Landauer Approach:

1D Resistor

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10/1/15
Landauer Approach

\[ f_1(E) = \frac{1}{1 + e^{(E-E_{F1})/k_BT}} \]

\[ f_2(E) = \frac{1}{1 + e^{(E-E_{F2})/k_BT}} \]

\[ I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) \, dE \]

transmission, modes (channels), differences in Fermi functions
1D Resistor

\[ M(E) = 1 \]
Ballistic current: $T = 0 \text{ K}$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) \, dE$$

$$I = \frac{2q}{h} \int (f_1 - f_2) \, dE$$

$$I = \frac{2q}{h} \left\{ \int_{0}^{+\infty} f_1(E) \, dE - \int_{0}^{+\infty} f_2(E) \, dE \right\}$$

$$I = \frac{2q}{h} \left\{ \int_{0}^{E_{F1}} 1 \, dE - \int_{0}^{E_{F2}} 1 \, dE \right\} = \frac{2q}{h} \left( E_{F1} - E_{F2} \right)$$

Ballistic transport:

$$\mathcal{T}(E) = 1$$

1D:

$$M(E) = 1$$
1D Resistor

\[ I = \frac{2q}{h} \left( E_{F1} - E_{F2} \right) \]

\[ I = \frac{2q^2}{h} V_A \]

\[ I = G_B V_A \]

\[ G_B = \frac{2q^2}{h} \]
Where does the voltage drop?

\[ \Delta V = I \left( \frac{R_B}{2} \right) \]

\[ R_B = \frac{1}{G_B} = \frac{h}{2q^2} \]

"quantum contact resistance"

Power dissipation in a ballistic resistor

Fermi level for $-k$ states

Power dissipated in right contact
Power dissipation in a ballistic resistor

power dissipated in left contact

Fermi level for $-k$ states
Power dissipation in a ballistic resistor

\[ \begin{align*}
E_F^1 & \quad \text{Fermi level for } -k \text{ states} \\
E_F^2 & \\
\end{align*} \]

*power dissipated equally in both contacts*

Lundstrom ECE-656 F15
1D Resistor: with scattering

\[ I = \frac{2q}{h} \int \mathcal{T}(E)M(E)(f_1 - f_2) dE \]

\[ \mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L} \]
Current: \( T = 0 \text{ K} \)

\[
I = \frac{2q}{h} \int \mathcal{T}(E) M(E)(f_1 - f_2) dE
\]

\[
I = \frac{2q}{h} \left( \frac{\lambda_0}{\lambda_0 + L} \right) \int (f_1 - f_2) dE
\]

\[
I = \frac{2q^2}{h} \left( \frac{\lambda_0}{\lambda_0 + L} \right) V_A = \frac{V_A}{R}
\]

\[
R = R_B \left( 1 + \frac{L}{\lambda_0} \right)
\]

Ballistic transport:

\[
\mathcal{T}(E) = \frac{\lambda_0}{\lambda_0 + L}
\]

1D:

\[
M(E) = 1
\]
1D Resistor: with scattering ($T = 0$ K)

\[
R = R_B \left(1 + \frac{L}{\lambda_0}\right) \\
R = R_B + \rho_{1D} L \\
R_B = \frac{h}{2q^2} \\
\rho_{1D} = \frac{R_B}{\lambda_0} = \frac{h}{2q^2 \lambda_0}
\]
1D Resistor: ballistic to diffusive

\[ R = R_B \left( 1 + \frac{L}{\lambda_0} \right) \]

\[ R = \rho_{1D} L \]