

$$n = N_{3D}\mathcal{F}_{1/2}\left(\eta_F\right) \text{cm}^{-3}.$$

For Maxwell Boltzmann statistics

$$n_L^{MB} = N_{1D}\exp\left(\eta_F\right) \text{cm}^{-1}$$
$$n^{MB} = N_{3D}\exp\left(\eta_F\right) \text{cm}^{-3}.$$

Compute the ratios, n_L/n_L^{MB} and n/n^{MB} for each of the following cases:

a) $\eta_F = -10$
b) $\eta_F = -3$
c) $\eta_F = 0$
d) $\eta_F = 3$
d) $\eta_F = 10$

Note that there is a Fermi-Dirac integral calculator available on nanoHUB.org. An iPhone app is also available.

5) Consider GaAs at room temperature doped such that $n = 10^{19}$ cm^{-3}. The electron density is related to the position of the Fermi level according to

$$n = N_C\mathcal{F}_{1/2}\left(\eta_F\right) \text{cm}^{-3},$$

where

$$N_C = 4.21\times 10^{17} \text{ cm}^{-3}.$$

Determine the position of the Fermi level relative to the bottom of the conduction band, E_C.

a) Assuming Maxwell-Boltzmann carrier statistics
b) NOT assuming Maxwell-Boltzmann carrier statistics