This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three questions. To receive full credit, you must show your work (scratch paper is attached).

The exam is designed to be taken in 50 minutes, but you have 75 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

50 points possible.

1) 10 points total – 2 points per part.
2) 20 points total – 10 points per part.
3) 20 points total – 10 points per part.

I understand that if I am caught cheating in this course, I will earn an F for the course and be reported to the Dean of Students.

Read and understood: ____________________________

signature
Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) If a resistor is 100 nm long and the mean-free-path for backscattering is 100 nm, then what is the **apparent** mean-free-path?
   a) 50 nm.
   b) 100 nm.
   c) 150 nm.
   d) 200 nm.
   e) 10,000 nm

1.2) What are the two most general driving forces for current?
   a) Differences in the electrostatic potential and temperature.
   b) Differences in the carrier concentration and temperature.
   c) Differences in the chemical potential and temperature.
   d) Differences in the electrostatic potential and carrier concentration.
   e) Differences in the electron density and electrostatic potential.

1.3) For a ballistic resistor, the power dissipated is \( P_D = IV = V^2/R \). Where is this power dissipated?
   a) Uniformly within the resistor.
   b) At the two ends of the resistor.
   c) Inside the contact with the most positive voltage.
   d) Inside the contact with the most negative voltage.
   e) Inside the two contacts equally.

1.4) For a ballistic resistor, with a voltage, \( V \), applied across it, where does the voltage drop?
   a) Uniformly across the resistor.
   b) At the two ends of the resistor.
   c) Inside the contact with the most positive voltage.
   d) Inside the contact with the most negative voltage.
   e) Inside the two contacts equally.

1.5) Under what conditions is \( (f_1 - f_2) = \left( -\frac{\partial f_0}{\partial E} \right) qV \)?
   a) Only under non-degenerate conditions.
   b) Only under fully degenerate conditions.
   c) Only at \( T = 0 \) K.
   d) When there is a small difference in the voltages of the two contacts.
   e) When there is a small difference in the temperatures of the two contacts.
2) Consider a metallic carbon nanotube, which has a linear dispersion, \( E(k_x) = \pm \hbar \nu_F k_x \).

The density of states in this case is a constant, independent of energy,

\[
D(E) = \frac{2g_v}{\pi \hbar \nu_F} \sqrt{E},
\]

where \( g_v = 2 \) is the valley degeneracy for a carbon nanotube, and \( \nu_F \) is the velocity. The numerical value of \( 2q^2 / \hbar \) is \( 7.73 \times 10^{-5} \) siemens.

2a) Determine the number of channels vs. energy, \( M(E) \).

2b) Consider a carbon nanotube that is one micrometer long. A small voltage of 0.01 V is applied, and a current of \( 3.86 \times 10^{-7} \) A is measured. What is the transmission? To keep the calculations simple, you should assume fully degenerate conditions, but you may assume that \( T = 300 \) K. Explain what you are doing beginning with the Landauer expression:

\[
I = \frac{2q}{\hbar} \int \mathcal{T}(E) M(E) (f_1 - f_2) dE.
\]
Consider a short, n-channel MOSFET with a small drain voltage applied. The inversion layer mobility of a similar wide and long channel MOSFET is measured as $250 \text{ cm}^2/(\text{V-s})$. In the on-state, the inversion layer density is $n_s = 10^{13} \text{ cm}^{-2}$. Only the lowest subband with $m^* = 0.19m_0$ and a valley degeneracy of 2 is occupied. Answer the following questions. To keep the calculations simple, you should assume that the temperature is $T = 0 \text{ K}$.

3a) Compute the number of channels in the Fermi window, $\langle M(E) \rangle$ assuming a minimum size transistor with $W = L = 20 \text{ nm}$.

3b) Compute the transmission for an $L = 20 \text{ nm}$ MOSFET. You may assume that the mean-free-path for backscattering is the same as it is for a long channel MOSFET, but you will first have to determine what its value is.
ECE-656 Key Equations (Weeks 1-7)

Physical constants:
\[ h = 1.055 \times 10^{-34} \quad [\text{J-s}] \quad m_0 = 9.109 \times 10^{-31} \quad [\text{kg}] \]
\[ k_B = 1.380 \times 10^{-23} \quad [\text{J/K}] \quad q = 1.602 \times 10^{-19} \quad [\text{C}] \quad \varepsilon_0 = 8.854 \times 10^{-14} \quad [\text{F/cm}] \]

Density of states in k-space:
1D: \( N_k = 2 \times \left( \frac{L}{2\pi} \right) = L/\pi \) \hspace{1cm} 2D: \( N_k = 2 \times \left( \frac{A}{4\pi^2} \right) = A/2\pi^2 \) \hspace{1cm} 3D:
\( N_k = 2 \times \left( \frac{\Omega}{8\pi^2} \right) = \frac{\Omega}{4\pi^3} \)

Density of states in energy (parabolic bands, per length, area, or volume):
\[ D_{1D}(E) = \frac{g_v}{\pi h^2} \frac{2m^*}{(E - \varepsilon_1)} \quad D_{2D}(E) = g_v \frac{m^*}{\pi h^2} \quad D_{3D}(E) = g_v \frac{m^* \sqrt{2m^*(E - E_C)}}{\pi^2 h^3} \]

Fermi function and Fermi-Dirac Integrals:
\[ f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} \]
\[ F_j(\eta) = \frac{1}{\Gamma(j + 1)} \int_0^\infty \frac{e^{-\eta} d\eta}{1 + e^{E - \eta}} \quad F_j(\eta_F) \rightarrow e^0 \quad \eta_F \ll 0 \quad \frac{dF_j}{d\eta_F} = F_{j-1} \]
\[ \Gamma(n) = (n - 1)! \quad (n \text{ an integer}) \quad \Gamma(1/2) = \sqrt{\pi} \quad \Gamma(p + 1) = p\Gamma(p) \]

Scattering:
\[ S(\vec{p}, \vec{p}') = \frac{2\pi}{\hbar} \left| H_{\vec{p}'\vec{p}} \right|^2 \delta(E' - E - \Delta E) \quad H_{\vec{p}'\vec{p}} = \frac{1}{\Omega} \int_{-\infty}^{\infty} e^{-i\vec{p}' \cdot \vec{r}/\hbar} U_S(\vec{r}) e^{i\vec{p} \cdot \vec{r}/\hbar} d\vec{r} \]
\[ \frac{1}{\tau(\vec{p})} = \sum_{\vec{p}', \vec{p}'} S(\vec{p}, \vec{p}') \quad \frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}', \vec{p}'} S(\vec{p}, \vec{p}') \frac{\Delta \rho_{\vec{p}'}}{\rho_{\vec{p}}} \quad \frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}', \vec{p}'} S(\vec{p}, \vec{p}') \frac{\Delta E}{E_0} \]
ADP: \( |K_{\beta}|^2 = \beta^2 D_A^2 \) \hspace{1cm} ODP: \( |K_{\beta}|^2 = D_O^2 \) \hspace{1cm} PZ: \( |K_{\beta}|^2 = \left( qe_{pz}/\kappa_s e_0 \right)^2 \) \hspace{1cm} POP: \( |K_{\beta}|^2 = \frac{p q^2 \omega_0^2}{\beta^2 \kappa_s e_0} \left( \frac{\kappa_0}{\kappa_{\omega}} - 1 \right) \)
\[ S(\vec{p}, \vec{p}') = \frac{\pi}{\Omega \rho} \left| K_{\beta} \right|^2 \left( N_0 + 1 + \frac{1}{2} \right) \delta_{\vec{p}', \vec{p}+\vec{\beta}} \delta(E' - E \mp \hbar \omega) \]
\[ \delta_{\vec{p}', \vec{p}+\vec{\beta}} \delta(E' - E \mp \hbar \omega) \rightarrow \frac{1}{\hbar v_B} \delta \left( \pm \cos \theta + \frac{\hbar \beta}{2 v_B} \mp \omega_0 / v_B \right) \]
\[ \frac{1}{\tau} = \frac{1}{\tau_{abs}} + \frac{1}{\tau_{ems}} = \frac{2\pi}{\hbar} \left( \frac{D_{3D}(E)}{c_t} \right) \frac{D_{3D}(E)}{2} \quad \text{(ADP)} \quad L_D = \sqrt{\frac{\kappa_s e_0 k_B T}{q^2 n_0}} \]
\[ \frac{1}{\tau} = \frac{1}{\tau_m} = \frac{2\pi}{\hbar} \left( \frac{h D_{3D}^2}{2 \rho_0} \right) \left( N_0 + 1 + \frac{1}{2} \right) \frac{D_{3D}(E \pm \hbar \omega_0)}{2} \quad N_0 = \frac{1}{e^{\hbar \omega_0 / \hbar v_B} - 1} \quad \text{(ODP)} \]
Isothermal Near-Equilibrium Transport: Summary of Landauer Approach

\[ I = \frac{2q}{h} \int T(E)M(E)(f_i - f_z)dE \rightarrow \text{small bias, isothermal:} \quad f_i(E) - f_z(E) = \left( -\frac{\partial f_0}{\partial E} \right)(qV) \]

Linear response (also called low bias, near-equilibrium):

\[ I = GV \quad G = \frac{2q^2}{h} \int T(E)M(E) \left( -\frac{\partial f_0}{\partial E} \right)dE \quad G = \frac{2q^2}{h} \left\langle \left\langle T(E) \right\rangle \right\rangle \langle M(E) \rangle \]

\[ \left\langle \left\langle T(E) \right\rangle \right\rangle \equiv \frac{\int T(E)M(E)(-\partial f_0/\partial E)dE}{\int M(E)(-\partial f_0/\partial E)dE} \quad \langle M(E) \rangle = \int M(E)(-\partial f_0/\partial E)dE \]

\[ R_{\text{ball}} = \frac{1}{M(E_f)} \frac{h}{2q^2} = \frac{12.8k\Omega}{M} \]

Modes / channels:

\[ M(E) \equiv \frac{h}{4} \langle v^+_s(E) \rangle D_{1D}(E) \quad \langle v^+_s(E) \rangle = v(E) \]

\[ M(E) = WM_{2D} \equiv W \frac{h}{4} \langle v^+_s(E) \rangle D_{2D}(E) \quad \langle v^+_s(E) \rangle = \frac{2}{\pi} v(E) \]

\[ M(E) = AM_{3D} \equiv A \frac{h}{4} \langle v^+_s(E) \rangle D_{3D}(E) \quad \langle v^+_s(E) \rangle = \frac{1}{2} v(E) \]

Parabolic bands: \( (v(k) = E_c + h^2k^2/2m^*) \) \quad Graphene (2D):

\[ M(E) = M_{1D}(E) = g_v \]

\[ M(E) = WM_{2D}(E) = g_v \sqrt{\frac{2m^*(E-E_c)}{\pi h}} \quad M(E) = W\left| E \right|/\pi h v_F \]

\[ M(E) = AM_{3D}(E) = g_v A \frac{m^*}{2\pi h^2} (E-E_c) \]

Transmission: \( T(E) = \frac{\lambda(E)}{\lambda(E)+L} \)

Mean-free-path for backscattering:

1D: \( \lambda(E) = 2v(E)\tau_m(E) \quad 2D: \lambda(E) = \frac{\pi}{2} v(E)\tau_m(E) \quad 3D: \lambda(E) = \frac{4}{3} v(E)\tau_m(E) \)

Diffusion coefficient: \( D_n = \langle v^+_s \rangle \left\langle \left\langle \lambda \right\rangle \right\rangle /2 \quad D_n = v_r \lambda_0 /2 \quad D_n(E) = \left\langle v^+_s(E) \right\rangle \lambda(E) /2 \)

Uni-directional thermal velocity: \( \left\langle v^+_s \right\rangle = v_r = \sqrt{2k_BT/\pi m^*} \quad (\eta_F << 0) \)