This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three equally weighted questions. To receive full credit, you must show your work (scratch paper is attached).

The exam is designed to be taken in 50 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

50 points possible.

1) 10 points total – 2 points per part.

2) 20 points total – 5 points per part.

3) 20 points total – 5 points per part.

I understand that if I am caught cheating in this course, I will earn an F for the course and be reported to the Dean of Students.

Read and understood: 

signature
Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) Which of the following is a reasonable estimate of the velocity for zone-center optical phonons in a typical semiconductor?
   a) $0 \text{ cm/sec.}$
   b) $10^3 \text{ cm/sec.}$
   c) $10^5 \text{ cm/sec.}$
   d) $10^7 \text{ cm/sec.}$
   e) $10^9 \text{ cm/sec.}$

1.2) For moderately doped ZnSe at room temperature, which of the follow scattering mechanisms will dominate?
   a) Acoustic deformation potential scattering.
   b) Optical deformation potential scattering.
   c) Polar optical phonon scattering.
   d) Ionized impurity scattering.
   e) Electron-plasmon scattering.

1.3) The transition rate for electron-phonon scattering contains a term, $(N_\omega + 1/2) \mp 1/2$.
   Which of the following statements is true?
   a) The phonon emission rate goes as $(N_\omega)$ and the absorption rate as $(N_\omega + 1)$
   b) The phonon emission rate goes as $(N_\omega)$ and the absorption rate as $(N_\omega - 1)$
   c) The phonon emission rate goes as $(N_\omega + 1)$ and the absorption rate as $(N_\omega)$
   d) The phonon emission rate goes as $(N_\omega - 1)$ and the absorption rate as $(N_\omega)$
   e) The phonon emission rate goes as $(N_\omega + 1/2)$ and the absorption rate as $(N_\omega - 1/2)$

1.4) We generally assume that POP scattering is not involved in intervalley scattering. Why?
   a) Because it is elastic, and intervalley scattering requires a large change in energy.
   b) Because it is inelastic and intervalley scattering requires no change in energy.
   c) Because polar materials have only a single conduction band valley.
   d) Because intervalley scattering requires a large change in momentum, and POP scattering favors small changes in momentum.
   e) Because intervalley scattering requires a small change in momentum, and POP scattering favors large changes in momentum.
Problem 1) continued:

1.5) For which of the following scattering mechanisms are the scattering rate and momentum relaxation rate the same and which of those has the largest energy relaxation rate?
   a) Ionized impurity scattering.
   b) POP scattering.
   c) Intervalley scattering.
   d) Alloy scattering.
   e) Acoustic deformation potential scattering.

2) The transition rate for alloy scattering in Si\textsubscript{x}Ge\textsubscript{1-x} is given by

\[ S(\bar{p}, \bar{p}') = \frac{2\pi}{\hbar^2} \left( \frac{3\pi^2}{16} \right) \frac{|\Delta U|^2}{NL} \delta(E' - E) \]

where

\[ \Delta U = x(1-x)(\chi_{Ge} - \chi_{Si}) \]

Answer the following questions for a 1D, SiGe nanowire with a density-of-states of

\[ D_{1D}(E) = \frac{\sqrt{2m^*}}{\pi \hbar} \frac{1}{\sqrt{E - E_C}} \]

Be sure to draw a box around your answers.

2a) Compute the scattering rate. (You may take short cuts if you explain what you are doing.)
2b) Compute the momentum relaxation rate.

2c) Compute the energy relaxation rate.

2d) The mobility is proportional to the momentum relaxation time. For what mole fraction, $x$, would you expect the lowest mobility in $\text{Si}_x\text{Ge}_{1-x}$?
3) Consider 2D electrons in a quantum well with 2 subbands. Assume parabolic energy bands and optical phonon scattering. Assume $\varepsilon_1 = 0.1 \, \text{eV}$, $\varepsilon_2 = 0.25 \, \text{eV}$, and $\hbar \omega_0 = 0.026 \, \text{eV}$. Answer the following questions.

3a) On the axes below, sketch the intra-subband scattering rate vs. energy for electrons in subband 1 with scattering by optical phonon absorption. Also sketch the intra-subband scattering rate vs. energy for electrons in subband 1 with scattering by optical phonon emission.

**Explain your reasoning.**
3b) On the axes below, sketch the \textbf{intra-subband} scattering rate vs. energy for electrons in subband 2 with scattering by optical phonon \textit{absorption}. Also sketch the \textbf{intra-subband} scattering rate vs. energy for electrons in subband 2 with scattering by optical phonon \textit{emission}.

\textbf{Explain your reasoning.}
3c) On the axes below, sketch the **inter-subband** scattering rate vs. energy for electrons in subband 1 to subband 2 with scattering by optical phonon **absorption and emission**.

**Explain your reasoning.**

\[
\frac{1}{\tau_{1-2}(E)}
\]

\[\varepsilon_1 = 0.1 \text{ eV} \quad \varepsilon_2 = 0.25 \text{ eV}\]
3d) On the axes below, sketch the **inter-subband** scattering rate vs. energy for electrons in subband 2 to subband 1 with scattering by optical phonon absorption and emission.

**Explain your reasoning.**
ECE-656 Key Equations (Weeks 1-4)

Physical constants:
\[ h = 1.055 \times 10^{-34} \text{ [J-s]} \]
\[ m_0 = 9.109 \times 10^{-31} \text{ [kg]} \]
\[ k_B = 1.380 \times 10^{-23} \text{ [J/K]} \]
\[ q = 1.602 \times 10^{-19} \text{ [C]} \]
\[ \epsilon_0 = 8.854 \times 10^{-14} \text{ [F/cm]} \]

Density of states in k-space:
1D: \[ N_k = 2 \times \left( \frac{L}{2\pi} \right) = \frac{L}{\pi} \]
2D: \[ N_k = 2 \times \left( \frac{A}{4\pi^2} \right) = \frac{A}{2\pi^2} \]
3D: \[ N_k = 2 \times \left( \frac{\Omega}{8\pi^2} \right) = \frac{\Omega}{4\pi^3} \]

Density of states in energy (parabolic bands, per length, area, or volume):
\[ D_{1D}(E) = \frac{g_v}{\pi h} \sqrt{\frac{2m^*}{E - E_1}} \]
\[ D_{2D}(E) = g_v \frac{m^*}{\pi h^2} \]
\[ D_{3D}(E) = g_v \frac{m^*}{\pi^2 h^3} \sqrt{\frac{2m^*(E - E_C)}{\pi^2 h^3}} \]

Fermi function and Fermi-Dirac Integrals:
\[ f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} \]
\[ F_j(\eta_F) = \frac{1}{\Gamma(j+1)} \int_0^{\infty} \frac{\eta^j d\eta}{1 + e^{\eta - \eta_F}} \]
\[ F_j(\eta_F) \rightarrow e^0 \eta_F << 0 \]
\[ \frac{dF_j}{d\eta_F} = F_{j-1} \]
\[ \Gamma(n) = (n-1)! \text{ (n an integer)} \quad \Gamma(1/2) = \sqrt{\pi} \quad \Gamma(p+1) = p\Gamma(p) \]

Scattering:
\[ S(\vec{p}, \vec{p}') = \frac{2\pi}{\hbar} |H_{\vec{p}', \vec{p}}|^2 \delta(E' - E - \Delta E) \]
\[ H_{\vec{p}', \vec{p}} = \frac{1}{\Omega} \int_{-\infty}^{\infty} e^{-i\vec{p}'\vec{r}/\hbar} U_S(\vec{r}) e^{i\vec{p}\vec{r}/\hbar} d\vec{r} \]
\[ \frac{1}{\tau(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \quad \frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}', \uparrow} S(\vec{p}, \vec{p}') \delta_{\vec{p}', \vec{p} \pm \hbar} \delta(E' - E + \hbar \omega) \]
\[ \delta_{\vec{p}', \vec{p} \pm \hbar} \delta(E' - E + \hbar \omega) \rightarrow \frac{1}{\hbar \nu \beta} \delta \left( \pm \cos \theta + \frac{\hbar \beta}{2p} \frac{\omega_\beta}{\nu \beta} \right) \]
\[ \frac{1}{\tau} = \frac{1}{\tau_{abs}} + \frac{1}{\tau_{ems}} = \frac{2\pi}{\hbar} \left( \frac{D_{1D}^2 k_B T}{c_i} \right) \frac{D_{3D}(E)}{2} \text{ (ADP)} \]
\[ L_D = \sqrt{\frac{\kappa_S \epsilon_0 k_B T}{q^2 \hbar \omega_0}} \]
\[ \frac{1}{\tau} = \frac{1}{\tau_m} = \frac{2\pi}{\hbar} \left( \frac{hD_{1D}^2}{2\rho \omega_0} \right) \left( N_0 + \frac{1}{2} \frac{1}{2} \frac{D_{3D}(E \pm \hbar \omega_0)}{2} \right) N_0 = \frac{1}{e^{\hbar \omega_0 / k_B T} - 1} \text{ (ODP)} \]