This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam.

There are three questions. To receive full credit, you must show your work (scratch paper is attached).

The exam is designed to be taken in 50 minutes, but you have 75 minutes.

Be sure to fill in your name and Purdue student ID at the top of the page and to sign the statement below.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last few pages list some equations. You may remove these pages if you want.

50 points possible.

1) 10 points total – 2 points per part.

2) 20 points total – 5, 10, 5

3) 20 points total – 5 points per part.

I understand that if I am caught cheating in this course, I will earn an F for the course and be reported to the Dean of Students.

Read and understood: ______________________________ signature
Answer the **five multiple choice questions** below by choosing the **one, best answer**.

1.1) What is the quantity, \( \sum_{\bar{p}'} S(\bar{p} \to \bar{p}') f(\bar{p}) [1 - f(\bar{p}')] \)?

   a) The collision integral.
   b) The in-scattering rate.
   c) **The out-scattering rate.**
   d) The relaxation time approximation.
   e) The momentum relaxation rate.

1.2) In the solution to the steady-state Boltzmann equation, \( \delta f = \tau_m (-\partial f/\partial E) \bar{v} \cdot \tilde{F} \), what effects are included in \( \tilde{F} \)?

   a) Gradients in electrostatic potential.
   b) Gradients in carrier density.
   c) Gradients in temperature.
   d) **All of the above.**
   e) Only a) and c) above.

1.3) What is the quantity: \( \frac{1}{A} \sum_k (E - E_C) \bar{v}(k) f(\bar{r},k) \)? (\( E \) is the total energy.)

   a) The total energy density in 2D.
   b) The kinetic energy density in 2D.
   c) The total energy flux in 2D.
   d) **The kinetic energy flux in 2D.**
   e) The heat flux in 2D.

1.4) Which of the following statements is true in equilibrium?

   a) The electrostatic potential is constant with position.
   b) The electrostatic potential and carrier density are constant with position.
   c) The electrostatic potential and temperature are constant with position.
   d) The carrier density and temperature are constant with position.
   e) **The electrochemical potential and temperature are constant with position.**

1.5) What are the proper boundary conditions for the 1D BTE?

   a) The carrier densities at the two contacts.
   b) The incident and emerging fluxes at each of the two contacts.
   c) The incident flux at either one of the two contacts.
   d) **The incident fluxes at each of the two contacts.**
   e) The carrier densities at the two contacts.
2) This question is about solving the BTE for 3D carriers.

\[
\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f + \vec{F}_e \cdot \nabla_p f = \hat{C}f
\]

Answer the following questions.

2a) Simplify the BTE for steady-state, no electric or magnetic field, the relaxation time approximation, and spatial variations only in the x-direction. Assume that the temperature is uniform too.

Solution:

\[
\frac{\partial f}{\partial t} = 0 \quad \vec{F}_e = 0 \quad \hat{C}f = -\frac{f - f_s}{\tau_m}
\]

\[
\vec{v} \cdot \nabla_x f = -\frac{f - f_s}{\tau_m}
\]

For spatial variations only in the x-direction:

\[
\nu_x \frac{df}{dx} = -\frac{f - f_s}{\tau_m(E)}
\]

2b) Solve the simplified BTE from 2a) (i.e. find \(f\)). You should assume that the scattering time depends on energy.

Solution:

Assume that we can approximate the spatial derivative on the LHS by using the large, symmetric component of the distribution.

\[
\nu_x \frac{df_s}{dx} = -\frac{f - f_s}{\tau_m(E)}
\]

Solve for \(f\)

\[
f = f_s - \tau_m(E)\nu_x \frac{df_s}{dx}
\]

Note that we could write \(f_s = 1/(e^{(E-F_s(x))/k_BT} + 1)\) and expand \(df_s/\,dx\) to get something proportional to \(dF_n/\,dx\), but there is no need to do that. If you did it that way, you would get \(\delta f = \tau_m\left(-\frac{\partial f_s}{\partial E}\right)\nu_x F_x\) where \(F_x = -dF_n/\,dx\).
2c) Using your solution to 2b), derive an expression for the x-directed electron flux. Your answer should be in terms of the carrier density and scattering time.

**Solution:**
We expect that the answer will be: \( F_x = -D_n \frac{dn}{dx} \), we just need to get it from our solution to the BTE and in the process determine \( D_n \). To begin, we obtain the electron flux by computing the sum:

\[
F_x = \frac{1}{\Omega} \sum_k v_x f = \frac{1}{\Omega} \sum_k v_x \left[ f_s - \tau_m(E) v_x \frac{df_s}{dx} \right]
\]

We recognize that

\[
\frac{1}{\Omega} \sum_k v_x f_s = 0 \]

because \( f_s \) is symmetric in x-directed velocity and \( v_x \) is anti-symmetric.

As a result

\[
F_x = -\frac{1}{\Omega} \sum_k v_x \left[ \tau_m(E) v_x \frac{df_s}{dx} \right]
\]

The spatial derivative can be moved outside the sum:

\[
F_x = -\frac{d}{dx} \left[ \frac{1}{\Omega} \sum_k v_x^2 \tau_m(E) f_s \right]
\]

We can write the term in brackets as \( D_n n \) where

\[
D_n \equiv \left\langle v_x^2 \tau_m(E) \right\rangle = \frac{1}{\Omega} \sum_k v_x^2 \tau_m(E) f_s \quad \text{and} \quad n = \frac{1}{\Omega} \sum_k f_s
\]

We conclude that:

\[
F_x = -\frac{d}{dx} \left[ D_n n \right]
\]

The diffusion coefficient, \( \left\langle v_x^2 \tau_m(E) \right\rangle \), is independent of position for a non-degenerate semiconductor (we are assuming that the bandstructure and scattering time are independent of position), so for a non-degenerate semiconductor, we can write the x-directed flux as

\[
F_x = -D_n \frac{dn}{dx}
\]

\[
D_n = \left\langle v_x^2 \tau_m(E) \right\rangle
\]
3) This problem is about a ballistic MOSFET whose energy band diagram is shown below.

3a) What is the probability that a positive velocity state located at \( x = x_1 \) with a total energy of \( E = E_{\text{max}} + dE \) is occupied? (Give an expression and explanation you do not need to provide a number).

**Solution:**
Positive velocity states above the top of the barrier can only be filled by contact 1. The answer is a Fermi function with the Fermi level of contact 1.

\[
f^+(E > E_{\text{max}}, x_1) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}
\]

\[
f^+(E_{\text{max}} + dE, x_1) = \frac{1}{1 + e^{(E_{\text{max}} + dE - E_{F1})/k_B T}}
\]

3b) What is the probability that a positive velocity state located at \( x = x_1 \) with a total energy of \( E = E_{\text{max}} - dE \) is occupied? (Give an expression and explanation you do not need to provide a number).

**Solution:**
Positive velocity states below the top of the barrier can only be filled by contact 2. The answer is a Fermi function with the Fermi level of contact 2.

\[
f^+(E < E_{\text{max}}, x_1) = \frac{1}{1 + e^{(E_{\text{max}} - dE - E_{F2})/k_B T}}
\]

\[
f^+(E = E_{\text{max}} - dE, x_1) = \frac{1}{1 + e^{(E_{\text{max}} - dE - E_{F2})/k_B T}}
\]
3c) What is the probability that a negative velocity state located at \( x = x_1 \) with a total energy of \( E = E_{\text{max}} + dE \) is occupied? (Give an expression and explanation you do not need to provide a number).

**Solution:**
Negative velocity states above the top of the barrier can only be filled by contact 2. The answer is a Fermi function with the Fermi level of contact 2.

\[
f^\text{\text{-}}(E > E_{\text{max}}, x_1) = \frac{1}{1 + e^{(E-E_{\text{F}})/k_BT}}
\]
\[
f^\text{\text{-}}(E_{\text{max}} + dE, x_1) = \frac{1}{1 + e^{(E_{\text{max}} + dE-E_{\text{F}})/k_BT}}
\]

3d) What is the local density of states vs. energy at \( x = x_1 \) that is fillable by contact 1? (Give an expression and explanation you do not need to provide a number).

**HINT:** For a bulk 2D semiconductor with a uniform potential, the DOS is

\[
D_{2D}(E) = g_v \frac{m^*}{\pi \hbar^2} \quad \text{if } E > E_C
\]
\[
D_{2D}(E) = 0 \quad \text{if } E \leq E_C
\]

**Solution:**

Only positive velocity states can be filled by contact 1, so that is one-half of the DOS, but contact 1 can only fill positive velocity states with \( E > E_{\text{max}} \), so

\[
D_{2D}(E, x_1) = g_v \frac{m^*}{2 \pi \hbar^2} \quad \text{if } E > E_{\text{max}}
\]
\[
D_{2D}(E, x_1) = 0 \quad \text{if } E < E_{\text{max}}
\]