

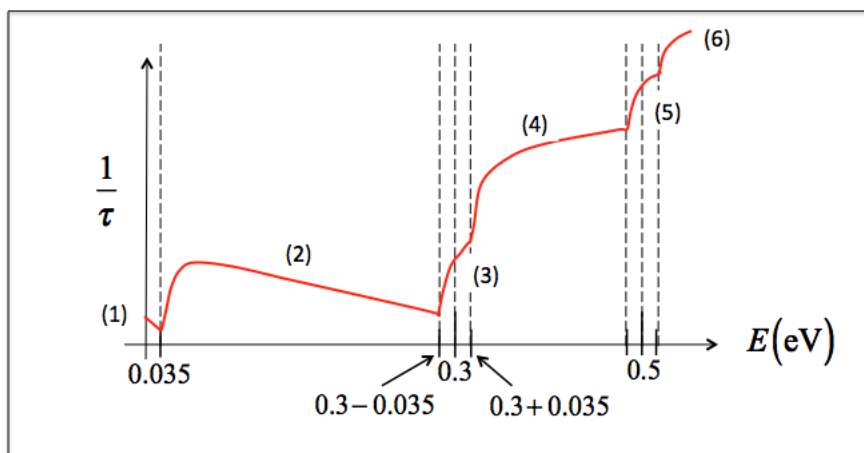
SOLUTIONS: ECE 656 Homework (Week 5)

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(Revised 9/23/13)

- 1) This problem concerns electron scattering in bulk (3D) GaAs. Assume that the optical phonon energy is $\hbar\omega_0 = 35$ meV. Recall that GaAs is a direct gap semiconductor and that the L valleys (along $\langle 111 \rangle$) have energy minima that are 0.3 eV above the Γ valley minimum. The four X valleys (along $\langle 100 \rangle$) have energy minima 0.5 eV above the Γ valley minimum. Recall that Γ valley electrons have a light effective mass and that the L and X valley electrons have a large (Si-like) effective mass. Answer the following questions.

Solution:

- a) Sketch the total electron scattering rate vs. energy for electrons **in the Γ valley**. Label all critical energies and give a brief explanation (label absorption and emission processes separately). All energies should be referred to the bottom of the Γ valley, i.e. $E_{\text{CT}} = 0$.

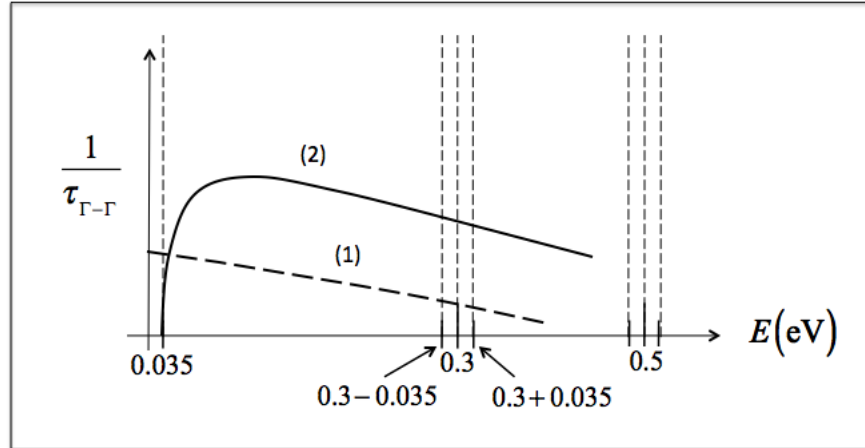


The scattering rate is the sum of 6 different processes; the last five of these have thresholds for their onset.

- 1) POP Absorption (ABS). $\Gamma - \Gamma$ intravalley scattering.
- 2) POP Emission (EMS). $\Gamma - \Gamma$ intravalley scattering plus 1)
- 3) OP ABS. $\Gamma - L$ intervalley scattering plus 1) and 2).
- 4) OP EMS. $\Gamma - L$ intervalley scattering plus 1), 2), and 3).
- 5) OP ABS. $\Gamma - X$ intervalley scattering plus 1), 2), 3), and 4).
- 6) OP EMS. $\Gamma - X$ intervalley scattering plus 1), 2), 3), 4), and 5).

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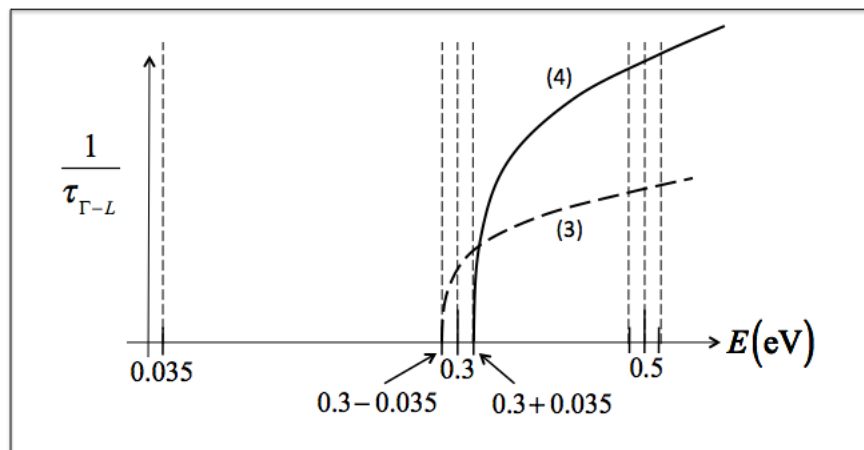
- b) Sketch the Γ to Γ electron scattering rate vs. energy. Label all critical energies and give a brief explanation (label absorption and emission processes separately).



- 1) POP Absorption (ABS). $\Gamma - \Gamma$ intravalley scattering.
2) POP Emission (EMS). $\Gamma - \Gamma$ intravalley scattering

Note that POP ABS is proportional to N_0 and POP EMS is proportional to $N_0 + 1$, so POP EMS is larger in magnitude.

- c) Sketch the Γ to L electron scattering rate vs. energy. Label all critical energies and give a brief explanation (label absorption and emission processes separately).

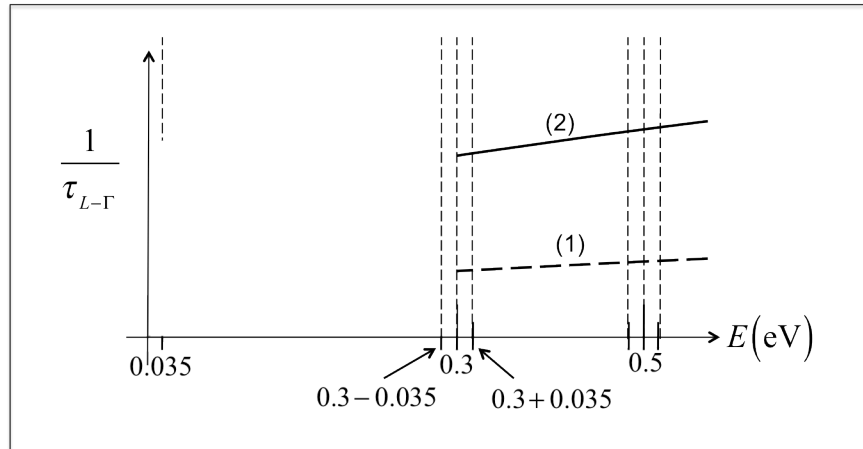


- 3) OP ABS. $\Gamma - L$ intervalley scattering.
4) OP EMS. $\Gamma - L$ intervalley scattering

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Note that EMS is, again, stronger than ABS. Also note that the scattering rates **increase** as the square root of energy because they go as the density of final states (no POP intervalley scattering because large phonon wavevectors are needed for this type of scattering).

- d) Sketch of the L to Γ electron scattering rate vs. energy. Label all critical energies and give a brief explanation (label absorption and emission processes separately).

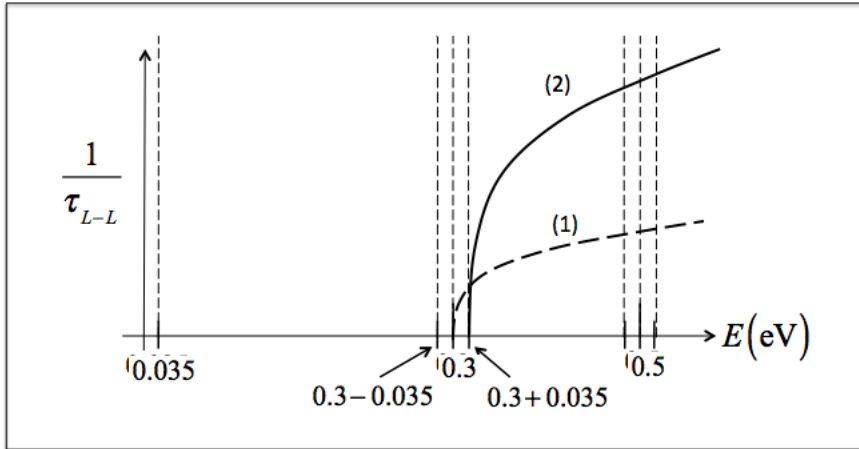


- 1) OP ABS. $L - \Gamma$ intervalley scattering.
- 2) OP EMS. $L - \Gamma$ intervalley scattering

Note that both ABS and EMS begin at the same energy because electrons at the bottom of the L-valley can scatter down to the Γ valley by either absorbing or emitting a phonon. Also note that the magnitude of the scattering rates is smaller than in c) because the final states in this case have a valley degeneracy of 1 instead of 4 as in c). Most importantly, note that the scattering rates do not begin at zero, because they are proportional to a DOS in the final valley, where the Γ valley DOS is quite high. $1/\tau_{\Gamma-L} \propto D_{\Gamma}(E \pm \hbar\omega_0)$ begins at an energy $E = 0.3 \pm \hbar\omega_0$, where the density-of-final states (in the Γ valley) is quite high.

- e) Sketch the L to L electron scattering rate vs. energy. Label all critical energies and give a brief explanation (label absorption and emission processes separately).

ECE 656 Homework 3 (Week 5) (continued)



An electron at the bottom of an L-valley can absorb a phonon and scatter to another L-valley, but an electron near the bottom of an L-valley cannot scatter to another L-valley by emission, because there are no L-valley states available at that energy.

- 2) The deformation potential scattering rate for optical phonon emission (ODP emission) is described by:

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \left(\frac{\hbar D_0^2}{2\rho\omega_0} \right) (N_0 + 1) \frac{D_{3D}(E - \hbar\omega_0)}{2}.$$

Obtain the **energy flux relaxation rate** for this scattering process.

Solution:

Begin with the definition of the energy flux relaxation rate:

$$\frac{1}{\tau_{F_W}} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \left[\frac{F_W(\vec{p}) - F_W(\vec{p}')}{F_W(\vec{p})} \right] \quad (*)$$

Work on the term in brackets and assume that the initial flux is oriented along the z-axis:

$$\left[\frac{F_W(\vec{p}) - F_W(\vec{p}')}{F_W(\vec{p})} \right] = \frac{Ev_z - (E - \hbar\omega_0)v'_z}{Ev_z} = 1 - \frac{(E - \hbar\omega_0)}{E} \left(\frac{v'_z}{v_z} \right) = 1 - \left(\frac{v'_z}{v_z} \right) + \frac{\hbar\omega_0}{E} \left(\frac{v'_z}{v_z} \right)$$

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$$\left[\frac{F_W(\vec{p}) - F_W(\vec{p}')}{F_W(\vec{p})} \right] = 1 - \left(\frac{v_z'}{v_z} \right) + \frac{\hbar\omega_0}{E} \left(\frac{v_z'}{v_z} \right) = \left(1 - \frac{v_z'}{v_z} \right) - \frac{\hbar\omega_0}{E} \left(1 - \frac{v_z'}{v_z} \right) + \frac{\hbar\omega_0}{E}$$

If we assume that the effective mass is constant (parabolic energy bands), then $m^* v_z = p_z$ and we find:

$$\left[\frac{F_W(\vec{p}) - F_W(\vec{p}')}{F_W(\vec{p})} \right] = \left(1 - \frac{p_z'}{p_z} \right) + \frac{\hbar\omega_0}{E} - \frac{\hbar\omega_0}{E} \left(1 - \frac{p_z'}{p_z} \right)$$

Now insert this expression in (*)

$$\begin{aligned} \frac{1}{\tau_{F_W}} &= \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \left[\left(1 - \frac{p_z'}{p_z} \right) + \frac{\hbar\omega_0}{E} - \frac{\hbar\omega_0}{E} \left(1 - \frac{p_z'}{p_z} \right) \right] \\ &= \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \left(1 - \frac{p_z'}{p_z} \right) + \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \left(\frac{\hbar\omega_0}{E} \right) - \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \frac{\hbar\omega_0}{E} \left(1 - \frac{p_z'}{p_z} \right) \end{aligned}$$

$$\frac{1}{\tau_{F_W}} = \frac{1}{\tau_m} + \left(\frac{\hbar\omega_0}{E} \right) \frac{1}{\tau} - \left(\frac{\hbar\omega_0}{E} \right) \frac{1}{\tau_m}$$

$$\boxed{\frac{1}{\tau_{F_W}} = \frac{1}{\tau_m} \left(1 - \frac{\hbar\omega_0}{E} \right) + \left(\frac{\hbar\omega_0}{E} \right) \frac{1}{\tau}}$$

Since we are assuming that phonon emission occurs, this only applies for $E > \hbar\omega_0$.

3) The ODP scattering rate for 2D electrons is:

$$\left(\frac{1}{\tau_{n,n'}} \right)^{a,e} = \frac{\pi}{\hbar} \left(\frac{\hbar D_0^2}{\rho \omega_0} \right) \left(N_0 + \frac{1}{2} \mp \frac{1}{2} \right) \frac{D_{2D}(E \pm \hbar\omega_0)}{2} \left(\frac{2 + \delta_{n,n'}}{2} \right)$$

Let $\hbar\omega_0 = 1.1 k_B T$, assume two subbands, and plot the absorption and emission scattering rates vs. energy for an electron in subband one.

ECE 656 Homework 3 (Week 5) (continued)**Solution:**

Let's write the scattering rate as

$$\left(\frac{1}{\tau_{n,n'}} \right)^{a,e} = \Gamma_0 \left(N_0 + \frac{1}{2} \mp \frac{1}{2} \right) \left(\frac{2 + \delta_{n,n'}}{2} \right)$$

where

$$\Gamma_0 = \frac{\pi}{\hbar} \left(\frac{\hbar D_0^2}{\rho \omega_0} \right) \frac{m^*}{2\pi \hbar^2}$$

$$N_0 = \frac{1}{e^{\hbar \omega_0 / k_B T} - 1} = \frac{1}{e^{1.1} - 1} = \frac{1}{3 - 1} = \frac{1}{2}$$

Now work out the various scattering rates:

$$\left(\frac{1}{\tau_{n,n'}} \right)^{a,e} = \Gamma_0 \left(\frac{1}{2} + \frac{1}{2} \mp \frac{1}{2} \right) \left(\frac{2 + \delta_{n,n'}}{2} \right) = \Gamma_0 \left(1 \mp \frac{1}{2} \right) \left(\frac{2 + \delta_{n,n'}}{2} \right)$$

$$\left(\frac{1}{\tau_{1,1}} \right)^a = \Gamma_0 \left(\frac{1}{2} \right) \left(\frac{3}{2} \right) = \frac{3}{4} \Gamma_0$$

$$\left(\frac{1}{\tau_{1,1}} \right)^e = \Gamma_0 \left(\frac{3}{2} \right) \left(\frac{3}{2} \right) = \frac{9}{4} \Gamma_0$$

$$\left(\frac{1}{\tau_{1,2}} \right)^a = \Gamma_0 \left(\frac{1}{2} \right) (1) = \frac{2}{4} \Gamma_0$$

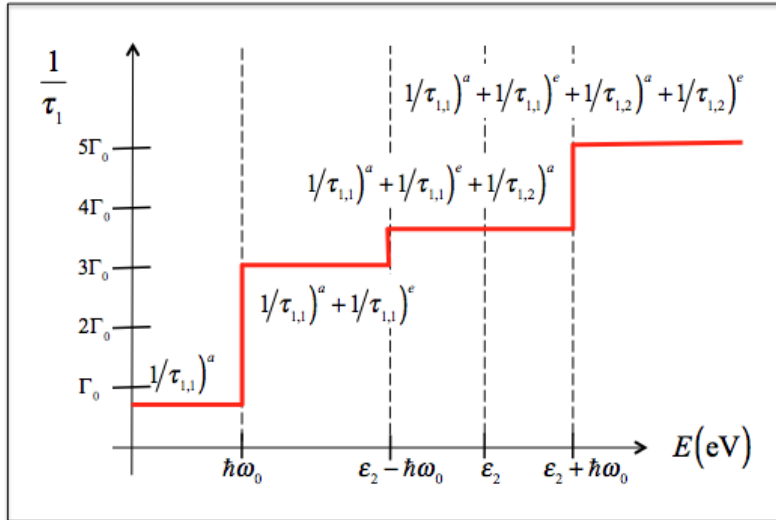
$$\left(\frac{1}{\tau_{1,2}} \right)^e = \Gamma_0 \left(\frac{3}{2} \right) (1) = \frac{6}{4} \Gamma_0$$

Total scattering rate for $E > \varepsilon_2$

$$\left(\frac{1}{\tau_1} \right)_{tot} = \Gamma_0 \left(\frac{3}{4} + \frac{9}{4} + \frac{2}{4} + \frac{6}{4} \right) = 5\Gamma_0$$

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The total scattering rate for electrons in subband 1 is plotted below. In this plot, we take the zero of energy to be the bottom of the first subband, $E = \varepsilon_1 = 0$.



- 4) Use arguments similar to those in Sec. 2.2 of FCT and evaluate the momentum relaxation time for piezoelectric scattering.
- a) Show that the scattering potential is

$$U_{PZ} = \frac{qe_{PZ}}{\kappa_S \varepsilon_0} u$$

where e_{PZ} is the piezoelectric constant and u is the elastic wave displacement.

HINT: Begin with

$$D = \kappa_S \varepsilon_0 \mathcal{E} + e_{PZ} \frac{\partial u}{\partial x}$$

- b) Use the scattering potential of part a) and evaluate the matrix element for PZ scattering. Show that the result is

$$\left| H_{\vec{p}', \vec{p}} \right|^2 = \left(\frac{qe_{PZ}}{\kappa_S \varepsilon_0} \right)^2 \frac{k_B T}{2c_s \beta^2 \Omega} \delta_{\vec{p}', \vec{p} \pm \hbar \vec{\beta}} = \left| K_\beta \right|^2 \left| A_\beta \right|^2 \delta_{\vec{p}', \vec{p} \pm \hbar \vec{\beta}}$$

- c) Write an expression for the transition rate, $S(\vec{p}, \vec{p}')$, and determine C_β .
- d) Evaluate $1/\tau_m$ assuming that the scattering is elastic.

ECE 656 Homework 3 (Week 5) (continued)**Solution:**

a) Begin with $D = \kappa_s \epsilon_0 \mathcal{E} + e_{PZ} \frac{\partial u}{\partial x}$ (*)

Assume space charge neutrality: $\nabla \cdot D = \kappa_s \epsilon_0 \mathcal{E} + e_{PZ} \frac{\partial u}{\partial x} = 0$

If, $D \propto e^{i\beta x}$, this implies that $D = 0$, so we find the electric field as:

$$\mathcal{E} = -\frac{e_{PZ}}{\kappa_s \epsilon_0} \frac{\partial u}{\partial x} = -i\beta \frac{e_{PZ}}{\kappa_s \epsilon_0} u$$

The scattering potential is

$$U_s = -q \int \mathcal{E} dx = \frac{e_{PZ}}{\kappa_s \epsilon_0} u$$

$$\boxed{U_{PZ} = \frac{e_{PZ}}{\kappa_s \epsilon_0} u}$$

b) Evaluate the matrix element

$$H_{\vec{p}', \vec{p}} = \int \frac{e^{-i\vec{p}' \cdot \vec{r}/\hbar}}{\sqrt{\Omega}} U_s \frac{e^{i\vec{p} \cdot \vec{r}/\hbar}}{\sqrt{\Omega}} d^3 r$$

Use the results from part a)

$$U_{PZ} = \frac{e_{PZ}}{\kappa_s \epsilon_0} u = K_\beta A_\beta e^{i\vec{\beta} \cdot \vec{r}}$$

$$K_\beta = \frac{e_{PZ}}{\kappa_s \epsilon_0} \quad (\text{eqn. 2.59d of FCT})$$

$$H_{\vec{p}', \vec{p}} = K_\beta A_\beta \frac{1}{\Omega} \int e^{i(\vec{p} - \vec{p}' \pm \hbar \vec{\beta}) \cdot \vec{r}/\hbar} d^3 r$$

Following the text, FCT, we find

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$$\left| H_{\vec{p}', \vec{p}} \right|^2 = \left| K_{\beta} \right|^2 \left| A_{\beta} \right|^2 \delta_{\vec{p}, \vec{p}' \pm \hbar \vec{\beta}} \quad \text{eqn. (2.60 of FCT)} \quad (**)$$

Now quantize the lattice vibrations according to eqn. (2.71c)

$$\left| A_{\beta} \right|^2 \rightarrow \frac{\hbar}{2\rho\Omega\omega_{\beta}} \left(N_{\beta} + \frac{1}{2} \mp \frac{1}{2} \right)$$

and we finally write the matrix element (**) as

$$\left| H_{\vec{p}', \vec{p}} \right|^2 = \left| K_{\beta} \right|^2 \left| A_{\beta} \right|^2 \delta_{\vec{p}, \vec{p}' \pm \hbar \vec{\beta}} = \left(\frac{e_{PZ}}{\kappa_s \epsilon_0} \right)^2 \frac{\hbar}{2\rho\Omega\omega_{\beta}} \left(N_{\beta} + \frac{1}{2} \mp \frac{1}{2} \right) \delta_{\vec{p}, \vec{p}' \pm \hbar \vec{\beta}} \quad (***)$$

c) Begin with the transition rate

$$S(\vec{p}, \vec{p}') = \frac{2\pi}{\hbar} \left| H_{\vec{p}', \vec{p}} \right|^2 \delta_{\vec{p}, \vec{p}' \pm \hbar \vec{\beta}} \delta(E' - E \mp \hbar\omega)$$

Now use (***) for the matrix element and eqn. (2.66) FCT for the two delta functions to find

$$S(\vec{p}, \vec{p}') = C_{\beta} \left(N_{\beta} + \frac{1}{2} \mp \frac{1}{2} \right) \delta(E' - E \mp \hbar\omega)$$

where

$$C_{\beta} = \frac{2\pi}{\hbar^2} \left(\frac{e_{PZ}}{\kappa_s \epsilon_0} \right)^2 \frac{\hbar}{2\rho\Omega\omega} \frac{1}{\hbar v \beta} = \frac{2\pi}{\hbar^2} \left(\frac{e_{PZ}}{\kappa_s \epsilon_0} \right)^2 \frac{\hbar}{2\rho\Omega\omega} \frac{m^*}{\hbar p \beta} \times \frac{\beta}{\beta}$$

$$C_{\beta} = \left(\frac{e_{PZ}}{\kappa_s \epsilon_0} \right)^2 \frac{\pi m^*}{\hbar \rho v_s \beta^2 p \Omega} \quad \text{(eqn. (2.73d) of FCT)}$$

Here, we have used: $\omega/\beta = v_s$ and $p = m^* v$ for the momentum of the electron.

d) Begin with eqn. (2.80) in FCT. Assume

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$\frac{\omega}{v\beta} = \frac{v_s}{v} \ll 1$ (average electron velocity much less than the phonon velocity, so (2.80) becomes

$$\frac{1}{\tau_m} = \frac{\Omega}{4\pi^2} \int_{\beta_{\min}}^{\beta_{\max}} \left(N_{\beta} + \frac{1}{2} \mp \frac{1}{2} \right) C_{\beta} \frac{\hbar\beta}{2p} \frac{\hbar\beta^3}{p} d\beta$$

Now assume equipartition:

$$N \approx N + 1 = \frac{k_B T}{\hbar\omega}$$

$$\frac{1}{\tau_m} = \frac{\pi m^* q^2 e_{pz}^2 k_B T}{c_{\ell} \kappa_s \epsilon_0 (2p^3)} \int_{\beta_{\min}}^{\beta_{\max}} \beta d\beta$$

Acoustic phonon scattering is elastic: $\beta_{\max} = 2p$ and $\beta_{\min} = 0$

$$\int_{\beta_{\min}}^{\beta_{\max}} \beta d\beta = \frac{\beta_{\max}^2}{2} - \frac{\beta_{\min}^2}{2} = 2p^2$$

Finally, the scattering rate becomes:

$$\boxed{\frac{1}{\tau_m} = \frac{\pi m^* q^2 e_{pz}^2 k_B T}{c_{\ell} \kappa_s \epsilon_0 p} \propto \frac{1}{\sqrt{E}}}$$

The momentum relaxation rate decreases with energy – as expected for an electrostatic scattering mechanisms.

- 5) For alloys of compound semiconductors like $\text{Al}_x\text{Ga}_{1-x}\text{As}$, microscopic fluctuations in the alloy composition, x , produce perturbations in the band edges. The transition rate for allow scattering is

$$S(\vec{p}, \vec{p}') = \frac{2\pi}{\hbar^2} \left(\frac{3\pi^2}{16} \right) \frac{|\Delta U|^2}{N\Omega} \delta(E' - E)$$

where N is the concentration of atoms and
ECE 656 Homework 3 (Week5) (continued)

$$\Delta U = x(1-x)(\chi_{\text{GaAs}} - \chi_{\text{AlAs}})$$

with χ being the electron affinity.

- a) Explain why the alloy scattering rate vanishes at $x = 0$ and at $x = 1$.
- b) Derive an expression for τ_m for alloy scattering.

Solution:

a) For $x = 0$, we have GaAs, which is not an alloy, so there is no alloy scattering. For $x = 1$, we have AlAs, which is not an alloy, so there is no alloy scattering. The strongest alloy scattering will occur when $x = 1/2$.

b)

$$S(\vec{p}, \vec{p}') = \frac{C}{\Omega} \delta(E' - E)$$

where

$$C = \frac{2\pi}{\hbar^2} \left(\frac{3\pi^2}{16} \right) \frac{|\Delta U|^2}{N}$$

Since there is no dependence on phonon wavevector, the scattering rate and momentum relaxation rates should be equal. Since there is no dependence on β , we do not need to worry about energy-momentum conservation and can do the sum simply.

$$\frac{1}{\tau} = \frac{1}{\tau_m} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') = C \frac{1}{\Omega} \sum_{\vec{p}', \uparrow} \delta(E' - E)$$

We recognize the sum as one-half of the density-of-states,

$$\boxed{\frac{1}{\tau(E)} = \frac{1}{\tau_m(E)} = C \frac{D_{3D}(E)}{2}}$$

$$\tau_m(E) \propto E^{-1/2}$$

Which is power law scattering with a characteristic exponent of $s = -1/2$.

ECE 656 Homework 3 (Week5) (continued)

- 6) Acoustic phonon scattering was assumed to be elastic when working out the momentum relaxation rate in eqn. (2.84) of FCT. Repeat the calculation but do not assume elastic scattering. Show that the result is nearly equal to eqn. (2.84) near room temperature.

Solution:

The transition rate for phonon scattering is:

$$S(\vec{p}, \vec{p}') = \frac{2\pi}{\hbar} |K_\beta|^2 |A_\beta|^2 \delta_{\vec{p}, \vec{p}' \pm \hbar\vec{\beta}} \delta(E' - E \mp \hbar\omega)$$

The two delta-functions can be replaced by one that expresses energy and momentum conservation (eqn. (2.66) of FCT):

$$\delta_{\vec{p}, \vec{p}' \pm \hbar\vec{\beta}} \delta(E' - E \mp \hbar\omega) \rightarrow \frac{1}{\hbar v \beta} \delta\left(\pm \cos\theta + \frac{\hbar\beta}{2p} \mp \frac{\omega_\beta}{v\beta}\right)$$

so the transition rate becomes:

$$S(\vec{p}, \vec{p}') = \frac{2\pi}{\hbar^2 v \beta} |K_\beta|^2 |A_\beta|^2 \delta\left(\pm \cos\theta + \frac{\hbar\beta}{2p} \mp \frac{\omega_\beta}{v\beta}\right),$$

which is eqn. (2.67) of FCT. For ADP scattering:

$$|K_\beta|^2 = \beta^2 D_A^2 \quad (\text{eqn. (2.59a) of FCT}) +$$

$$|A_\beta|^2 \rightarrow \frac{\hbar}{2\rho\Omega\omega_\beta} \left(N_{\omega_\beta} + \frac{1}{2} \mp \frac{1}{2}\right) \quad (\text{eqn. (2.71c) of FCT})$$

So we find:

$$S(\vec{p}, \vec{p}') = \frac{\pi m^* D_A^2}{\hbar \rho p v_s} \frac{1}{\Omega} \left(N_{\omega_\beta} + \frac{1}{2} \mp \frac{1}{2}\right) \delta\left(\pm \cos\theta + \frac{\hbar\beta}{2p} \mp \frac{\omega_\beta}{v\beta}\right)$$

or

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$$S(\vec{p}, \vec{p}') = C_\beta \left(N_{\omega_\beta} + \frac{1}{2} \mp \frac{1}{2} \right) \delta \left(\pm \cos \theta + \frac{\hbar \beta}{2p} \mp \frac{\omega_\beta}{v\beta} \right) \quad (\text{eqn. (2.72 FCT)})$$

$$C_\beta = \frac{\pi m^* D_A^2}{\hbar \rho p v_s} \frac{1}{\Omega} \quad (\text{eqn. (2.73a) FCT})$$

Following FCT, we write the momentum relaxation rate as:

$$\frac{1}{\tau_m} = \sum_{\vec{\beta}, \uparrow} S(\vec{p}, \vec{p}') \frac{(\mp \hbar \beta \cos \theta)}{p} = \frac{\Omega}{8\pi^3} \int_0^{2\pi} d\phi \int_0^{\infty+1} S(\vec{p}, \vec{p}') \frac{(\mp \hbar \beta \cos \theta)}{p} d(\cos \theta) \beta^2 d\beta$$

or

$$\frac{1}{\tau_m} = \frac{\Omega}{4\pi^2} \int_0^{\infty+1} C_\beta \left(N_{\omega_\beta} + \frac{1}{2} \mp \frac{1}{2} \right) \delta \left(\pm \cos \theta + \frac{\hbar \beta}{2p} \mp \frac{\omega_\beta}{v\beta} \right) \frac{(\mp \hbar \beta \cos \theta)}{p} d(\cos \theta) \beta^2 d\beta$$

Now assume equipartition: $N_{\omega_\beta} \approx N_{\omega_\beta} + 1 \approx \frac{k_B T}{\hbar \omega_\beta}$ and use the expression for C_β .

$$\frac{1}{\tau_m} = \frac{m^* D_A^2 k_B T}{4\pi \hbar \rho p^2 v_s^2} \int_0^{\infty+1} \int_{-1}^1 \delta \left(\pm \cos \theta + \frac{\hbar \beta}{2p} \mp \frac{\omega_\beta}{v\beta} \right) (\mp \cos \theta) d(\cos \theta) \beta^2 d\beta$$

Integrate over β first:

$$\frac{1}{\tau_m} = \frac{m^* D_A^2 k_B T}{4\pi \hbar \rho p^2 v_s^2} \int_{\beta_{\min}}^{\beta_{\max}} \left(\frac{\hbar \beta}{2p} \mp \frac{\omega_\beta}{v\beta} \right) \beta^2 d\beta = \frac{m^* D_A^2 k_B T}{4\pi \hbar \rho p^2 v_s^2} \int_{\beta_{\min}}^{\beta_{\max}} \left(\frac{\hbar \beta}{2p} \mp \frac{v_s}{v} \right) \beta^2 d\beta$$

where we have used the acoustic phonon dispersion for the last step.

$$\frac{1}{\tau_m} = \frac{m^* D_A^2 k_B T}{4\pi \hbar \rho p^2 v_s^2} \left[\frac{\hbar}{2p} \left(\frac{\beta_{\max}^4}{4} - \frac{\beta_{\min}^4}{4} \right) \mp \frac{v_s}{v} \left(\frac{\beta_{\max}^3}{3} - \frac{\beta_{\min}^3}{3} \right) \right]$$

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Assume $\beta_{\max} \gg \beta_{\min}$ and $\beta_{\max}^4 \gg \beta_{\max}^3$

$$\frac{1}{\tau_m} = \frac{m^* D_A^2 k_B T}{8\pi \rho p^3 v_s^2} \left(\frac{\beta_{\max}^4}{4} \right)$$

Energy and momentum conservation gives (see FCT, eqn. 2.54)

$$\hbar \beta_{\max} = 2p \left(1 \pm \frac{v_s}{v} \right)$$

$$\frac{\beta_{\max}^4}{4} = \frac{4p^4}{\hbar^4} \left(1 \pm \frac{v_s}{v} \right)^4$$

so the momentum relaxation rate becomes:

$$\frac{1}{\tau_m} = \frac{m^* p D_A^2 k_B T}{2\pi \hbar^4 \rho v_s^2} \left(1 \pm \frac{v_s}{v} \right)^4 = \frac{m^* p D_A^2 k_B T}{2\pi \hbar^4 c_\ell} \left(1 \pm \frac{v_s}{v} \right)^4$$

Recall that the 3D density-of-states is:

$$D_{3D}(E) = \frac{(2m^*)^{3/2}}{4\pi^2 \hbar^3} E^{1/2}$$

which can be used to re-express the momentum relaxation rate as:

$$\boxed{\frac{1}{\tau_m(E)} = \frac{m^* D_A^2 k_B T}{\hbar c_\ell} \frac{D_{3D}(E)}{2} \left(1 \pm \frac{v_s}{v} \right)^4},$$

which is almost exactly the result for elastic scattering (eqn. (2.84) of FCT) since for a typical electron $v \ll v_s$.