

SOLUTIONS: ECE 656 Homework (Week 6)
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- 1) In Lecture 14, we discussed $M(E)$ for a 3D semiconductor with parabolic energy bands.
 Answer the following two questions about a 3D semiconductor with non-parabolic energy bands.

- a) Assume that the non-parabolicity can be described by

$$E(1+\alpha E) = \frac{\hbar^2 k^2}{2m^*(0)} .$$

Derive an expression for the corresponding $M(E)$.

- b) Using the following numbers for GaAs

$$m^*(0) = 0.067 m_0$$

$$\alpha = 0.64 ,$$

plot $M(E)$ from the bottom of the Γ valley to $E = 0.3$ eV comparing results from the non-parabolic expression derived in part a) to the parabolic expression quoted in the lecture.

Solutions:

a)

Begin with the definition:

$$M_{3D}(E) = \frac{\hbar}{4} \langle v_x^+ \rangle D_{3D}(E) \quad (1)$$

$$\langle v_x^+ \rangle = (1/2) v(E) \quad (2)$$

Step 1: compute $D_{3D}(E)$ for non-parabolic bands

Step 2: compute $v(E)$ for non-parabolic bands

Step 3: multiply the two to get the answer

Step 1:

$$D_{3D}(E)dE = \frac{N_{3D}(k)}{\Omega} 4\pi k^2 dk = \frac{1}{4\pi^3} 4\pi k^2 dk = \frac{1}{\pi^2} k^2 dk \quad (3)$$

$$E(1+\alpha E) = \frac{\hbar^2 k^2}{2m^*(0)} \quad (4)$$

Solve for k :

$$k = \frac{\sqrt{2m^*(0)E(1+\alpha E)}}{\hbar} \quad (5)$$

differentiate to find:

$$k dk = \frac{m^*(0)}{\hbar^2} (1+2\alpha E) dE \quad (6)$$

put the two together:

$$k^2 dk = \frac{m^*(0)}{\hbar^3} \sqrt{2m^*(0)E(1+\alpha E)} (1+2\alpha E) dE$$

Insert in (3)

$$\begin{aligned} D_{3D}(E)dE &= \frac{1}{\pi^2} k^2 dk = \frac{m^*(0)}{\pi^2 \hbar^3} \sqrt{2m^*(0)E(1+\alpha E)} (1+2\alpha E) dE \\ D_{3D}(E) &= \frac{1}{\pi^2} k^2 dk = \frac{m^*(0)}{\pi^2 \hbar^3} \sqrt{2m^*(0)E(1+\alpha E)} (1+2\alpha E) \end{aligned} \quad (7)$$

Step 2:

From (6)

$$\frac{dE}{dk} = \frac{\hbar^2 k}{m^*(0)} \frac{1}{(1+2\alpha E)} \rightarrow \frac{1}{\hbar} \frac{dE}{dk} = v = \frac{\hbar k}{m^*(0)} \frac{1}{(1+2\alpha E)}$$

Now use (5)

$$v(E) = \frac{\hbar k}{m^*(0)} \frac{1}{(1+2\alpha E)} = \sqrt{\frac{2E(1+\alpha E)}{m^*(0)}} \frac{1}{(1+2\alpha E)} \quad (8)$$

Step 3:

Now use (1), (2), (7), and (8)

$$\begin{aligned} M_{3D}(E) &= \frac{h}{4} \left(\frac{v(E)}{2} \right) D_{3D}(E) = \frac{h}{4} \left(\frac{v(E)}{2} \right) D_{3D}(E) = \frac{h}{4} \sqrt{\frac{E(1+\alpha E)}{2m^*(0)}} \frac{1}{(1+2\alpha E)} D_{3D}(E) \\ M_{3D}(E) &= \frac{h}{4} \sqrt{\frac{E(1+\alpha E)}{2m^*(0)}} \frac{1}{(1+2\alpha E)} \left\{ \frac{m^*(0)}{\pi^2 \hbar^3} \sqrt{2m^*(0)E(1+\alpha E)(1+2\alpha E)} \right\} \\ M_{3D}(E) &= \frac{m^*(0)}{2\pi\hbar^2} E(1+\alpha E) \end{aligned}$$

We have been assuming that the bottom of the conduction band is at $E_C = 0$. Let's put E_C back in explicitly.

$$M_{3D}(E) = \frac{m^*(0)}{2\pi\hbar^2} (E - E_C) [1 + \alpha(E - E_C)]$$

b)

The following Matlab script is used to produce the plot below.

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```
function ece656_hw6_1b()

% ECE 656 (Fall 2013) HW#6, 1b
% This script plots and compares the density of modes for GaAs assuming parabolic
% non-parabolic energy bands
```

Parameters

```
m_star = 0.067; % (1) effective mass coefficient
alpha = 0.64; % (/eV) non-parabolicity factor

m_o = 0.511e6; % (eV/c^2) electron rest mass
hbar = 6.582e-16; % (eV-s)
m = m_star*m_o; % (eV/c^2) electron effective mass

E = linspace(0,0.3,100); % (eV) range of energy
```

Calculation of Modes

```
M_3D_nonp = m./(2*pi*hbar^2).*E.*(1+alpha.*E).*1e-4; % (/cm^2)
M_3D_p = m./(2*pi*hbar^2).*E.*1e-4; % (/cm^2)
```

Plots

```
figure(1)

plot(E, M_3D_nonp, 'LineWidth',2)
hold on
plot(E, M_3D_p, '--', 'LineWidth',2)

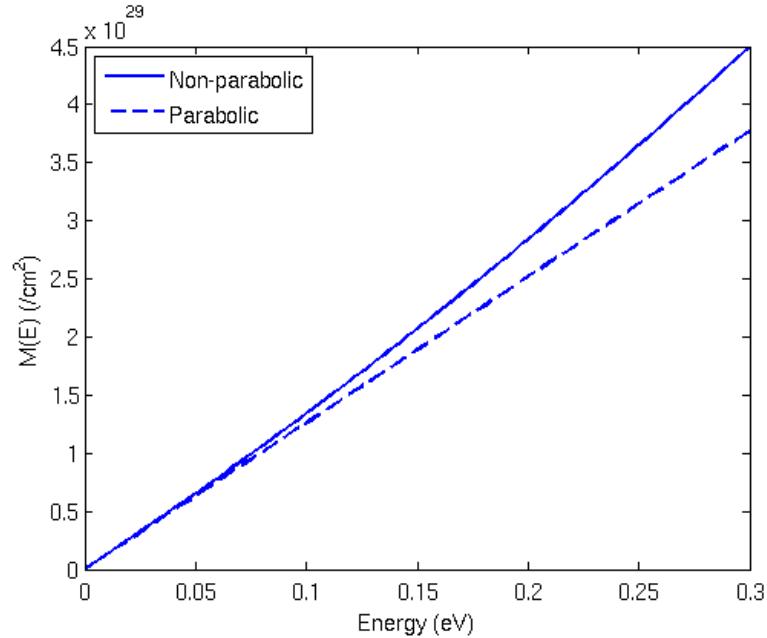
xlabel('Energy (eV)');
ylabel('M(E) (/cm^2)');
legend('Non-parabolic', 'Parabolic', 'Location', 'NorthWest');

xlim([0 max(E)])

font_size = 12;

set(gcf, 'color', 'white');
set(gca, 'FontSize', font_size);
set(get(gca,'title'), 'FontSize',font_size);
set(get(gca,' xlabel'), 'FontSize',font_size);
```

```
set(get(gca,'ylabel'),'FontSize',font_size);
```



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