Week 6 Summary:

Landauer Approach

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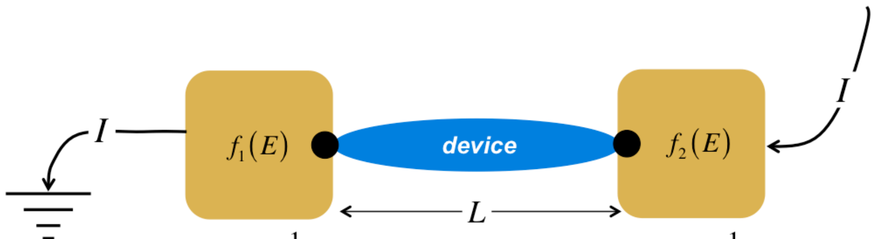
nano to macro device

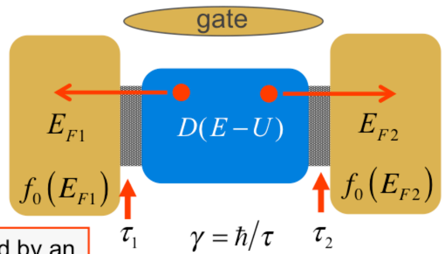


$$f_1(E)=\frac{1}{1+e^{(E-E_{F1})/k_BT_1}} \qquad f_2(E)=\frac{1}{1+e^{(E-E_{F2})/k_BT_2}}$$

assumptions

2) **Contacts** are large with strong inelastic scattering, always near equilibrium

3) U is the self-consistent (mean-field) potential. (For “strongly correlated” transport, see Datta.)



5) ***Reflectionless*** (“absorbing”) contacts.

1) Device is described by an $E(k)$. For the more general case, see Datta.

4) All **inelastic scattering** takes place in the contacts. Electrons flow from left to right (or right to left) in **independent** energy channels.

Landauer-Boltzmann-NEGF (LBN) Approach

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)\,dE$$

1) Transmission

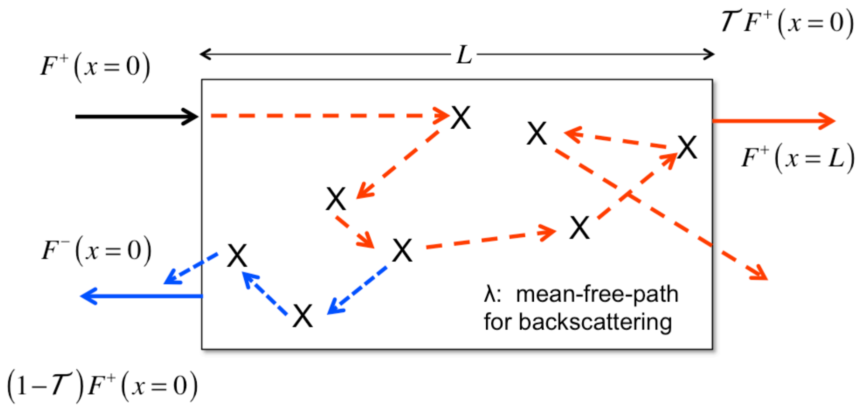
2) modes (channels)

3) differences in Fermi-levels

For a discussion of fundamentals, see:

https://nanohub.org/groups/lnebook/questions

transmission



Lundstrom ECE-656 F13

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transmission

$$I = \frac{2q}{h}\int T(E)M(E)(f_1 - f_2)\,dE$$

1) Diffusive: $L >> \lambda \quad T = \frac{\lambda}{L} << 1$

2) Ballistic: $L << \lambda \quad T = 1$

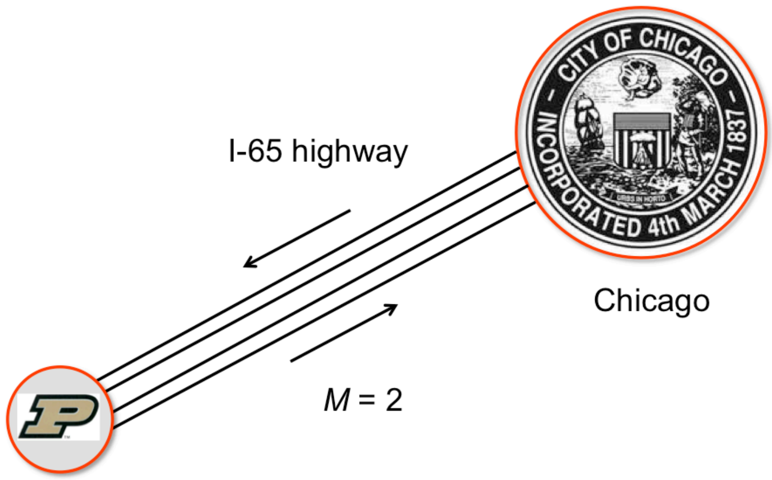
3) Quasi-ballistic: $L \approx \lambda \quad T < 1$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

λ is the "mean-free-path for backscattering"

This expression can be derived with relatively few assumptions.

modes (channels)



modes (channels)

$$M(E) = \frac{h}{4}\langle \upsilon_x^+ \rangle D(E)$$

parabolic energy bands: $\frac{1}{2} m^* \upsilon^2(E) = E - E_C$

$$\upsilon(E) = \sqrt{2(E - E_C)/m^*}$$

x-directed velocity:

1D: $\langle \upsilon_x^+ \rangle = \upsilon(E)$

2D: $\langle \upsilon_x^+ \rangle = (2/\pi)\upsilon(E)$

3D: $\langle \upsilon_x^+ \rangle = (1/2)\upsilon(E)$

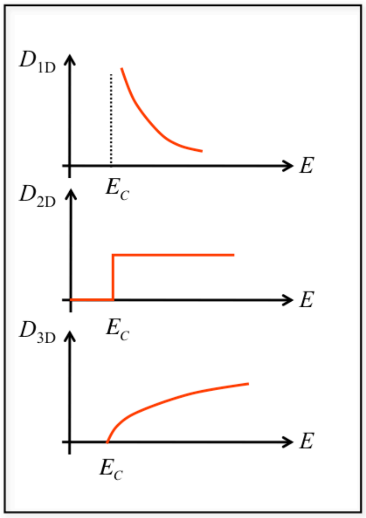
density of states (for parabolic energy bands)

$$D(E) = L\,D_{1D}(E) = \frac{L}{\pi\hbar}\sqrt{\frac{2m^*}{(E-E_C)}}\,H(E-E_C)$$

$$D(E) = A D_{2D}(E) = A\frac{m^*}{\pi\hbar^2}H(E-E_C)$$

$$D(E) = \Omega\, D_{3D}(E) = \Omega\frac{m^*\sqrt{2m^*(E-E_C)}}{\pi^2\hbar^3}H(E-E_C)$$

$$\left(E(k) = E_C + \hbar^2k^2/2m^*\right)$$



number of modes (for parabolic energy bands)

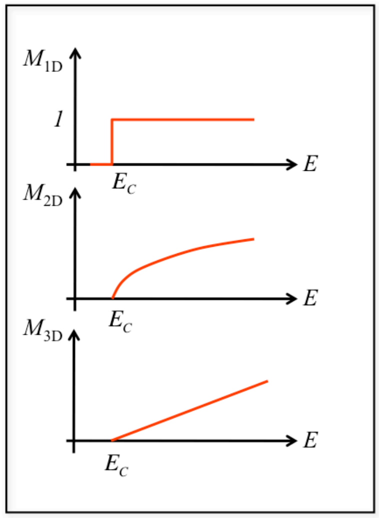
$$M(E) = \frac{h}{4}\langle \upsilon_x^+ \rangle D(E)$$

$$M(E) = M_{1D}(E) = H(E - E_C)$$

$$M(E) = W\,M_{2D}(E) = W\frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar}H(E - E_C)$$

$$M(E) = A\,M_{3D}(E) = A\frac{m^*}{2\pi\hbar^2}(E - E_C)H(E - E_C)$$

$$\left(E(k) = E_C + \hbar^2 k^2 / 2m^*\right)$$

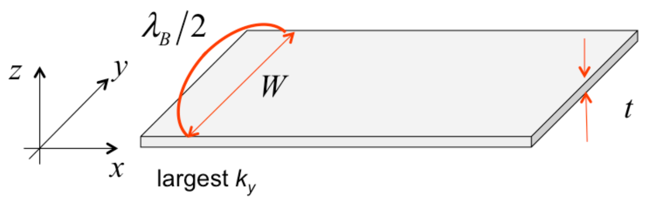


waveguide modes

Assume that there is **one** subband associated with confinement in the z-direction. How many subbands (channels) are there associated with confinement in the y-direction?

$$M(E) = \frac{W}{\lambda_B(E)/2}$$

When $M(E) = 1 \rightarrow \lambda_B(E)/2 = W$



largest k_y

$E = \hbar^2 k_y^2 / 2m^*$

$\psi(x, y) \propto e^{ik_x x} \sin k_y y$

$k_y = m\pi/W \quad m = 1, 2, \ldots$

LBN expression for current

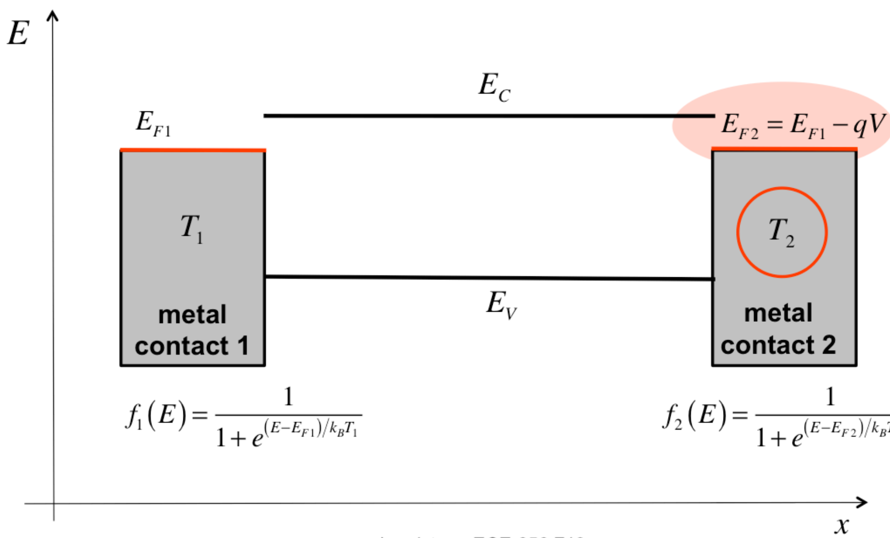
$$I = \frac{2q}{h}\int \mathcal{T}(E) M(E)\left(f_1 - f_2\right) dE$$

1) Transmission

2) Modes (channels)

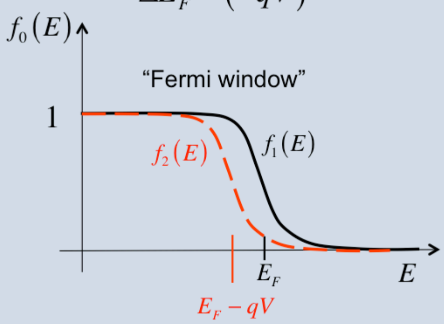
3) Difference in Fermi-levels

differences in the Fermi levels



differences in the Fermi levels (constant *T*)

$$\Delta E_F = (-qV)$$



$f_1 > f_2$ and current flows

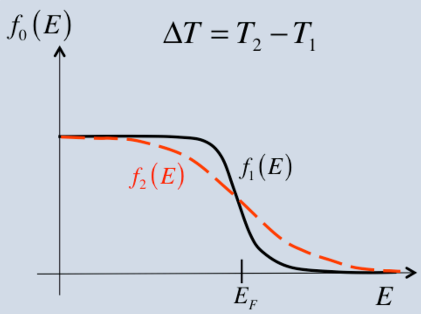
near-equilibrium
(small applied bias)

$$\boxed{(f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) qV}$$

$$(f_1 \approx f_2 \approx f_0)$$

ΔE_F

differences in the Fermi levels (constant V)



$|f_1 - f_2| > 0$ so current flows, but the sign depends on whether the states are located above or below E_F (n-type or p-type).

near-equilibrium
(small temperature difference)

$$\boxed{(f_1 - f_2) \approx -\left(-\frac{\partial f_0}{\partial E}\right)\frac{(E - E_F)}{T}\Delta T}$$

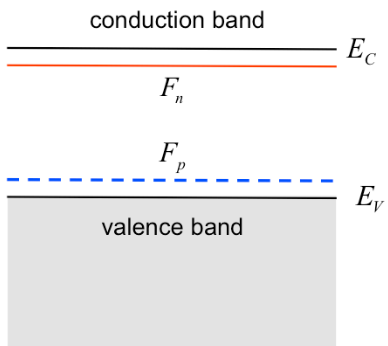
$$(f_1 \approx f_2 \approx f_0)$$

re-cap

Now that we understand transmission, modes, and $f_1 - f_2$, we can compute the coupled electrical and heat currents.

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)dE$$

what about ~~holes~~?



n-type

$$I = G_n V$$

$$G_n = \frac{2q^2}{h}\int \mathcal{T}(E) M_C(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$f_0(E) = \frac{1}{1 + e^{(E-F_n)/k_B T_L}}$$

p-type

$$I = G_p V$$

$$G_p = \frac{2q^2}{h}\int \mathcal{T}(E) M_V(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$f_0(E) = \frac{1}{1 + e^{(E-F_p)/k_B T_L}}$$

All of these expressions refer to ***electrons*** in the conduction and valence bands

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