This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three questions. To receive full credit, you must show your work (scratch paper is attached).

The exam is designed to be taken in 50 minutes, but you have 75 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove, if you want.

50 points possible.

1) 10 points total – 2 points per part.

2) 20 points total – 10 points per part.

3) 20 points total – a) 8 pts, b) 8 pts, c) 4 pts.

I understand that if I am caught cheating in this course, I will earn an F for the course and be reported to the Dean of Students.

Read and understood:  

______________________________

signature
Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) What is a plot of $\hbar \omega(\vec{q})$ vs. $\vec{q}$ for lattice vibrations called?
   
   a) The Einstein approximation.
   
   b) The Debye approximation.
   
   c) The gray approximation.
   d) **The phonon dispersion.**
   
   e) The Brillouin zone.

1.2) What is the biggest difference between the electron dispersion and the phonon dispersion of a material?
   
   a) The size in q-space of the Brillouin zone for phonons is smaller than the Brillouin zone for electrons.
   
   b) The size in q-space of the Brillouin zone for phonons is larger than the Brillouin zone for electrons.
   
   c) **The bandwidth in energy of the phonon dispersion is much less than the bandwidth of the electron dispersion.**
   
   d) The bandwidth in energy of the phonon dispersion is much greater than the bandwidth of the electron dispersion.
   
   e) The slope of the dispersion (the group velocity of the particle) is much larger for phonons than for electrons.

1.3) Comparing the electrical conductivity to the lattice thermal conductivity, which of the following statements is true?
   
   a) The electrical conductivity can be positive or negative, but the lattice thermal conductivity is always positive.
   
   b) The lattice thermal conductivity varies over many orders of magnitude.
   
   c) **The electrical conductivity varies over many orders of magnitude, the lattice thermal conductivity only by a few orders of magnitude.**
   
   d) The two are related by the Wiedemann-Franz Law.
   
   e) The two are related by the Lorenz number.

1.4) Why are “four probe” measurements used to determine the resistivity of a sample?
   
   a) To eliminate the Nearnst effect.
   
   b) **To eliminate the effect of the contact resistance.**
   
   c) To minimize the influence of the Seebeck effect.
   
   d) To minimize the influence of the Peltier effect.
   
   e) To obtain two independent measurements of the resistivity.
1.5) Which of the following would be considered to be a reasonable, but good interfacial contact resistivity, $\rho_C$?

a) $\rho_C = 10^2 \ \Omega \cdot \text{cm}^2$.
b) $\rho_C = 10^{-1} \ \Omega \cdot \text{cm}^2$.
c) $\rho_C = 10^{-4} \ \Omega \cdot \text{cm}^2$.
\[ \boxed{d) \rho_C = 10^{-7} \ \Omega \cdot \text{cm}^2.} \]
e) $\rho_C = 10^{-10} \ \Omega \cdot \text{cm}^2$.

2) Assume that we measure a Seebeck coefficient of 700 $\mu$V/K in Ge at $T = 300$ K. Assume the properties of Ge listed below, and answer the following questions.

$N_C = 1.04 \times 10^{19} \ \text{cm}^{-3}$  
$N_V = 5.35 \times 10^{18} \ \text{cm}^{-3}$
$\mu_n = 3500 \ \text{cm}^2/\text{V-s}$  
$\mu_p = 1500 \ \text{cm}^2/\text{V-s}$

2a) Estimate the doping density (and specify if the sample is n-type or p-type). Make reasonable assumptions, but clearly state your assumptions.

**Solution:**

The positive Seebeck coefficient means that we are dealing with a **p-type material**. **Assume** non-degenerate carrier statistics and parabolic energy bands, so the current flows $2k_BT$ below the top of the valence band.

$$S = + \frac{E_F - E_J}{qT} = \left( \frac{k_B}{q} \right) \left( \frac{E_F - E_V}{k_BT} + 2 \right)$$

Insert the measured Seebeck coefficient:

$$700 = 86 \left( \frac{E_F - E_V}{k_BT} + 2 \right) \Rightarrow 8.14 - 2 = \left( \frac{E_F - E_V}{k_BT} \right) \quad \left( \frac{E_F - E_V}{k_BT} \right) = 6.14$$

Since the Fermi level is several $k_BT$ above the top of the valence band, our assumption of non-degenerate carrier statistics is valid.

Now deduce the hole density:

$$p = N_V \exp \left( \frac{E_V - E_F}{k_BT} \right) = 5.35 \times 10^{18} \exp(-6.14) = 1.15 \times 10^{16}$$

Assume full ionization of dopants, so $N_A = p$

$N_A = 1.15 \times 10^{16} \ \text{cm}^{-3}$
2b) Estimate the electronic thermal conductivity, $\kappa_e$ (Be sure to included units.)

**Solution:**

Use the Weidemann-Franz law:

$$\frac{\kappa_e}{\sigma} = TL$$

Assume nondegenerate carrier statistics: $L = 2 \left( \frac{k_B}{q} \right)^2$

$$\kappa_e = TL\sigma = T \left( 2 \left( \frac{k_B}{q} \right)^2 \right) \sigma$$

$$\sigma = \sigma_p = pq\mu_p = \left(1.15 \times 10^{16}\right)\left(1.6 \times 10^{-19}\right)\left(1500\right) = 2.76 \frac{1}{\Omega\cdot\text{cm}} = 276 \frac{1}{\Omega\cdot\text{m}}$$

$$\kappa_e = TL\sigma = T \left( 2 \left( \frac{k_B}{q} \right)^2 \right) \sigma = 300 \left( 2 \left( \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} \right)^2 \right) \frac{276}{1} = 1.2 \times 10^{-3} \text{ W/m-K}$$

$$\kappa_e = 1.2 \times 10^{-3} \text{ W/m-K}$$
3) Consider a material for which all of the channels are at the same energy, $E = E_c$, i.e. $M(E) = M_0(E_c)$. Answer the following questions.

3a) Derive an expression for the Seebeck coefficient, $S$.

Solution:

$$S = -\frac{k_B}{q} \int \left( \frac{E - E_F}{k_B T} \right) \sigma'(E) dE$$

$$\sigma'(E) = \frac{2q^2}{h} \lambda(E) \frac{M(E)}{A} \left( -\frac{\partial f_0}{\partial E} \right) = \frac{2q^2}{h} \lambda(E) \frac{M_0 \delta(E_c)}{A} \left( -\frac{\partial f_0}{\partial E} \right)$$

$$S = \text{num/demon} = -\frac{k_B}{q} \int \frac{E - E_F}{k_B T} \sigma'(E) dE = \frac{2q^2}{h} \int \left( \frac{E - E_F}{k_B T} \right) \lambda(E) \frac{M_0 \delta(E_c)}{A} \left( -\frac{\partial f_0}{\partial E} \right)$$

$$S = \text{num/demon} = -\frac{k_B}{q} \left( \frac{E_c - E_F}{k_B T} \right)$$

$$S = -\frac{k_B}{q} \left( \frac{E_c - E_F}{k_B T} \right)$$

3b) Derive an expression for the electronic thermal conductivity under open circuit condition, $\kappa_e$.

Solution:

Begin with the definition of $\kappa_e$:

$$\kappa_e = \kappa_0 - \pi S \sigma$$

Need to evaluate $\kappa_0$

$$\kappa_0 = T \left( \frac{k_B}{q} \right)^2 \int \left( \frac{E - E_F}{k_B T} \right)^2 \sigma'(E) dE$$
Proceeding as in 3a), we find:

\[
\kappa_0 = T \left( \frac{k_B}{q} \right)^2 \frac{2q^2}{h} \left( \frac{E_C - E_F}{k_BT} \right)^2 \lambda(E_C) \frac{M_0}{A} \left( -\frac{\partial f_0}{\partial E} \right)_{E_C}
\]

\[
\kappa_e = \kappa_0 - \pi S\sigma = \kappa_0 - TS^2\sigma
\]

\[
\kappa_e = T \left( \frac{k_B}{q} \right)^2 \frac{2q^2}{h} \left( \frac{E_C - E_F}{k_BT} \right)^2 \lambda(E_C) \frac{M_0}{A} \left( -\frac{\partial f_0}{\partial E} \right)_{E_C} - T \left( \frac{k_B}{q} \right) \left( \frac{E_C - E_F}{k_BT} \right)^2 \frac{2q^2}{h} \lambda(E_C) \frac{M_0}{A} \left( -\frac{\partial f_0}{\partial E} \right)_{E_C}
\]

\[
\kappa_e = 0
\]

3c) Explain why your answers to 3a) and 3b) should be obvious without doing any math.

**Solution:**

We know that the Seebeck coefficient is related to the average energy at which the current flows, \( E_J \):

\[
S = -\frac{E_J - E_F}{qT}
\]

Since all the channels are at the same energy, \( E = E_C \), we must have \( E_J = E_C \), so the answer to 3a) is obtained directly. QED

We only have channels at one energy. If the device is open-circuited, then no electrons are flowing from contact 1 to contact 2. If no electrons are flowing between the hot and cold contacts, then \( \kappa_e = 0 \). as derived in 3b). QED