

Week 7 Summary:

Resistance: ballistic to diffusive

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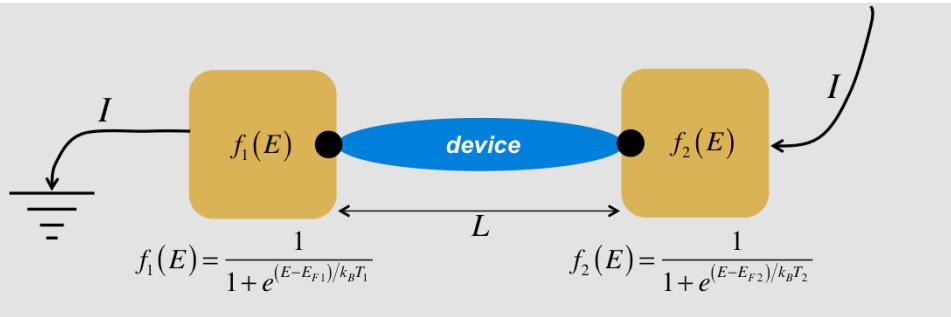


10/4/13



nano to macro device

$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

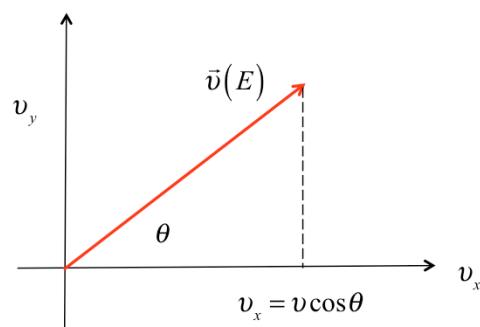


$$M_{3D}(E) A = A \frac{h}{4} \langle v_x^+ \rangle D_{3D}(E) \quad T(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

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2D $\langle v_x^+ \rangle$



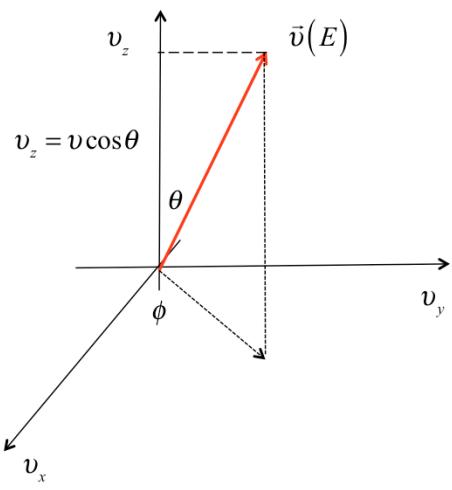
$$\langle v_x^+ \rangle = v \langle \cos \theta \rangle$$

$$\langle \cos \theta \rangle = \frac{\int_{-\pi/2}^{+\pi/2} \cos \theta d\theta}{\int_{-\pi/2}^{+\pi/2} d\theta} = \frac{2}{\pi}$$

$$\langle v_x^+ \rangle = \frac{2}{\pi} v$$

$$v = \sqrt{\frac{2(E - E_C)}{m^*}}$$

3D $\langle v_x^+ \rangle$



$$\langle v_z^+ \rangle = v \langle \cos \theta \rangle$$

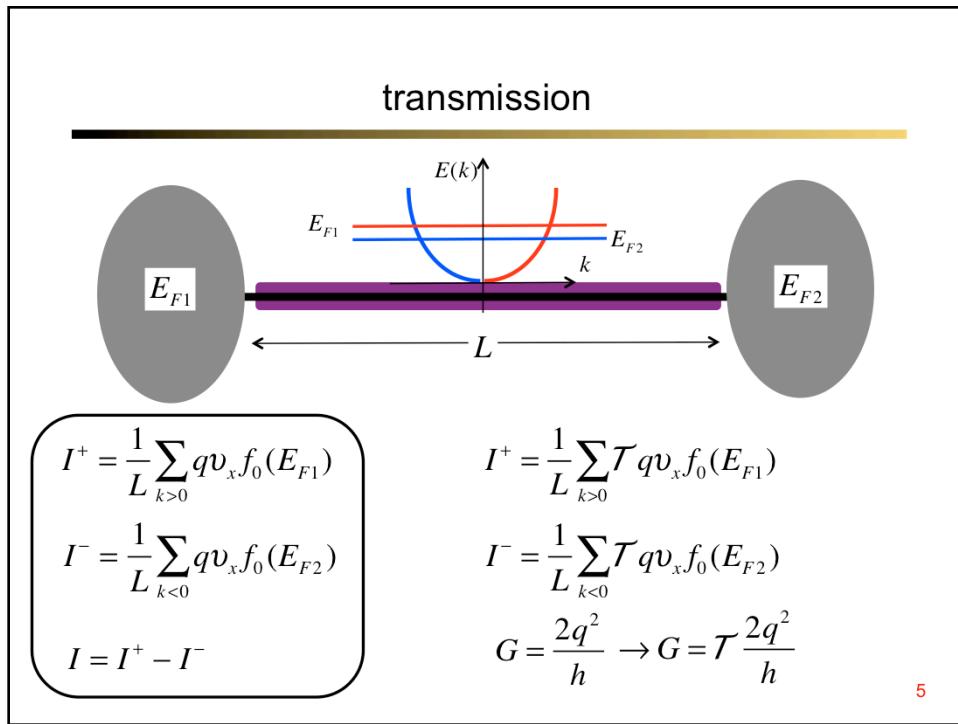
$$\langle \cos \theta \rangle = \frac{\int_0^{+\pi/2} \cos \theta \sin \theta d\theta \int_0^{2\pi} d\phi}{2\pi}$$

$$\langle \cos \theta \rangle = \frac{-\frac{\cos^2 \theta}{2} \Big|_0^{\pi/2} \times 2\pi}{2\pi} = \frac{1}{2}$$

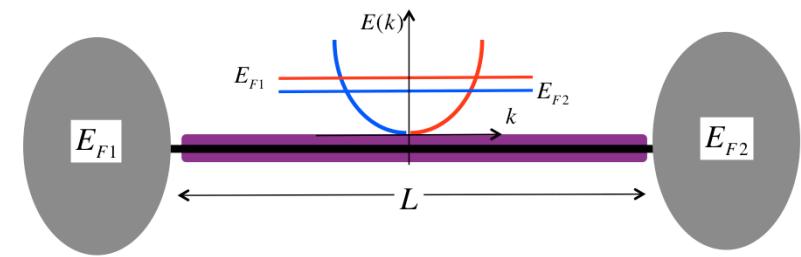
$$\langle v_z^+ \rangle = \langle v_x^+ \rangle = \frac{v}{2}$$

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for more discussion



S. Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge, 1995, pp. 57-59.

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transport in the bulk

$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

$$f_1 - f_2 \approx \left(-\frac{\partial f_1}{\partial E} \right) qV$$

$$T(E) = \frac{\lambda}{\lambda + L} \rightarrow \frac{\lambda}{L}$$

$$I = \left\{ \frac{2q}{h} \int \lambda(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right\} \frac{qV}{L}$$

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transport in the bulk

$$I = \left\{ \frac{2q}{h} \int \lambda(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right\} \frac{qV}{L}$$

$$M(E) = W \frac{h}{4} \langle v_x^+ \rangle D_{3D}(E) \quad J_x = -I/A \quad D_n(E) = \frac{\langle v_x^+ \rangle \lambda(E)}{2}$$

$$J_x = \left\{ \int q D_n(E) D_{3D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right\} \frac{\Delta F_n}{L}$$

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transport in the bulk

$$J_x = \left\{ \int q D_n(E) D_{3D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \right\} \frac{\Delta F_n}{L}$$

$$J_x = \sigma \frac{dF_n/q}{dx} \quad \sigma = \int q^2 D_n(E) D_{3D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

Fundamental description of near-equilibrium transport.

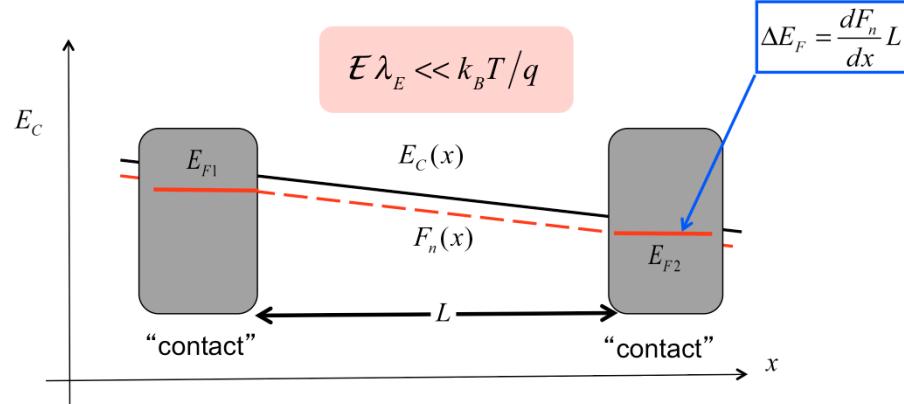
assumptions:

- 1) near-equilibrium
- 2) $L \gg$ mfp
- 3) temperature constant

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physical picture

uniformly doped n -type resistor

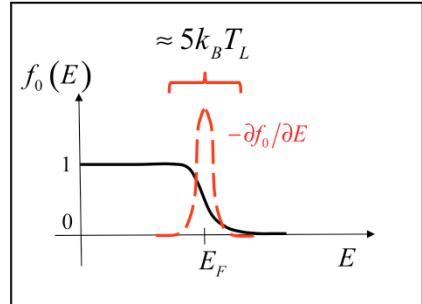


$F_n(x)$ is the *electrochemical potential* (or “quasi-Fermi level”) which we now regard as slowly varying across the sample.

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conductivity of a metal



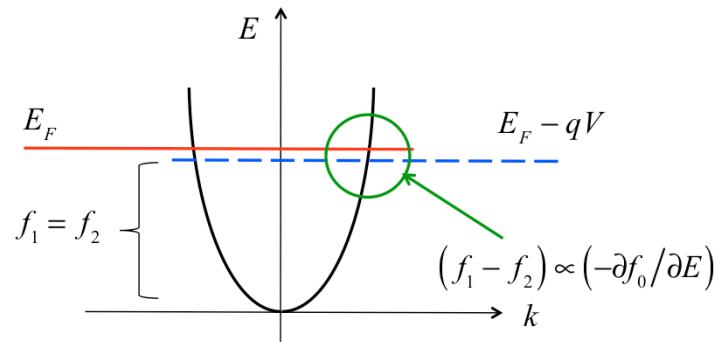
$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T_L}}$$

$$\left(-\frac{\partial f_0}{\partial E} \right) \approx \delta(E_F)$$

$$\sigma = q^2 D_n(E_F) D_{3D}(E_F)$$

Conductivity is not proportional to n but rather to the DOS at the Fermi energy.

most of the carriers do not carry current (metal)



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conductivity

$$\sigma = \frac{2q^2}{h} M_{3D}(E_F) \lambda(E_F)$$

$$\sigma = q^2 D_{2D}(E_F) D_n(E_F)$$

$$\sigma = q^2 D_{3D}(E_F) \frac{v^2(E_F) \tau(E_F)}{2}$$

$$\sigma = nq\mu_n = nq \left(\frac{q\tau(E_F)}{m^*} \right)$$

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mobility

$$\sigma = \frac{2q^2}{h} \int \lambda(E) M_{3D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\sigma = nq\mu_n$$

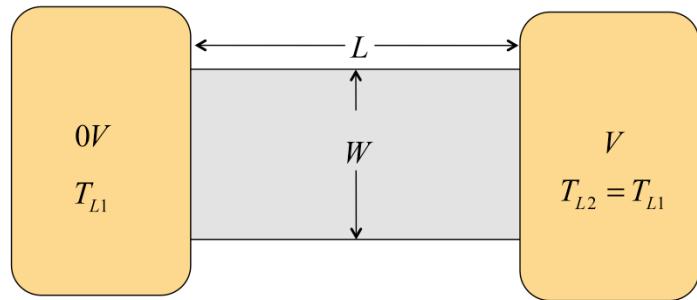
$$\mu \equiv \frac{1}{n} \frac{2q}{h} \int \lambda(E) M_{3D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

Kubo-Greenwood formula

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an isothermal 2D resistor



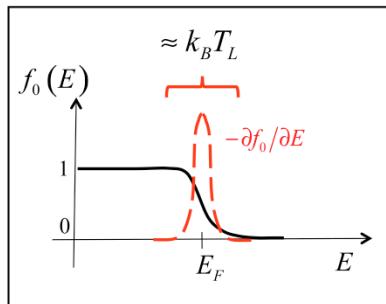
$$G = \frac{1}{R} = \frac{I}{V} = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$T = 0 \text{ K}$

$$G = \frac{2q^2}{h} \int_{-\infty}^{+\infty} \mathcal{T}(E) M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$G = \frac{2q^2}{h} \mathcal{T}(E_F) M(E_F)$$

$$f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_B T_L}}$$



$$\left(-\frac{\partial f_0}{\partial E} \right) \approx \delta(E_F)$$

resistance vs. length

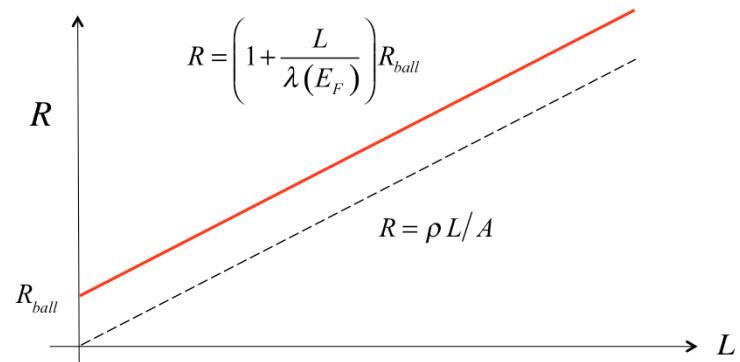
$$G = \frac{2q^2}{h} \mathcal{T}(E_F) M_{2D}(E_F) \quad \mathcal{T}(E_F) = \frac{\lambda(E_F)}{\lambda(E_F) + L}$$

$$G = \frac{\lambda(E_F)}{\lambda(E_F) + L} G_{ball} \quad R = \left(1 + \frac{L}{\lambda(E_F)} \right) R_{ball}$$

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differences in the Fermi levels



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finite temperature

$$G = \frac{1}{R} = \frac{I}{V} = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$G = \frac{2q^2}{h} \langle \langle T(E) \rangle \rangle \langle M(E) \rangle$$

$$\langle M \rangle \equiv \frac{\int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int \left(-\frac{\partial f_0}{\partial E} \right) dE}$$

$$\langle \langle T \rangle \rangle = \frac{\int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}$$

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discussion

- power dissipation
- voltage drop
- n-type vs. p-type
- “apparent” mobility

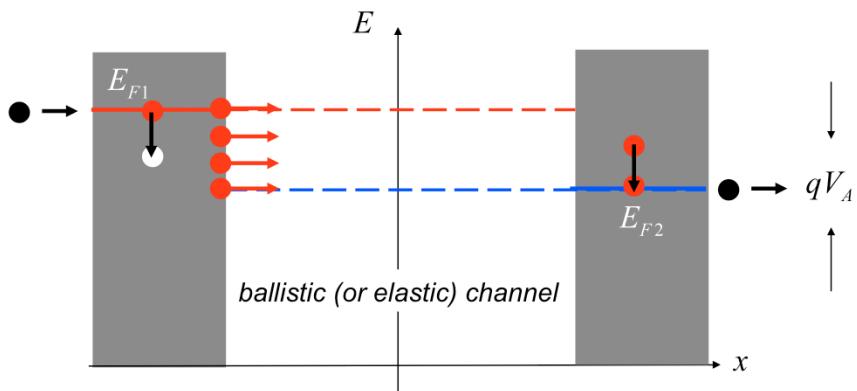
power dissipation in a ballistic resistor

$$P_D = IV = GV^2 = V^2/R$$

Where is the power dissipated in a ballistic resistor?

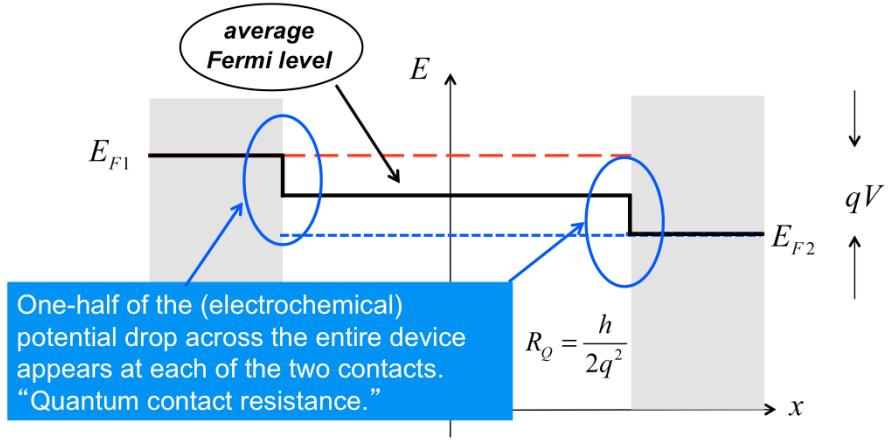
Answer: In the two contacts.

power dissipation in a ballistic resistor



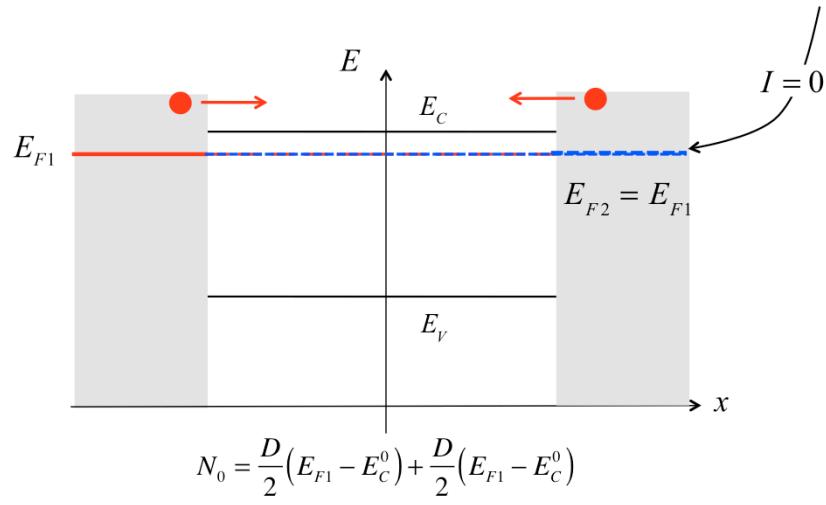
dissipation occurs in the contacts

voltage drop in a ballistic resistor



S. Datta, *Electronic Conduction in Mesoscopic Systems*, Chpt. 2, Cambridge, 1995
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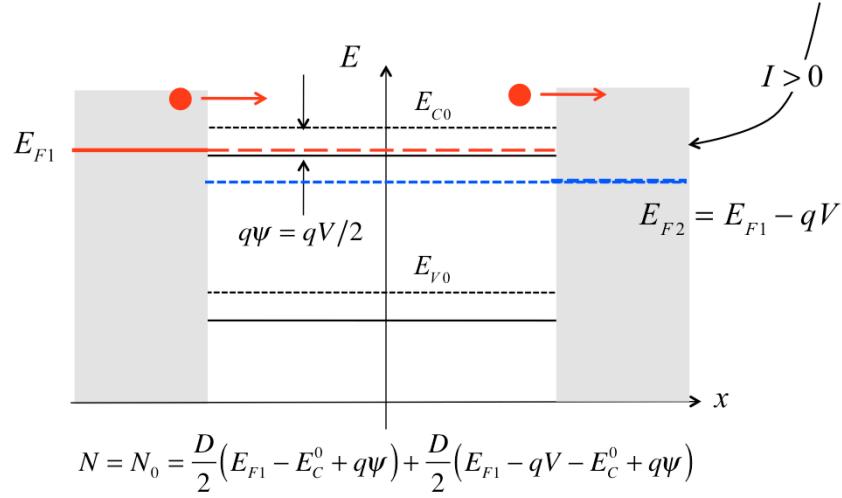
another view



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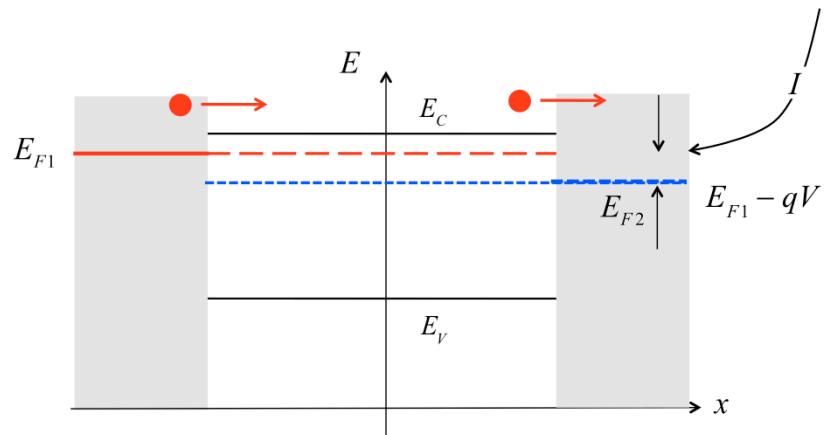
another view



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n-type conduction

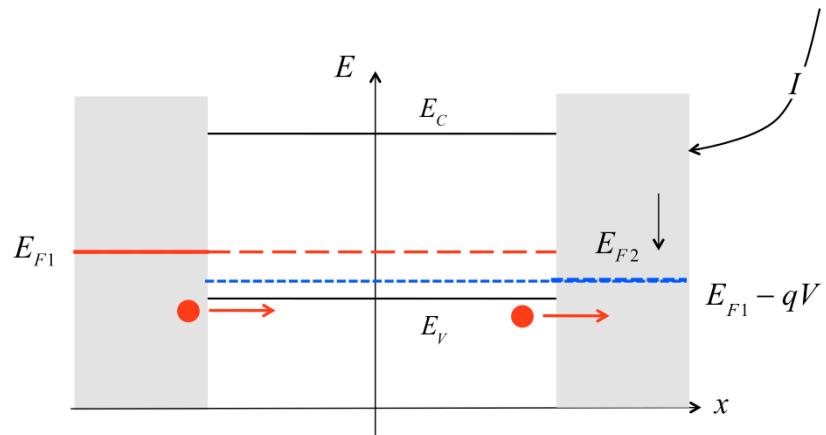


current is due to electrons flowing in the conduction band

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p-type conduction



current is due to electrons flowing in the valence band

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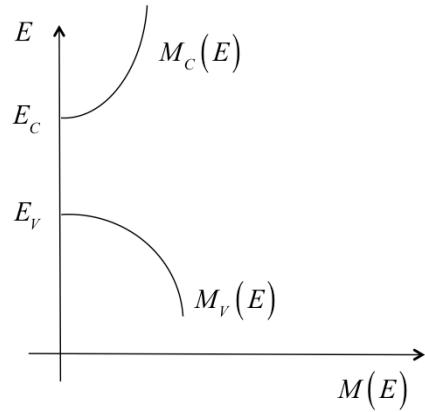
bipolar conduction

$$I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) dE$$

$$M(E) = M_V(E_V - E) + M_C(E - E_C)$$

$$M_C(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar}$$

$$M_V(E) = g_V W \frac{\sqrt{2m^*(E_V - E)}}{\pi\hbar}$$



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mobility

We usually like to think of mobility as a material dependent quantity.

In bulk materials with a large electric field, we generalize this to a field-dependent quantity.

More generally, “mobility” can be device dependent.

how should we define mobility?

definition of “apparent” mobility:

$$\mu_{app} \equiv \frac{1}{n_S} \frac{2q}{h} \int \mathcal{T}(E) L M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

not:

$$\mu_n = \frac{q\tau_m}{m^*}$$

in the diffusive limit

definition of mobility:

$$\mu_n \equiv \frac{1}{n_s} \frac{2q}{h} \int T(E) L M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$T(E) = \frac{\lambda(E)}{\lambda(E) + L} \rightarrow \frac{\lambda(E)}{L}$$

$$\mu_n \equiv \frac{1}{n_s} \frac{2q}{h} \int \lambda(E) M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\mu_n = \frac{q\tau_m}{m^*}$$

in the ballistic limit

definition of mobility:

$$\mu_n \equiv \frac{1}{n_s} \frac{2q}{h} \int T(E) L M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$T(E) = 1$$

$$\mu_{ball} \equiv \frac{1}{n_s} \frac{2q}{h} \int L M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\mu_{ball} \neq \frac{q\tau_m}{m^*} \quad \lambda(E) \rightarrow L$$

ballistic to diffusive...

definition of **apparent** mobility:

$$\mu_{app} \equiv \frac{1}{n_s} \frac{2q}{h} \int \mathcal{T}(E) L M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\mathcal{T}(E)L = \frac{\lambda(E)L}{\lambda(E)+L} \rightarrow \lambda_{app}(E)$$

$$\mu_{app} \equiv \frac{1}{n_s} \frac{2q}{h} \int \lambda_{app}(E) M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\frac{1}{\lambda_{app}(E)} \equiv \frac{1}{\lambda(E)} + \frac{1}{L}$$

$T_L = 0\text{K}$ apparent mobility

$$\mu_{app} \equiv \frac{1}{n_s} \frac{2q}{h} \int \lambda_{app}(E) M_{2D}(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\mu_{app}(T_L = 0\text{K}) = \frac{1}{n_s} \frac{2q}{h} \lambda_{app}(E_F) M_{2D}(E_F)$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ball}}$$

$$\frac{1}{\lambda_{app}(E)} = \frac{1}{\lambda(E)} + \frac{1}{L}$$

$$M_{2D} = \frac{h}{4} \langle v_x^+ \rangle D_{2D} \quad \langle v_x^+ \rangle = \frac{2}{\pi} v_F$$

$$n_s = g_V \frac{m^*}{\pi \hbar^2} (E_F - E_C) \\ = D_{2D} (E_F - E_C)$$

“ballistic mobility” in 2D at $T_L = 0\text{K}$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ball}} \quad \mu_n = \frac{\langle v_x^+ \rangle \lambda(E_F)_n}{2(E_F - E_C)/q} \quad \mu_{ball} = \frac{\langle v_x^+ \rangle L}{2(E_F - E_C)/q}$$

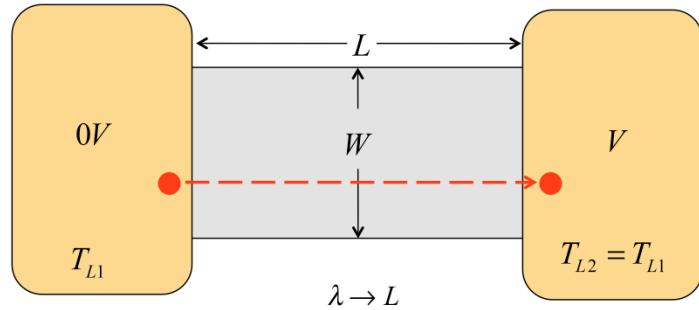
$$G_{2D} = \frac{2q^2}{h} T(E_F) W M_{2D}(E_F) = n_s q \mu_{app} \frac{W}{L}$$

M.S. Shur, “Low Ballistic Mobility in GaAs HEMTs,” *IEEE Electron Dev. Lett.*, **23**, 511-513, 2002

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physical interpretation



$$G_{2D} = n_s q \mu_{app} \frac{W}{L}$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_{ball}}$$

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1D, 2D, and 3D resistors

$$G = \frac{2q^2}{h} \int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$G = \frac{2q^2}{h} \langle \langle T \rangle \rangle \langle M \rangle$$

$$\langle M \rangle = \int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\langle \langle T \rangle \rangle = \frac{\int T(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}$$

For a constant mfp:

$$\langle \langle T \rangle \rangle = \frac{\lambda_0}{\lambda_0 + L}$$

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