This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three equally weighted questions. To receive full credit, you must show your work (scratch paper is attached).

The exam is designed to be taken in 50 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

30 points possible, 10 per question

1) 2 points per part – 10 points total

2) 10 points

3a) 5 points
3b) 5 points

I understand that if I am caught cheating in this course, I will earn an F for the course and be reported to the Dean of Students.

Read and understood: ________________________________

signature
Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) What is the physical meaning of the quantity, \( \frac{1}{\hbar}\frac{\partial E}{\partial k_x} \)?
   a) The effective mass of an electron.
   b) The phase velocity of an electron.
   c) The phase velocity of a hole.
   **d) The group velocity of an electron.**
   e) The group velocity of a hole.

1.2) This course focuses on semiclassical transport. Which of the following statements is true about semiclassical transport?
   a) Parabolic energy bands are assumed.
   b) The effective mass must be independent of energy.
   **c) Electrons (and holes) are treated as particles with sharply defined position and momentum.**
   d) All of the above.
   e) None of the above.

1.3) Consider electrons in the conduction band of silicon. If these electrons are confined in a quantum well with the confinement direction being [010]. Which of the following statements is true about the confinement mass (which determines the subband energy) and the valley degeneracy of the lowest subband(s)?
   a) Confinement mass = \( m_i^* \) and valley degeneracy = 2.
   b) Confinement mass = \( m_t^* \) and valley degeneracy = 4.
   c) Confinement mass = \( m_i^* \) and valley degeneracy = 2.
   d) Confinement mass = \( m_t^* \) and valley degeneracy = 4.
   e) Confinement mass = \( \sqrt{m_i^* m_t^*} \) and valley degeneracy = 4.

1.4) In general, how does conduction band nonparabolicity affect the density of states (DOS) in k-space?
   a) It increases the DOS.
   b) It decreases the DOS.
   **c) It has no effect on the DOS.**
   d) It causes the DOS to become non-uniform in k-space.
   e) It causes extended states to become localized states.
Problem 1) continued:

1.5) Which of the following is generally true of the characteristic times? (Scattering time, \( \tau \), momentum relaxation time, \( \tau_m \), and energy relaxation time, \( \tau_E \).)

a) \( \tau > \tau_m > \tau_E \).
b) \( \tau > \tau_m < \tau_E \).
c) \( \tau < \tau_m > \tau_E \).
d) \( \tau < \tau_m < \tau_E \).
e) \( \tau \approx \tau_m \approx \tau_E \).

2) Work out the following integral: \( I = \int_{E_C}^{\infty} \left( E - E_C \right)^{3/2} \left( -\partial f_0 / \partial E \right) dE \).

Draw a box around your answer.

Solution:

\[
I = \left[ \frac{\partial}{\partial E_F} \int_{E_C}^{\infty} (E - E_C)^{3/2} f_0(E) dE \right] = \frac{1}{k_BT} \int_{E_C}^{\infty} \left( \frac{E - E_C}{k_BT} \right)^{3/2} \frac{1}{1 + e^{(E-E_C)/k_BT}} dE \\

\text{Change variables: } \eta = \frac{E - E_C}{k_BT} \text{ and } d\eta = \frac{dE}{k_BT} \\
I = (k_BT)^{3/2} \int_{0}^{\infty} \frac{\eta^{3/2} d\eta}{1 + e^{\eta - E_F/k_BT}} \text{ where } \eta_F = \frac{E_F - E_C}{k_BT} \text{ and } d\eta_F = dE_F/k_BT \\
I = (k_BT)^{3/2} \frac{\partial}{\partial \eta_F} \int_{0}^{\infty} \eta^{3/2} d\eta \\

\text{From the formula sheet: } F_{3/2}(\eta_F) = \frac{1}{\Gamma(5/2)} \int_{0}^{\infty} \eta^{3/2} d\eta \\
I = (k_BT)^{3/2} \Gamma(5/2) \frac{\partial}{\partial \eta_F} F_{3/2}(\eta_F) = \Gamma(5/2)(k_BT)^{3/2} F_{1/2}(\eta_F) \\
\Gamma(5/2) = \frac{3}{2} \Gamma(3/2) = \frac{3}{2} \times \frac{1}{2} \Gamma(1/2) = \frac{3}{4} \sqrt{\pi} \\
I = \frac{3\sqrt{\pi}}{4} (k_BT)^{3/2} F_{1/2}(\eta_F) 
\]
3) Consider a hypothetical 1D conductor with the following dispersion for $E > 0$.

$$E(k_x)[1 + \alpha E(k_x)] = \hbar \nu_F k_x,$$

where $\alpha \geq 0$. Answer the following questions. **Draw a box around your answers.**

3a) Assume $\alpha = 0$ and compute the density-of-states, $DOS(E)$. Be sure to include units in your answer.

**Solution:**

$$E(k_x) = \hbar \nu_F k_x \quad dE = \hbar \nu_F dk_x$$

$$N_k dk_x = \left(\frac{L}{2\pi}\right) \times 2 \times \frac{L}{\pi} dk_x \times DOS(E) dE \times L$$

$$DOS(E) = \left(\frac{1}{\pi}\right) \frac{dk_x}{dE} = \left(\frac{1}{\pi}\right) \frac{1}{\hbar \nu_F}$$

Note: we are only considering $k_x > 0$ here. There are an equal number of states for $k_x > 0$, so we will need to multiply the answer by 2.

$$DOS(E) = \left(\frac{2}{\pi \hbar \nu_F}\right) J^{-1} m^{-1}$$

3b) Assume $\alpha > 0$ and compute the density-of-states, $DOS(E)$. Be sure to include units in your answer.

**Solution:**

$$E + \alpha E^2 = \hbar \nu_F k_x \quad dE(1 + 2\alpha E) = \hbar \nu_F dk_x$$

$$\frac{dk_x}{dE} = \frac{1 + 2\alpha E}{\hbar \nu_F}$$

$$DOS(E) = \left(\frac{1}{\pi}\right) \frac{dk_x}{dE} = \left(\frac{1}{\pi}\right) \frac{1 + 2\alpha E}{\hbar \nu_F}$$

Finally, multiply by 2 to get the minus $k_x$ branch of the dispersion.

$$DOS(E) = \left(\frac{2}{\pi \hbar \nu_F}\right)(1 + 2\alpha E) J^{-1} m^{-1}$$
ECE-656 Key Equations (Weeks 1-2)

Physical constants:
\[ \hbar = 1.055 \times 10^{-34} \text{ [J-s]} \quad m_0 = 9.109 \times 10^{-31} \text{ [kg]} \]
\[ k_B = 1.380 \times 10^{-23} \text{ [J/K]} \quad q = 1.602 \times 10^{-19} \text{ [C]} \quad \varepsilon_0 = 8.854 \times 10^{-14} \text{ [F/cm]} \]

Density of states in k-space:
1D: \( N_k = 2 \times \left( \frac{L}{2\pi} \right) = \frac{L}{\pi} \)
2D: \( N_k = 2 \times \left( \frac{A}{4\pi^2} \right) = \frac{A}{2\pi^2} \)
3D: \( N_k = 2 \times \left( \frac{\Omega}{8\pi^2} \right) = \frac{\Omega}{4\pi^3} \)

Density of states in energy (parabolic bands, per length, area, or volume):
\[ D_{1D}(E) = \frac{g_v}{\pi \hbar} \sqrt{\frac{2m^*}{E - E_1}} \quad D_{2D}(E) = g_v \frac{m^*}{\pi \hbar^2} \quad D_{3D}(E) = g_v \frac{m^*}{\pi^2 h^3} \sqrt{\frac{2m^*(E - E_C)}{\pi^2 h^3}} \]

Fermi function and Fermi-Dirac Integrals:
\[ f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}} \]
\[ F_j(\eta_F) = \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{\eta^j d\eta}{1 + e^{\eta - \eta_F}} \quad F_j(\eta_F) \to e^n \quad \eta_F \ll 0 \quad \frac{dF_j}{d\eta_F} = F_{j-1} \]
\[ \Gamma(n) = (n-1)! \quad (n \text{ an integer}) \quad \Gamma(1/2) = \sqrt{\pi} \quad \Gamma(p+1) = p\Gamma(p) \]