

**ECE 656 Homework 2 (Week 2)**

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- 1) Assume  $T = 0\text{K}$  and work out the electron density per unit area for two cases:
  - i) A 2D semiconductor with parabolic energy bands and an effective mass of  $m^*$ . (Assume a valley degeneracy of 2.)
  - ii) Graphene, where we consider  $E > 0$  to be the conduction band. ( $E = 0$  is where the bands cross, the so-called Dirac point.)
  - 1a) Express your two answers in terms of the Fermi energy, and show that they are **different**.
  - 1b) Express your two answers in terms of the Fermi wavevector and show that they are **the same**.
  
- 2) Assume a finite temperature and work out the sheet carrier densities,  $n_s$ , for:
  - 2a) Electrons in the conduction band of a 2D parabolic band semiconductor
  - 2b) Electrons in the conduction band ( $E > 0$ ) of graphene.
  
- 3) Assume  $T = 0\text{K}$  and work out the average +x-directed velocity for electrons in:
  - 3a) A 2D semiconductor with a parabolic conduction band and
  - 3b) The conduction band ( $E > 0$ ) of graphene.

Your answer should be in terms of the Fermi energy,  $E_F$ .

- 4) Assume a nonparabolic, 1D energy bandstructure described by:

$$E(k_x)[1 + \alpha E(k_x)] = \frac{\hbar^2 k_x^2}{2m^*(0)}$$

where

$$\frac{1}{m^*(0)} = \frac{1}{\hbar^2} \left. \frac{d^2 E(k_x)}{dk_x^2} \right|_{k_x=0}$$

**ECE 656 Homework 2 (Week 2) (continued)**

4a) Sketch (or produce a Matlab plot) of  $E(k)$  vs.  $k$  for two cases: i)  $\alpha = 0$  and ii)  $\alpha > 0$ . If you are producing a Matlab plot, the energy range should be from 0 to 1 eV, and you can assume  $\alpha = 0.5$  eV.

4b) For this bandstructure, derive an expression for the velocity,  $v_x(k_x)$  as a function of  $k_x$ .

5) For parabolic energy bands, the 2D density of states is

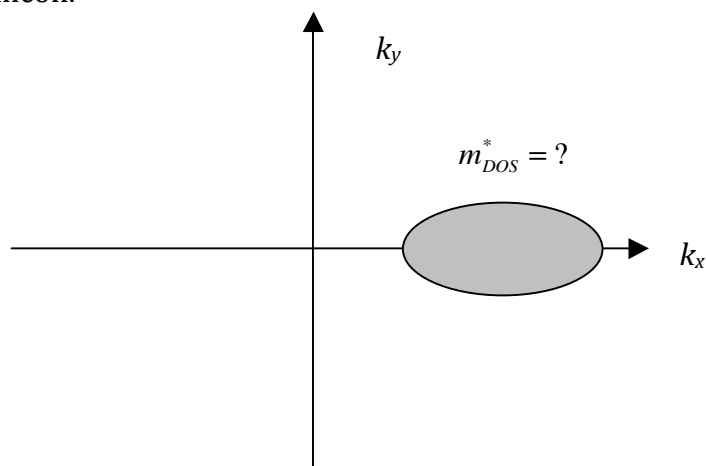
$$D_{2D}(E) = \frac{m^*}{\pi \hbar^2} \Theta(E - \epsilon_1)$$

Assume a non-parabolic band described by the so-called Kane dispersion,

$$E(k) \left[ 1 + \alpha E(k) \right] = \frac{\hbar^2 k^2}{2m^*(0)},$$

and derive the density of states.

6) Derive an expression for the 2D density of states for one of the conduction band ellipsoids in silicon.



HINT: You may find the discussion in Pierret (Advanced Semiconductor Fundamentals) on pp. 94-95 helpful.

**ECE 656 Homework 2 (Week 2) (continued)**

- 7) Assume an ultra thin body (100) silicon structure with a thickness of 3 nm. Assume no bandbending within the structure and infinitely high energy barriers at the oxide-silicon interfaces. Compute and plot the 2D density of states vs. energy.