ECE 656 Homework 1: Week 1
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1) Working out Fermi-Dirac integrals just takes some practice. For practice, work out the integral

\[ I_1 = \int_{-\infty}^{\infty} M(E) f_0(E) dE \]

where

\[ f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}} \]

and

\[ M(E) = W \sqrt{\frac{2m^*(E-E_C)}{\pi \hbar}} H(E-E_C) \]

where

\[ H(E-E_C) \] is the unit step function.

2) For more practice, work out the integral in problem 1) assuming non-degenerate carrier statistics.

3) For still more practice, work out this integral:

\[ I_2 = \int_{E_C}^{\infty} M(E) \left( -\frac{\partial f_0}{\partial E} \right) dE, \]

where \( M(E) \) is as given in problem 1).
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4) It is important to understand when Fermi-Dirac statistics must be used and when non-degenerate (Maxwell-Boltzmann) statistics are good enough. The electron density in 1D is

\[ n_L = N_{1D} \mathcal{F}_{1/2} \left( \eta_F \right) \text{cm}^{-1}, \]

where \( N_{1D} \) is the 1D effective density of states and \( \eta_F = \left( E_F - E_C \right)/k_B T \). In 3D,

\[ n = N_{3D} \mathcal{F}_{1/2} \left( \eta_F \right) \text{cm}^{-3}. \]

For Maxwell Boltzmann statistics

\[ n_L^{MB} = N_{1D} \exp \left( \eta_F \right) \text{cm}^{-1} \]
\[ n^{MB} = N_{3D} \exp \left( \eta_F \right) \text{cm}^{-3}. \]

Compute the ratios, \( n_L/n_L^{MB} \) and \( n/n^{MB} \) for each of the following cases:

a) \( \eta_F = -10 \)

b) \( \eta_F = -3 \)

c) \( \eta_F = 0 \)

d) \( \eta_F = 3 \)

d) \( \eta_F = 10 \)

Note that there is a Fermi-Dirac integral calculator available on nanoHUB.org. An iPhone app is also available.

5) Consider GaAs at room temperature doped such that \( n = 10^{19} \text{ cm}^{-3} \). The electron density is related to the position of the Fermi level according to

\[ n = N_C \mathcal{F}_{1/2} \left( \eta_F \right) \text{cm}^{-3}, \]

where

\[ N_C = 4.21 \times 10^{17} \text{ cm}^{-3}. \]

Determine the position of the Fermi level relative to the bottom of the conduction band, \( E_C \).

a) Assuming Maxwell-Boltzmann carrier statistics

b) NOT assuming Maxwell-Boltzmann carrier statistics