This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three questions. To receive full credit, you must show your work (scratch paper is attached).

The exam is designed to be taken in 50 minutes, but you have 75 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

50 points possible.

1) 10 points total – 2 points per part.

2) 20 points total – 10 points per part.

3) 20 points total – 10 points per part.

I understand that if I am caught cheating in this course, I will earn an F for the course and be reported to the Dean of Students.

Read and understood: ________________________________

signature
Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) If a resistor is 100 nm long and the mean-free-path for backscattering is 100 nm, then what is the **apparent** mean-free-path?
   - a) **50 nm.**
   - b) 100 nm.
   - c) 150 nm.
   - d) 200 nm.
   - e) 10,000 nm

1.2) What are the two most general driving forces for current?
   - a) Differences in the electrostatic potential and temperature.
   - b) Differences in the carrier concentration and temperature.
   - c) **Differences in the electrochemical potential and temperature.**
   - d) Differences in the electrostatic potential and carrier concentration.
   - e) Differences in the electron density and electrostatic potential.

1.3) For a ballistic resistor, the power dissipated is \( P_D = IV = V^2/R \). Where is this power dissipated?
   - a) Uniformly within the resistor.
   - b) At the two ends of the resistor.
   - c) Inside the contact with the most positive voltage.
   - d) Inside the contact with the most negative voltage.
   - e) **Inside the two contacts equally.**

1.4) For a ballistic resistor, with a voltage, \( V \), applied across it, where does the voltage drop?
   - a) Uniformly across the resistor.
   - b) **At the two ends of the resistor.**
   - c) Inside the contact with the most positive voltage.
   - d) Inside the contact with the most negative voltage.
   - e) Inside the two contacts equally.

1.5) Under what conditions is \( (f_1 - f_2) = (-\partial f/\partial E)qV \)?
   - a) Only under non-degenerate conditions.
   - b) Only under fully degenerate conditions.
   - c) Only at \( T = 0 \) K.
   - d) **When there is a small difference in the voltages of the two contacts.**
   - e) When there is a small difference in the temperatures of the two contacts.
2) Consider a metallic carbon nanotube, which has a linear dispersion, \( E(k_x) = \pm \hbar v_F k_x \). The density of states in this case is a constant, independent of energy, 
\( D(E) = \frac{2g_v}{\pi \hbar v_F} \), where \( g_v = 2 \) is the valley degeneracy for a carbon nanotube, and \( v_F \) is the velocity. The numerical value of \( \frac{2q^2}{\hbar} \) is \( 7.73 \times 10^{-5} \) siemens.

2a) Determine the number of channels vs. energy, \( M(E) \).

Solution:

For a 1D dispersion like this, the number of channels is 1 times the valley degeneracy, so the answer is:

\[
M(E) = 2
\]

We can also work this out from the expression for \( M(E) \):

\[
M_{1D}(E) = \frac{\hbar}{4} \left( \frac{\nu_F}{\pi \hbar v_F} \right) = \frac{\hbar}{4} \nu_F \frac{2g_v}{\pi \hbar v_F} = g_v = 2
\]

2b) Consider a carbon nanotube that is one micrometer long. A small voltage of 0.01 V is applied, and a current of \( 3.86 \times 10^{-7} \) A is measured. What is the transmission? To keep the calculations simple, you should assume fully degenerate conditions, but you may assume that \( T = 300 \) K. Explain what you are doing beginning with the Landauer expression:

\[
I = \frac{2q}{\hbar} \int T(E) M(E)(f_1 - f_2) dE.
\]

Solution:

\[
I = \frac{2q}{\hbar} \int T(E) M(E)(f_1 - f_2) dE = \left[ \frac{2q}{\hbar} \int T(E_F) g_v \left( -\frac{\partial f_0}{\partial E} \right) dE \right] qV
\]

(fully degenerate conditions)

\[
I = \left( \frac{2q^2}{\hbar} T(E_F) g_v \right) V \quad \text{(fully degenerate, which is like } T = 0 \text{ K)}
\]
Now compute the ballistic current:

\[ I_{\text{ball}} = \left( g_v \frac{2q^2}{h} \right) V = \left( 2 \times 7.73 \times 10^{-3} \right) \times 0.01 = 15.6 \times 10^{-7} \text{ A} \]

We see that:

\[ \mathcal{T}(E_F) = \frac{I}{I_{\text{ball}}} = \frac{3.86}{15.6} = 0.25 \]

\[ \mathcal{T}(E_F) = 0.25 \]

3) Consider a short, n-channel MOSFET with a small drain voltage applied. The inversion layer mobility of a similar wide and long channel MOSFET is measured as 250 cm²/(V·s). In the on-state, the inversion layer density is \( n_s = 10^{13} \text{ cm}^{-2} \). Only the lowest subband with \( m^* = 0.19m_0 \) and a valley degeneracy of 2 is occupied. Answer the following questions. To keep the calculations simple, you should assume that the temperature is \( T = 0 \text{ K} \).

3a) Compute the number of channels in the Fermi window, \( \langle M(E) \rangle \) assuming a minimum size transistor with \( W = L = 20 \text{ nm} \).

Solution:

Begin with the conductance:

\[ G = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE = \frac{2q}{h} \left( \langle \mathcal{T}(E) \rangle \right) \langle M(E) \rangle \]

At \( T = 0 \text{ K} \), the Fermi window is a delta-function at the Fermi energy, so

\[ G = \frac{2q}{h} \mathcal{T}(E_F) M(E_F) = \frac{2q}{h} \left( \langle \mathcal{T}(E) \rangle \right) \langle M(E) \rangle \]

so

\[ \langle M(E) \rangle = M(E_F) \]

For parabolic energy bands: \( M_{2D}(E) = g_v \sqrt{\frac{2m^*(E-E_C)}{\pi h}} \),

so

\[ \langle M(E) \rangle = WM_{2D}(E_F) = Wg_v \sqrt{\frac{2m^*(E_F-E_C)}{\pi h}} \quad (*) \]
and we only need to find $E_F$. We are given the carrier density, and we can find the Fermi level from the carrier density

$$n_s = \int_{E_C}^{\infty} D_2D(E)f_0(E)dE = \int_{E_C}^{E_F} \left( g_v \frac{m^*}{\pi \hbar^2} \right)(1)dE = g_v \frac{m^*}{\pi \hbar^2} (E_F - E_C)$$

$$E_F - E_C = \frac{n_s}{\left( g_v m^*/\pi \hbar^2 \right)}.$$ 

Now use this result for the Fermi level in the expression for modes (*) :

$$M(E_F) = WG_v \sqrt{\frac{2m^* n_s}{\left( g_v m^*/\pi \hbar^2 \right)}} = W \sqrt{\frac{2g_v n_s}{\pi}}$$

Putting in numbers:

$$M(E_F) = W \sqrt{\frac{2g_v n_s}{\pi}} = 20 \times 10^{-7} \sqrt{4 \times 10^{13}/\pi} = 7.1$$

$$\boxed{M(E_F) = 7.1}$$

This number is very small... We should probably be counting modes instead, so the answer would be 7.

3b) Compute the transmission for the $L=20$ nm MOSFET. You may assume that the mean-free-path for backscattering is the same as it is for a long channel MOSFET, but you will first have to determine what its value is.

**Solution:**

From the numbers given, we can compute the sheet conductance for a long channel MOSFET:

$$\sigma_s = n_s q \mu_n = 10^{13} \times 1.6 \times 10^{-19} \times 250 = 4 \times 10^{-4} \ \Omega$$

From the previous problem, we know:

$$M(E_F)/W = \sqrt{2g_v n_s/\pi} = 3.56 \times 10^6 \ \text{cm}^{-1}$$

We also know:

The numerical value of $2q^2/h$ is $7.73 \times 10^{-5}$ siemens.
The Landauer expression gives us the conductance:

\[ G = \frac{2q^2}{\hbar} \int \mathcal{T}(E) M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE = \frac{2q^2}{\hbar} \left\langle \mathcal{T}(E) \right\rangle \left\langle M(E) \right\rangle \]

Assume fully degenerate conditions:

\[ G = \frac{2q^2}{\hbar} \mathcal{T}(E_F) M(E_F) \]

We write the conductance in the **diffusive limit** in the following form:

\[ G = \sigma_s \frac{W}{L} = \left( n_s q \mu_n \right) \frac{W}{L} \]

Now equate the two expressions:

\[ G = \frac{2q^2}{\hbar} \mathcal{T}(E_F) M(E_F) = \left( n_s q \mu_n \right) \frac{W}{L} \]

Assume the diffusive limit:

\[ \frac{2q^2}{\hbar} \frac{\lambda(E_F)}{L} M(E_F) = \left( n_s q \mu_n \right) \frac{W}{L} \]

Simplify:

\[ \frac{2q^2}{\hbar} \frac{\lambda(E_F)}{L} M(E_F) / W = \left( n_s q \mu_n \right) \]

Solve for the mfp:

\[ \lambda(E_F) = \frac{\left( n_s q \mu_n \right)}{\left( 2q^2 / h \right) M(E_F) / W} \]

Insert numbers:

\[ \lambda(E_F) = \frac{\left( 4 \times 10^{-4} \ \Omega^{-1} \right)}{\left( 7.73 \times 10^{-5} \ \Omega^{-1} \right) \left( 3.56 \times 10^6 \ \text{cm}^{-1} \right)} = 14.5 \times 10^{-7} \text{cm} \]

Finally, for the \( L = 20 \) nm MOSFET

\[ \mathcal{T}(E_F) = \frac{\lambda(E_F)}{\lambda(E_F) + L} = \frac{14.5}{14.5 + 20} = 0.42 \]

\[ \mathcal{T}(E_F) = 0.42 \]