Landauer Approach:

Fermi Window and Current

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Landauer Approach

\[ f_1(E) = \frac{1}{1 + e^{(E-E_{F1})/k_BT}} \]

\[ f_2(E) = \frac{1}{1 + e^{(E-E_{F2})/k_BT}} \]

\[ I = \frac{2q}{h} \int T(E)M(E)(f_1 - f_2) \, dE \]

transmission, modes (channels), differences in Fermi functions
Channels (modes)

\[ M_{1D}(E) = \frac{\hbar}{4} \langle \nu^+_x \rangle D_{1D}(E) \]

\[ M_{2D}(E)W = W \frac{\hbar}{4} \langle \nu^+_x \rangle D_{2D}(E) \]

\[ M_{3D}(E)A = A \frac{\hbar}{4} \langle \nu^+_x \rangle D_{3D}(E) \]
Transmission

\[ \mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L} \]

\( \lambda \) is the “mean-free-path for backscattering”

1) Diffusive: \( L \gg \lambda \) \( \mathcal{T} = \frac{\lambda}{L} \ll 1 \)

2) Ballistic: \( L \ll \lambda \) \( \mathcal{T} = 1 \)

3) Quasi-ballistic: \( L \approx \lambda \) \( \mathcal{T} < 1 \)
Fermi window

\[ I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) \, dE \]

Fermi window:
The range of energies over which \((f_1 - f_2) \neq 0\)
Differences in the Fermi levels

\[ f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/kT_1}} \]

\[ f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/kT_2}} \]

\[ E_{F2} = E_{F1} - qV \]

T_2 = T_1

metal contact 1

metal contact 2
Differences in the Fermi levels

\[ E_{F2} = E_{F1} - qV \]

\[ f_1 \neq f_2 \]

\[ f_1 = f_2 \]
Fermi window: Large bias

\[ f(E) = \begin{cases} 1 & \text{for } E < E_{F2} \\ 0 & \text{for } E_{F2} < E < E_{F1} \\ 1 & \text{for } E > E_{F1} \end{cases} \]

\[ f(E) = \begin{cases} 1 & \text{for } E_{F2} < E < E_{F1} \end{cases} \]

\[ T = 0 \text{ K} \]

\[ T > 0 \text{ K} \]
Fermi window: small bias

\[ T = 0 \text{ K} \]

\[ T > 0 \text{ K} \]
Small voltage (linear response)

\[ I = \frac{2q}{h} \int \mathcal{F}(E) M(E) (f_1(E) - f_2(E)) \, dE \]

\[ f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \]

\[ f_2(E) \approx f_1(E) + \frac{\partial f_1}{\partial E_F} \delta E_F \]

\[ f_2(E) \approx f_1(E) + \left( -\frac{\partial f_1}{\partial E} \right)(qV) \]

\[ f_2(E) \approx f_1(E) + \left( -\frac{\partial f_1}{\partial E} \right)(-qV) \]
Fermi window: small bias

\[ W_F(E) = \left( -\frac{\partial f_0}{\partial E} \right) \]

\[ \int W_F(E)\,dE = 1 \]

\[ f_1(E) - f_2(E) = W_F(E)(qV) \]

\[ T > 0 \text{ K} \]
Near-equilibrium conductance

\[ I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left( f_1(E) - f_2(E) \right) dE \]

\[ f_1(E) - f_2(E) = \left( -\frac{\partial f_1}{\partial E} \right)(qV) \]

\[ I = GV \quad A \]

\[ G = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \quad S \]
Quantum of conductance

\[ G = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE \]

\[ G(T = 0 \text{ K}) = \frac{2q^2}{h} \mathcal{T}(E_F) M(E_F) \]

\[ M(E) = W M_{2D}(E) = W g_v \frac{\sqrt{2m^* E}}{\pi \hbar} \]

For large \( W \), \( M \) is \( \sim W \)

For small \( W \), \( M \)
comes in discrete units.
Quantized conductance

Differences in the Fermi levels (constant $V$)

near-equilibrium
(small temperature difference)

$|f_1 - f_2| > 0$ so current flows, but the sign depends on whether the states are located above or below $E_F$ (n-type or p-type).

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What about holes?

Landauer expression for electrons:

\[ I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) dE \]

Do we need a Landauer expression for holes?
N-type conduction

current is due to electrons flowing in the conduction band
P-type conduction

current is due to electrons flowing in the valence band
What about holes?

All of these expressions refer to electrons in the conduction and valence bands.

**n-type**

\[ I = G_n V \]

\[ G_n = \frac{2q^2}{h} \int \mathcal{T}_C(E) M_C(E) \left(-\frac{\partial f_0}{\partial E}\right) dE \]

\[ f_0(E) = \frac{1}{1 + e^{(E - F_n)/k_B T}} \]

**p-type**

\[ I = G_p V \]

\[ G_p = \frac{2q^2}{h} \int \mathcal{T}_V(E) M_V(E) \left(-\frac{\partial f_0}{\partial E}\right) dE \]

\[ f_0(E) = \frac{1}{1 + e^{(E - F_p)/k_B T}} \]
Bipolar conduction

\[ I = \frac{2q}{h} \int_{E_1}^{E_2} \mathcal{T}(E) M(E)(f_1 - f_2) dE \]

\[ M(E) = M_V(E_V - E) + M_C(E - E_C) \]

\[ M_C(E) = A \frac{m_n^*}{2\pi\hbar^2} (E - E_C) \]

\[ M_V(E) = A \frac{m_p^*}{2\pi\hbar^2} (E_V - E) \]
Landauer Approach

\[ f_1(E) = \frac{1}{1 + e^{(E-E_{F1})/k_BT}} \]

\[ f_2(E) = \frac{1}{1 + e^{(E-E_{F2})/k_BT}} \]

\[ I = \frac{2q}{h} \int T(E) M(E) (f_1 - f_2) \, dE \]

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