Temperature gradients give rise to an open circuit voltage, which is known as the Seebeck effect. We expect a **positive** voltage for an n-type semiconductor (and **negative** voltage for a p-type semiconductor.)

\[
V_{oc} = -S \Delta T
\]
Peltier effect

\[ J_n < 0 \quad J_n/(-q) > 0 \quad J_Q = \pi_n J_n > 0 \]

\[ \Delta J_Q \]

\[ \text{heat flux} \]

\[ \text{n-type semiconductor} \]

\[ T_1 \quad x \quad T_2 = T_1 \]

\[ \text{contacts maintained at same temperature} \]

cool

hot
Questions:

Why does $J_Q = \pi_n J_n$? 
(when the two contacts are at the same temperature)

What determines the Peltier coefficient, $\pi_n$?

Answer: We should draw an energy band diagram.
N-type semiconductor: equilibrium, $V = 0$

\[ n(x) = N_C e^{(E_F - E_C)/k_B T} \approx N_D^+ \]

ideal contacts  (no band bending)
N-type semiconductor: isothermal, $V > 0$

Electrons flow at an energy a little above the bottom of the conduction band.

$E_{F2} = E_{F1} - qV$

$T_2 = T_1$

(elastic scattering only)
N-type semiconductor: isothermal, $V > 0$

Energy absorbed per electron

$$Q = E_C(0) + \Delta_n - E_{F1}$$

Energy dissipated

$$Q = E_C(L) + \Delta_n(L) - E_{F2}$$

Heat is absorbed (emitted) when the average energy at which the heat current flows increases (decreases)
Peltier coefficient

1) Electrons flow from left to right when $V_2 > V_1$.

2) The flux of electrons from left to right is $J_{nx}/(-q)$

3) Each electron absorbs and then carries an amount of heat: $Q = E_C(0) + \Delta_n - E_{F1}$

4) So the heat flux from left to right is:

$$J_{Q1} = \left[ E_C(0) + \Delta_n - E_{F1} \right] \times J_{nx}/(-q) = \pi_n J_{nx}$$

$$\pi_n = -\frac{E_C(0) + \Delta_n - E_{F1}}{q}$$

(less than zero for an n-type semiconductor)
The Peltier coefficient is proportional to the difference between the energy at which current flows and the Fermi energy – just as the Seebeck coefficient was.
physics of Peltier cooling

electrons absorb thermal energy, \( E - E_{F1} \)
electrons enter contact 1 at the Fermi energy, \( E_{F1} \)

energy channel

Net power dissipated: \( P_D = IV \)

electrons dissipate energy, \( E - E_{F2} \)
electrons leave contact 2 at the Fermi energy, \( E_{F2} \)
electrons leave contact 2 at the Fermi energy, \( E_{F2} = E_{F1} - qV \)

contact 1

contact 2

\( T_1 \)

\( T_2 \)

\( V \)
Mathematics: electric and heat currents

Electrons carry charge, so there is an electrical current.

\[ I'(E) = \frac{2q}{h} \mathcal{T}(E) M(E) (f_1 - f_2) \]

But electrons also carry heat (thermal energy), so there is a heat current too.

\[ q \rightarrow (E - E_F) \]

Note: if \( E_C > E_{F1} \), then electrons in the contact must absorb energy to flow in one of the energy channels in the device.
Heat current

\[ I'_{Q_1} (E) = \frac{2(E - E_{F1})}{h} T(E) M(E) (f_1 - f_2) \]

\[ I'_{Q_2} (E) = \frac{2(E - E_{F2})}{h} T(E) M(E) (f_1 - f_2) \]
the math

\[ I'_Q(E) = \frac{2(E - E_{F1})}{h} T(E) M(E) (f_1 - f_2) \]

\[ (f_1 - f_2) \approx \left(-\frac{\partial f_0}{\partial E}\right) q\Delta V - \left(-\frac{\partial f_0}{\partial E}\right) \left(\frac{E - E_F}{T}\right) \Delta T \]

\[ I'_Q(E) = -T S'_T(E) \Delta V - K'_0(E) \Delta T \]

\[ K'_0(E) = \frac{2}{h} \frac{(E - E_F)^2}{T} T(E) M(E) \left(-\frac{\partial f_0}{\partial E}\right) \]
The result

\[
I = G\Delta V + S_T\Delta T
\]

\[
I_Q = -TS_T\Delta V - K_0\Delta T
\]

\[
\Delta V = RI - S\Delta T
\]

\[
I_Q = -\pi I - K_e\Delta T
\]

\[
G = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE
\]

\[
S_T = -\int \frac{(E - E_F)}{qT} G'(E) dE
\]

\[
K_0 = \int \frac{(E - E_F)^2}{q^2 T} G'(E) dE
\]

\[
S = \frac{S_T}{G}
\]

\[
K_e = K_0 - \pi SG
\]
for bulk 3D semiconductors

\[ J = \sigma \mathcal{E} - s_T \, dT / dx \]
\[ J_Q = T S_T \, \mathcal{E} - \kappa_0 \, dT / dx \]
\[ \mathcal{E} = \rho J + S \frac{dT}{dx} \]
\[ J_Q = \pi J - \kappa_e \frac{dT}{dx} \]

(diffusive transport)

\[ \sigma = \int \sigma'(E) \, dE \]
\[ \sigma'(E) = \frac{2q^2}{h} \lambda(E) \frac{M(E)}{A} \left( -\frac{\partial f_0}{\partial E} \right) \]
\[ s_T = -\int \frac{(E - E_F)}{qT} \sigma'(E) \, dE \]
\[ \kappa_0 = \int \frac{(E - E_F)^2}{q^2 T} \sigma'(E) \, dE \]
\[ S = s_T / \sigma \quad \pi = TS \]
\[ \kappa_e = \kappa_0 - \pi S \sigma \]
lattice thermal conductivity

\[ J_Q = \pi J - \kappa \frac{dT}{dx} \quad \kappa = \kappa_e + \kappa_L \]

Both electrons and lattice vibrations carry heat – we have been discussing the electronic part.

In metals, heat conduction by electrons dominates: \( \kappa_e >> \kappa_L \)

In semiconductors, lattice vibrations dominate: \( \kappa_L >> \kappa_e \)
Example: TE transport parameters of n-Ge

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_n$</td>
<td>$\Omega\cdot m$</td>
</tr>
<tr>
<td>$S_n$</td>
<td>V/K</td>
</tr>
<tr>
<td>$\pi_n$</td>
<td>W/A</td>
</tr>
<tr>
<td>$\kappa_n$</td>
<td>W/m-K</td>
</tr>
</tbody>
</table>

$E = \rho_n J_n + S_n \frac{dT}{dx} \left( \frac{V}{m} \right)$

$J_Q = \pi_n J_n - \kappa \frac{dT}{dx} \left( W \right)$

$N_D = 10^{15} \, \text{cm}^{-3}$

$T = 300 \, \text{K}$

$\mu_n = 3200 \, \text{cm}^2/\text{V-s}$

$n_0 = N_C e^{(E_F - E_c)/k_B T_L} \approx N_D$

$N_C = 1.04 \times 10^{19} \, \text{cm}^{-3}$
TE transport parameters of n-Ge: resistivity

\[ \rho_n \quad \Omega \text{-cm} \]
\[ S_n \quad V/K \]
\[ \pi_n \quad W/A = V \]
\[ \kappa_n \quad W/m-K \]

\[ E = \rho_n J_n + S_n \frac{dT}{dx} \quad \left( \frac{V}{m} \right) \]
\[ J_Q = \pi_n J_n - \kappa \frac{dT}{dx} \quad (W) \]

\[ N_D = 10^{15} \text{ cm}^{-3} \approx n_0 \]
\[ \mu_n = 3200 \text{ cm}^2/\text{V-s} \]
\[ \sigma_n = n_0 q \mu_n \quad \text{S/cm} \]

\[ \rho_n = \frac{1}{n_0 q \mu_n} \approx 2 \Omega \text{-cm} \]
TE transport parameters of n-Ge: Seebeck coeff.

\[
\rho_n = 2 \ \Omega \cdot \text{cm} \\
S_n = \text{V/K} \\
\pi_n = W/A = V \\
\kappa_n = W/m \cdot \text{K}
\]

\[
E = \rho_n J_n + S_n \frac{dT}{dx} \left( \frac{V}{m} \right) \\
J_Q = \pi_n J_n - \kappa \frac{dT}{dx} \left( \text{W} \right)
\]

\[
N_D = 10^{15} \ \text{cm}^{-3} \approx n_0 \\
n_0 = N_C e^{(E_F - E_c)/k_B T} \\
N_C = 1.04 \times 10^{19} \ \text{cm}^{-3} \\
T = 300 \ \text{K}
\]

\[
\left( E_c - E_F \right)/k_B T \approx \ln \left( N_C / n_0 \right) \approx 9.3 \\
\delta_n \approx 2 \quad \text{(non-degenerate, 3D)}
\]

\[
S_n = \left( \frac{k_B}{-q} \right) \left\{ \frac{\left( E_c - E_F \right)}{k_B T} + \delta_n \right\} \approx -970 \ \mu \text{V/K}
\]
TE transport parameters of n-Ge: Peltier coeff.

\[
\rho_n = 2 \ \Omega \text{-cm}
\]
\[
S_n = -970 \ V/K
\]
\[
\pi_n \ W/A = V
\]
\[
\kappa_n \ W/m-K
\]

\[
E = \rho_n J_n + S_n \frac{dT}{dx} \left( \frac{V}{m} \right)
\]
\[
J_Q = \pi_n J_n - \kappa \frac{dT}{dx} \ (W)
\]

\[
\pi_n = TS_n \approx -0.3 \ V
\]
TE transport parameters of n-Ge: Peltier coeff.

\[ \rho_n = 2 \ \text{Ω-cm} \]
\[ S_n = -970 \ \text{V/K} \]
\[ \pi_n = -0.3 \ \text{W/A} = \text{V} \]
\[ \kappa_n = \text{W/m-K} \]

\[ \mathcal{E} = \rho_n J_n + S_n \frac{dT}{dx} \left( \frac{\text{V}}{\text{m}} \right) \]
\[ J_Q = \pi_n J_n - \kappa \frac{dT}{dx} \left( \text{W} \right) \]

\[ \frac{\kappa_n}{T \sigma_n} = L \quad \text{(Lorenz number)} \]

\[ L \approx 2 \left( \frac{k_B}{q} \right)^2 \quad \text{(non-degenerate, 3D)} \]

\[ \sigma_n = \frac{1}{\rho_n} \]

\[ \kappa_n = 2.2 \times 10^{-4} \ \text{W/m-K} \]
TE transport parameters of n-Ge:

\[ \rho_n = 2 \ \Omega\text{-cm} \]
\[ S_n = -970 \ \text{V}/\text{K} \]
\[ \pi_n = -0.3 \ \text{W/A} = \text{V} \]
\[ \kappa_n = 2.2 \times 10^{-4} \ \text{W/m-K} \]

\[ E = \rho_n J_n + S_n \frac{dT}{dx} \left( \frac{\text{V}}{\text{m}} \right) \]
\[ J_Q = \pi_n J_n - \kappa \frac{dT}{dx} \left( \text{W} \right) \]

All of these parameters depend on the temperature and carrier concentration (Fermi level).

Note also:

\[ \kappa_L = 58 \ \text{W/m-K} \gg \kappa_n \]