1) For electrons, the bandstructure is a plot of energy, \( E(\vec{k}) \), vs. wavevector, \( \vec{k} \). For phonons, the dispersion is a plot of phonon energy, \( \hbar\omega(\vec{q}) \), vs. phonon wavector, \( \vec{q} \). For electrons, we often approximate the bandstructure with simple, parabolic bands,
\[
E(\vec{k}) = \frac{\hbar^2 k^2}{2m^*}
\]
For phonons, we can sometimes approximate the phonon dispersion with the Debye approximation,
\[
\hbar\omega = \hbar\nu_D q,
\]
where \( \nu_D \) is the Debye velocity (an average of the longitudinal and transverse acoustic velocities.)

1a) Compute the density-of-states, \( D_{ph}(\hbar\omega) \), for phonons in the Debye model.

**Solution:**

Equate the DOS in q-space to energy space:
\[
\frac{1}{\Omega} \int_q N_q dq = D_{ph}(\hbar\omega) d(\hbar\omega) \quad (i)
\]
\[
\frac{1}{\Omega} \int_q N_q dq = \frac{1}{8\pi^3} \times 3(4\pi q^2) dq = D_{ph}(\hbar\omega) d(\hbar\omega) \quad (ii)
\]
Note that there is no factor of 2 for spin in this case, but we have a factor of three because of the three polarizations, longitudinal and two transverse acoustic phonons.

From the dispersion,
\[
\hbar\omega = \hbar\nu_D q \quad (iii)
\]
we have
\[
q^2 = \left( \frac{\hbar\omega}{\hbar\nu_D} \right)^2 \quad (iv)
\]
and
\[
dq = \frac{d(\hbar\omega)}{\hbar\nu_D} \quad (v)
\]
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Using (iv) and (v) in (ii), we find

\[
D_{ph}(\hbar\omega) d(\hbar\omega) = \frac{3}{2\pi^2} q^2 dq = \frac{3}{2\pi^2} \left( \frac{\hbar\omega}{\hbar \nu_D} \right)^2 d(\hbar\omega)
\]

(vi)

so the final answer is

\[
D_{ph}(\hbar\omega) = \frac{3(\hbar\omega)^2}{2\pi^2 (\hbar \nu_D)^3} \left( Jm^3 \right)^{-1}
\]

(vii)

1b) Compute the distribution of channels, \( M_{ph}(\hbar\omega) \), for phonons in the Debye model.

Solution:

Begin with the definition:

\[
M_{ph}(\hbar\omega) = \frac{h}{2} \langle v^+_x \rangle D_{ph}(\hbar\omega)
\]

(viii)

From the dispersion, (iii), we find

\[
\nu(\hbar\omega) = \nu_D
\]

(ix)

and the average over angles in 3D gives

\[
\langle v^+_x \rangle = \frac{\nu_D}{2}
\]

(x)

Now using (x) in (viii), we find

\[
M_{ph}(\hbar\omega) = \frac{h}{2} \langle v^+_x \rangle D_{ph}(\hbar\omega) = \frac{h \nu_D}{2} \frac{3(\hbar\omega)^2}{2\pi^2 (\hbar \nu_D)^3}
\]

(xi)

\[
M_{ph}(\hbar\omega) = \frac{3(\hbar\omega)^2}{4\pi (\hbar \nu_D)^3}
\]