SOLUTIONS: ECE 656 Homework (Week 10)
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1) Show that the following relation (eqn. (4.119) in Fundamentals of Carrier Transport) is true:
\[ e^{-\frac{\pi \rho_{MN,OP}}{p_s}} + e^{-\frac{\pi \rho_{NO,PM}}{p_s}} = 1 \]

Solution:
From slide 6 in Lecture 18, Fall 2011 (or eqns. (4.117) and (4.118) of FCT):
\[ R_{MN,OP} = \frac{\rho_s}{\pi} \ln \left( \frac{(a+b)(b+c)}{b(a+b+c)} \right) \Rightarrow \frac{\pi R_{MN,OP}}{\rho_s} = \ln \left( \frac{(a+b)(b+c)}{b(a+b+c)} \right) \] (i)
\[ R_{NO,PM} = \frac{\rho_s}{\pi} \ln \left( \frac{(a+b)(b+c)}{ac} \right) \Rightarrow \frac{\pi R_{NO,PM}}{\rho_s} = \ln \left( \frac{(a+b)(b+c)}{ac} \right) \] (ii)

From (i) and (ii)
\[ e^{-\frac{\pi \rho_{MN,OP}}{p_s}} + e^{-\frac{\pi \rho_{NO,PM}}{p_s}} = \frac{b(a+b+c)}{(a+b)(b+c)} + \frac{ac}{ab+ac+b^2+bc} = 1 \]

\[ e^{-\frac{\pi \rho_{MN,OP}}{p_s}} + e^{-\frac{\pi \rho_{NO,PM}}{p_s}} = 1 \]

2) For n-type, bulk silicon doped at \( N_D = 10^{17} \) cm\(^{-3} \) the room temperature mobility is 800 cm\(^2\)/V-s. Answer the following questions. Some potentially useful information is:
\[ N_C = 3.23 \times 10^{19} \text{ cm}^{-3} \quad N_V = 1.83 \times 10^{19} \text{ cm}^{-3} \quad E_G = 1.11 \text{ eV} \quad \nu_T = 1.05 \times 10^7 \text{ cm/s} \]
Estimate the Seebeck coefficient. Make reasonable assumptions, but clearly state them.

Solution:
\[ S = -\left( \frac{k_B}{q} \right) \left[ \frac{\Delta n}{k_BT_i} - \eta_F \right] \]
\[ n_0 = N_C e^{\eta_F} \rightarrow \eta_F = \ln \left( \frac{n_0}{N_C} \right) = -5.78 \quad \text{(non-degenerate semiconductor)} \]
ECE 656 Homework Solutions (Week 10) (continued)

Assume $\Delta_n = 2k_B T_L$ (non-degenerate, constant mfp)

$$S = -(86)[2 + 5.78] = -669 \, \mu V/K$$

3) A Hall effect experiment is performed on an n-type semiconductor with a length of 2.65 cm, a width of 1.70 cm, and a thickness of 0.0520 cm, in a magnetic field of 0.5 T. The current in the sample along its length is 200 $\mu$A. The potential difference along the length of the sample is 195 mV and across the width is 21.4 mV.

3.a) What is the carrier concentration of the sample?

Solution:

Recall that in 2D:

$$\vec{E} = \rho_S \vec{J}_n + \left( \rho_S \mu_n r_H \right) \vec{J}_n \times \vec{B}$$

$$E_y = \rho_S J_y - (\rho_S \mu_n r_H) J_x B_z = -(\rho_S \mu_n r_H) J_x B_z$$

$$V_H = -W E_y = (\rho_S \mu_n r_H) I_x B_z = \left( \frac{1}{n_S q \mu_n} \mu_n r_H \right) I_x B_z = \left( \frac{r_H}{q n_S} \right) I_x B_z$$

which is the same as eqn. (4.110) in FCT

$$V_H = \frac{r_H}{q n_S} B z I$$

$$\frac{n_S}{r_H} = \frac{B z I}{q V_H} = \frac{0.5 \times 200 \times 10^{-6}}{1.6 \times 10^{-19} \left(21.4 \times 10^{-3}\right)} = 2.92 \times 10^{16} \text{ cm}^2$$

$$\frac{n_S}{r_H} = \frac{2.92 \times 10^{16} \text{ cm}^2}{0.0520 \text{ cm}} = 5.62 \times 10^{17} \text{ cm}^{-3}$$

$$n_H = \frac{n}{r_H} = 5.62 \times 10^{17} \text{ cm}^{-3}$$

Not the carrier concentration, but the “Hall carrier concentration.”
ECE 656 Homework Solutions (Week 10) (continued)

3b) What is the mobility?

Solution:

\[ R_{xx} = \frac{195 \times 10^{-3}}{200 \times 10^{-6}} = 975 \ \Omega \]

\[ R_{xx} = \rho_S \frac{L}{W} = 975 \ \Omega \]

\[ \rho_S = \frac{W}{L} 975 \ \Omega = \frac{1.70 \times 975}{2.65} = 625 = \frac{1}{n_s q \mu_n} \]

\[ \mu_n = \frac{1}{n_s q \rho_S} = \frac{1}{r_H \times 2.92 \times 10^{16} \times 1.6 \times 10^{-19} \times 625} \]

\[ r_H \mu_n = \mu_H = \frac{1}{2.92 \times 10^{16} \times 1.6 \times 10^{-19} \times 625} = 0.3 \text{ cm}^2/\text{V-s} \]

\[ \mu_H = 0.3 \text{ cm}^2/\text{V-s} \]

Not the mobility, but the “Hall mobility.”

3c) If the scattering time is 1 ps, find the magnetic field for which this classical analysis of Hall effect is no longer valid?

Solution:

We require:

\[ \omega_c \tau \ll 1 \]

\[ \frac{qB}{m^*} \tau \ll 1 \] (i)

\[ \mu_n = \frac{q \tau}{m^*} \quad \tau = \frac{m^* \mu_n}{q} \] (ii)

With (ii), (i) becomes

\[ \frac{qB}{m^*} \frac{m^* \mu_n}{q} \ll 1 \rightarrow B \mu_n \ll 1 \]
ECE 656 Homework Solutions (Week 10) (continued)

Using the Hall mobility as an estimate of the real mobility

\[ B_z \mu_n = 0.5 \times 0.3 = 0.15 \ll 1 \]

so this experiment is in the low B-field regime. The largest the B-field can be to be in the low field regime is

\[ B_z = \frac{1}{\mu_n} = \frac{1}{0.3} = 3.33 \text{ T} \]

\[ [B_z = 3.33 \text{ T}] \]

4) Contact resistances are important. They can complicate measurements of semiconductor transport parameters, and they can degrade device performance. The constant resistance is specified by the interfacial contact resistivity, \( \rho_C \), in \( \Omega \text{-cm}^2 \). A very good value is \( \rho_C = 10^{-8} \Omega \text{-cm}^2 \). Consider n+ Si at room temperature and doped to \( N_D = 10^{20} \text{ cm}^{-3} \). What is the lower limit to \( \rho_C \)? (Assume a fully degenerate semiconductor and use appropriate effective masses for the conduction band of Si.)

Solution:

The lower limit resistance must be the ballistic contact resistance:

\[ R_B = \frac{1}{G_B} = \frac{1}{\left( \frac{2q^2}{h} \right) \left\langle \frac{T}{M} \right\rangle} \]

Assuming that one-half of the ballistic resistance is associated with each of the two contacts:

\[ \rho_C = \frac{R_B A}{2} = \frac{h}{4q^2} \left\langle \frac{T}{M} \right\rangle \frac{1}{M/A} \]

Assume a strongly degenerate semiconductor:

\[ \rho_C = \frac{h}{4q^2} \frac{T}{E_F} \frac{1}{M(E_F)/A} \]
The lower limit occurs when the transmission is one

\[ \rho_C^{\text{min}} = \frac{\hbar}{4q^2} M(E_F) \frac{1}{A} \]

Need to find the Fermi level. Recall that at 0 K,

\[ n_o = \int_{E_c}^{E_f} D_{3D}(E) dE \]

\[ D_{3D}(E) = \left( \frac{m_{\text{DOS}}^*}{\hbar^2} \right)^{3/2} \frac{\sqrt{2(E - E_c)}}{\pi^2} \]

\[ n_o = \int_{E_c}^{E_f} D_{3D}(E) dE = \int_{E_c}^{E_f} \left( \frac{m_{\text{DOS}}^*}{\hbar^2} \right)^{3/2} \frac{\sqrt{2(E - E_c)}}{\pi^2} dE = \frac{\sqrt{2}}{\pi^2} \int_{E_c}^{E_f} (E - E_c)^{1/2} dE \]

\[ n_o = \frac{2\sqrt{2}}{3\pi^2} \left( \frac{m_{\text{DOS}}^*}{\hbar^2} \right)^{3/2} (E_F - E_C)^{3/2} \]

\[ (E_F - E_C) = \frac{1}{m_{\text{DOS}}^*} \left( \frac{3\pi^2 h^3}{2\sqrt{2}} \right)^{2/3} (n_o)^{2/3} \]

\[ M_{3D}(E_F) = \frac{m_{\text{DOM}}^*}{2\pi^2} (E_F - E_C) \]

\[ M_{3D}(E_F) = \frac{m_{\text{DOM}}^*}{m_{\text{DOS}}^*} \frac{1}{2\pi^2} \left( \frac{3\pi^2 h^3}{2\sqrt{2}} \right)^{2/3} (n_o)^{2/3} = \frac{m_{\text{DOM}}^*}{m_{\text{DOS}}^*} \left( \frac{3\sqrt{\pi}}{8} \right)^{2/3} n_o^{2/3} \]

For Si, we have to consider the ellipsoidal bandstructure:

\[ m_{\text{DOS}}^* = \left( 6 \right)^{2/3} \left( m_i^2 m_r \right)^{1/3} = 1.06 m_0 \]

\[ m_{\text{DOM}}^* = 2m_i^* + 4\sqrt{m_i^* m_r} = 2.04 m_0 \]

(See: Jeong, Changwook; Kim, Raseong; Luisier, Mathieu; Datta, Supriyo; and Lundstrom, Mark S., "On Landauer versus Boltzmann and full band versus effective mass evaluation of thermoelectric transport coefficients," J. Appl. Phys., 107, 023707, 2010.)
\[ M_{3D}(E_F) = \frac{m^*_{DOM}}{m^*_{DOS}} \left( \frac{3 \sqrt{\pi}}{8} \right)^{2/3} n_0^{2/3} = \frac{2.04}{1.06} \left( 0.762 \right) \left( 10^{20} \right)^{2/3} = 3.16 \times 10^{13} \text{ cm}^{-2} \]

and finally:
\[ \rho_C^{\text{min}} = \frac{h}{4q^2} \frac{1}{M(E_F) / A} = 6.48 \times 10^3 \frac{1}{3.16 \times 10^{13}} = 2.1 \times 10^{-10} \text{ } \Omega\text{-cm}^2 \]

\[ \boxed{\rho_C^{\text{min}} = 2.1 \times 10^{-10} \text{ } \Omega\text{-cm}^2} \]

So even for a very good interfacial contact resistivity, \( T(E_F) \approx 0.01 \)

For more on this topic, see: