ECE 656 Homework 3 (Week 5)

Mark Lundstrom
Purdue University
(Revised 9/17/13)

1) This problem concerns electron scattering in bulk (3D) GaAs. Assume that the optical phonon energy is $\hbar\omega_0 = 35$ meV. Recall that GaAs is a direct Gap semiconductor and that the L valleys (along <111>) have energy minima that are 0.3 eV above the Γ valley minimum. The four X valleys (along <100>) have energy minima 0.5 eV above the Γ valley minimum. Recall that Γ valley electrons have a light effective mass and that the L and X valley electrons have a large (Si-like) effective mass. Answer the following questions.

a) Sketch the total electron scattering rate vs. energy for electrons in the Γ valley. Label all critical energies and give a brief explanation (label absorption and emission processes separately). All energies should be referred to the bottom of the Γ valley, i..e. $E_{C\Gamma} = 0$.
b) Sketch the Γ to Γ electron scattering rate vs. energy. Label all critical energies and give a brief explanation (label absorption and emission processes separately).
c) Sketch the Γ to L electron scattering rate vs. energy. Label all critical energies and give a brief explanation (label absorption and emission processes separately).
d) Sketch of the L to Γ electron scattering rate vs. energy. Label all critical energies and give a brief explanation (label absorption and emission processes separately).
e) Sketch the L to L electron scattering rate vs. energy. Label all critical energies and give a brief explanation (label absorption and emission processes separately).

2) The deformation potential scattering rate for optical phonon emission (ODP emission) is described by:

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\left(\frac{\hbar D_0^2}{2\rho\omega_0}\right)\left(N_0 + 1\right)\frac{D_{3D}\left(E - \hbar\omega_0\right)}{2} .$$

Obtain the energy flux relaxation rate for this scattering process.

3) The ODP scattering rate for 2D electrons is:

$$\left.\frac{1}{\tau_{n,n'}}\right)^{a,e} = \frac{\pi}{\hbar}\left(\frac{\hbar D_0^2}{\rho\omega_0}\right)\left(N_0 + \frac{1}{2} \mp \frac{1}{2}\right)\frac{D_{2D}\left(E \pm \hbar\omega_0\right)}{2}\left(\frac{2 + \delta_{n,n'}}{2}\right)$$

ECE 656 Homework 3 (Week 5) (continued)

Let $\hbar\omega_0 = 1.1 k_B T$, assume two subbands, and plot the absorption and emission scattering rates vs. energy for an electron in subband one.

4) Use arguments similar to those in Sec. 2.2 of FCT and evaluate the momentum relaxation time for piezoelectric scattering.

a) Show that the scattering potential is

$$U_{PZ} = \frac{q e_{PZ}}{\kappa_S \varepsilon_0} u$$

where e_{PZ} is the piezoelectric constant and u is the elastic wave displacement.
HINT: Begin with

$$D = \kappa_S \varepsilon_0 \mathcal{E} + e_{PZ} \frac{\partial u}{\partial x}$$

b) Use the scattering potential of part a) and evaluate the matrix element for PZ scattering. Show that the result is

$$\left| H_{\vec{p}',\vec{p}} \right|^2 = \left(\frac{q e_{PZ}}{\kappa_S \varepsilon_0} \right)^2 \frac{k_B T}{2 c_\ell \beta^2 \Omega} \delta_{\vec{p}',\vec{p} \pm \hbar\vec{\beta}} = \left| K_\beta \right|^2 \left| A_\beta \right|^2 \delta_{\vec{p}',\vec{p} \pm \hbar\vec{\beta}}$$

c) Write an expression for the transition rate, $S(\vec{p},\vec{p}')$, and determine C_β.

d) Evaluate $1/\tau_m$ assuming that the scattering is elastic.

5) For alloys of compound semiconductors like $\mathrm{Al_x Ga_{1\text{-}x} As}$, microscopic fluctuations in the alloy composition, x, produce perturbations in the band edges. The transition rate for allow scattering is

$$S(\vec{p},\vec{p}') = \frac{2\pi}{\hbar^2} \left(\frac{3\pi^2}{16} \right) \frac{\left| \Delta U \right|^2}{N \Omega} \delta(E' - E)$$

where N is the concentration of atoms and

$$\Delta U = x(1-x)\left(\chi_{\mathrm{GaAs}} - \chi_{\mathrm{AlAs}} \right)$$

with χ being the electron affinity.

ECE 656 Homework 3 (Week5) (continued)

a) Explain why the alloy scattering rate vanishes at $x = 0$ and at $x = 1$.

b) Derive an expression for τ_m for alloy scattering.

6) Acoustic phonon scattering was assumed to be elastic when working out the momentum relaxation rate in eqn. (2.84) of FCT. Repeat the calculation but do not assume elastic scattering. Show that the result is nearly equal to eqn. (2.84) near room temperature.