ECE 656: Electronic Transport in Semiconductors Fall 2017

Ionized Impurity Scattering:

Conwell-Weisskoph Approach

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9/5/15

BH summary

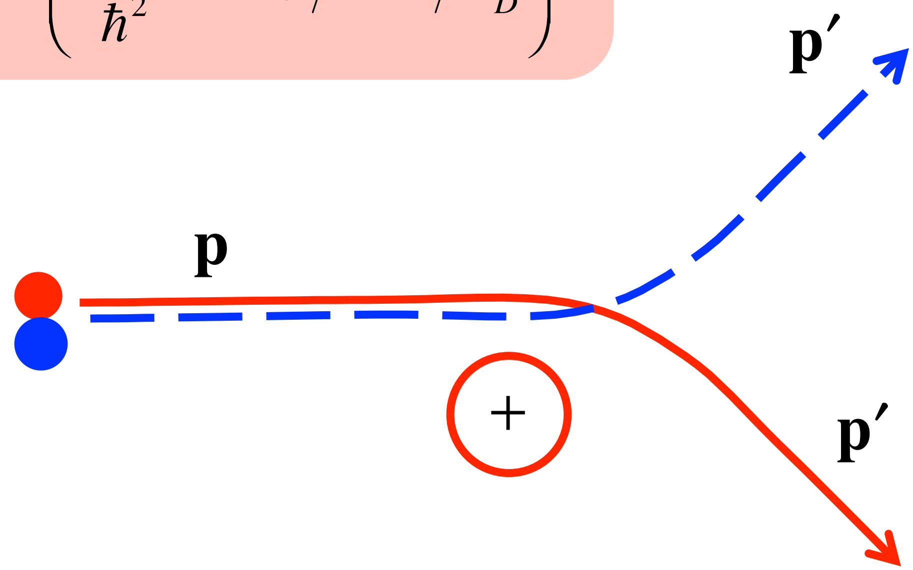
$$S(\vec{p},\vec{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_S^2 \Omega} \frac{\delta(E'-E)}{\left(\frac{4p^2}{\hbar^2}\sin^2\alpha/2 + 1/L_D^2\right)^2}$$

1) $S(\vec{p},\vec{p}') \sim N_I$

2) $S(\vec{p},\vec{p}') \sim q^4$

3) $S(\vec{p},\vec{p}') \sim 1/E^2$

4) Favors small angle scattering



All true for CW too.

BH summary (ii)

Momentum relaxation time:

$$\tau_m(E) = \frac{16\sqrt{2m^*}\pi\kappa_S^2\varepsilon_0^2}{N_I q^4}\left[\ln\left(1+\gamma^2\right) - \frac{\gamma^2}{1+\gamma^2}\right]E^{3/2} >> \tau(E)$$

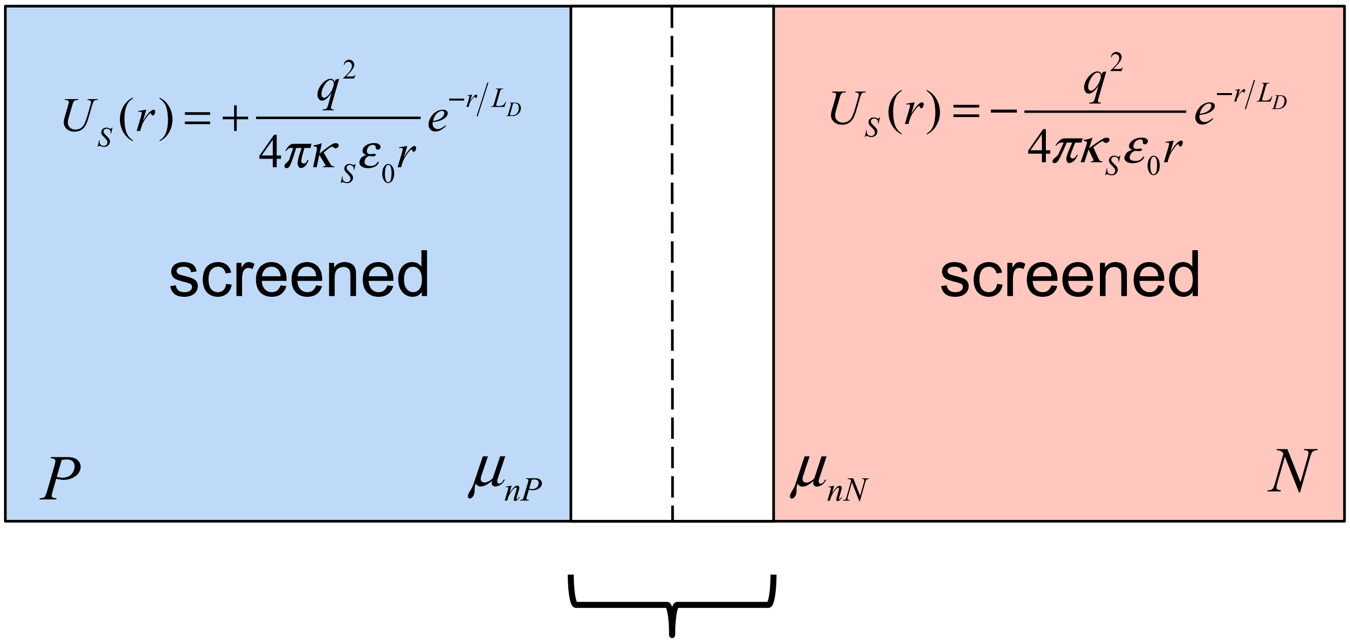
$$\gamma^2 = 8m^* E L_D^2 / \hbar^2$$

$$\tau_m(E) \sim E^{3/2}$$

$$\tau_m(E) \approx \tau_0\left(E/k_B T\right)^s \qquad \tau_0 \sim T^{3/2} \qquad s = 3/2$$

(True for CW too)

Screened vs. unscreened



$$U_S(r) = +\frac{q^2}{4\pi\kappa_S\varepsilon_0 r}$$

unscreened

CW approach treats unscreened Coulomb scattering.

Outline

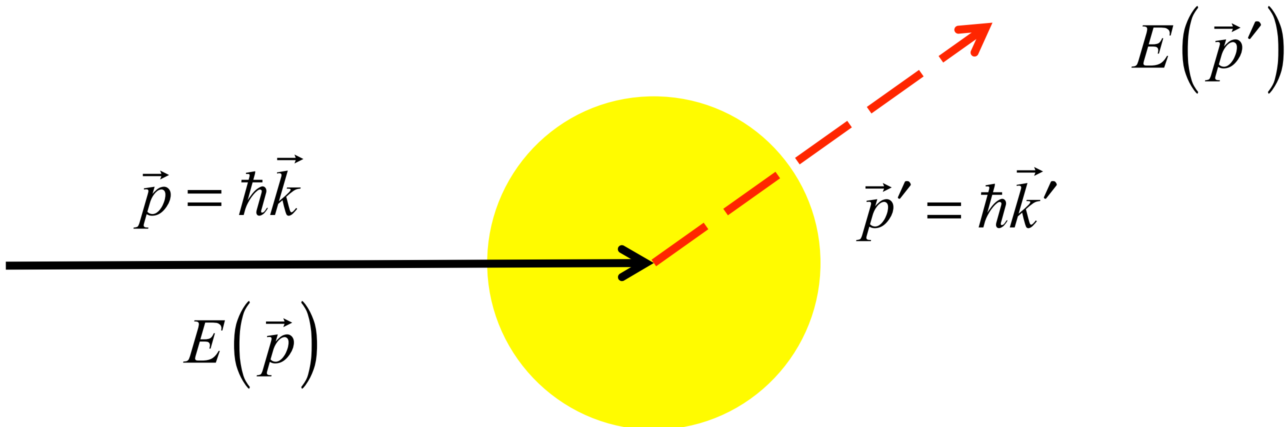
1) Introduction

2) Transition rate

3) Characteristic times

Fermi's Golden Rule

$$S(\vec{p},\vec{p}') = S(\vec{p} \rightarrow \vec{p}')$$



$$S(\vec{p} \rightarrow \vec{p}') = \frac{2\pi}{\hbar}\left|H_{\vec{p}',\vec{p}}\right|^2 \delta(E' - E)$$

$$H_{\vec{p}',\vec{p}} = \int_{-\infty}^{+\infty} \psi_f^* U_S(\vec{r})\psi_i d\vec{r}$$

$$U_S(\vec{r}) = \pm\frac{q^2}{4\pi\kappa_S\varepsilon_0 r}$$

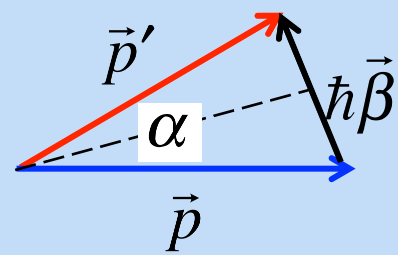
Unscreened Coulomb potential

Recall BH transition rate

$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_0^2 \Omega} \frac{\delta(E' - E)}{\left(\beta^2 + 1/L_D^2\right)^2}$$

$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_S^2 \Omega} \frac{\delta(E' - E)}{\left(\frac{4p^2}{\hbar^2}\sin^2\alpha/2 + 1/L_D^2\right)^2}$$

$$\vec{p}\,' = \vec{p} + \hbar\vec{\beta}$$

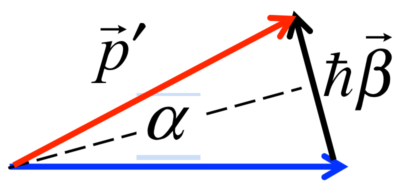


$$\frac{\hbar\beta}{2} = p\sin\alpha/2$$

CW transition rate

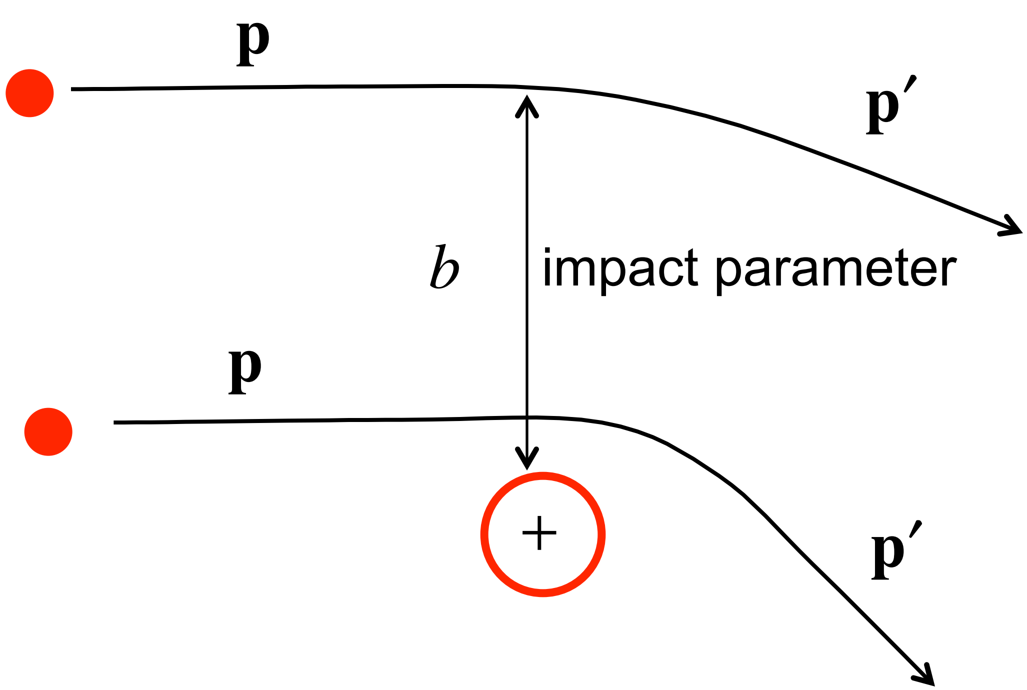
$$L_D = \sqrt{\frac{\kappa_S \varepsilon_0 k_B T_L}{q^2 n_0}} \rightarrow \infty$$

$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_0^2 \Omega} \frac{\delta(E'-E)}{\left(\beta^2 + 1/L_D^2\right)^2} \rightarrow S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_0^2 \Omega} \frac{\delta(E'-E)}{\beta^4}$$



$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_S^2 \Omega} \frac{\delta(E'-E)}{\left(\frac{4p^2}{\hbar^2} \sin^2 \alpha/2\right)^2}$$

Question about impact parameter



Is there a maximum b?

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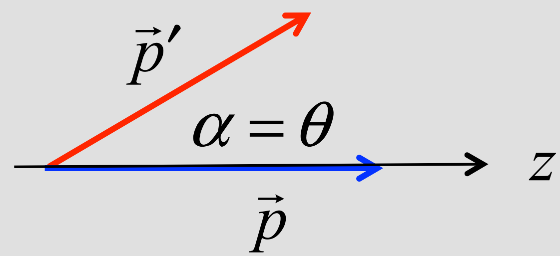
Momentum relaxation time

$$S\left(\vec{p},\vec{p}'\right)=\frac{2\pi q^4 N_I}{\hbar\kappa_S^2\varepsilon_0^2\Omega}\frac{\delta\left(E'-E\right)}{\left(\frac{4p^2}{\hbar^2}\sin^2\alpha/2\right)^2}$$

unscreened Coulomb potential

$$S\left(\vec{p},\vec{p}'\right)\rightarrow\infty \quad \text{as} \quad \alpha\rightarrow 0$$

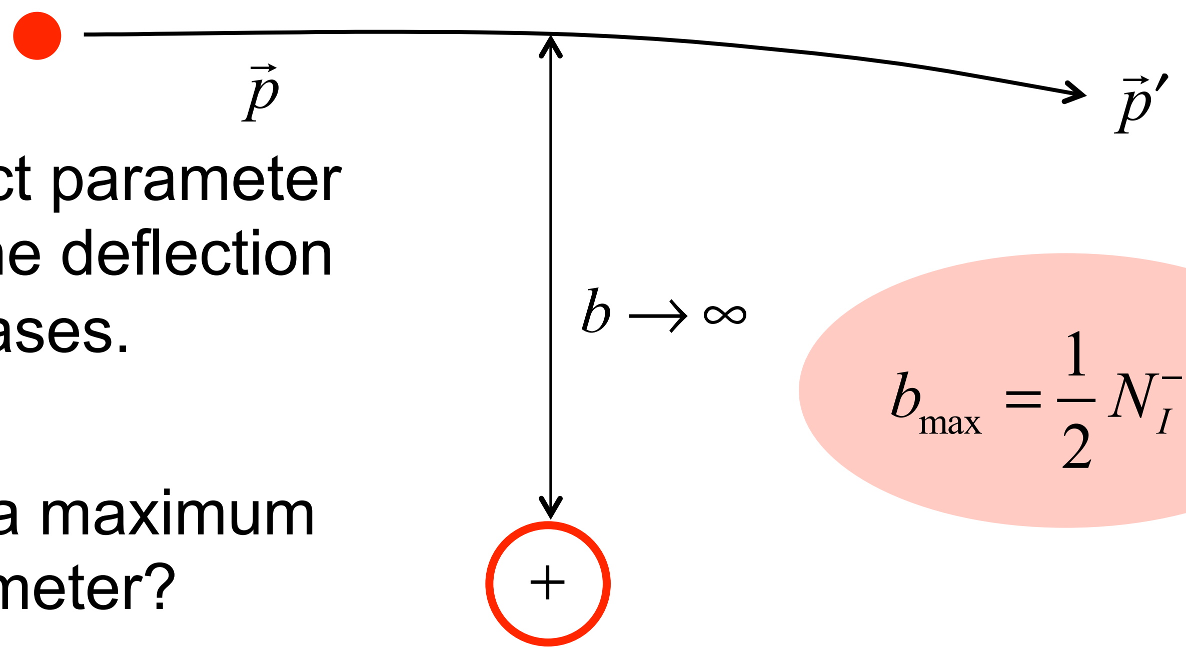
$$\frac{1}{\tau_m}=\sum_{\vec{p}'}S\left(\vec{p},\vec{p}'\right)\left(1-\cos\alpha\right)$$



Can we specify a minimum angle, so that the integral does not blow up?

Maximum impact factor

$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_0^2 \Omega} \frac{\delta(E' - E)}{\left(\frac{4p^2}{\hbar^2} \sin^2 \alpha/2 \right)^2}$$



As the impact parameter increases, the deflection angle decreases.

But there is a maximum impact parameter?

$$b_{\max} = \frac{1}{2} N_I^{-1/3}$$

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CW Momentum relaxation time

$$\frac{1}{\tau_m} = \sum_{\vec{p}'} S(\vec{p},\vec{p}')(1-\cos\alpha)$$

$$\frac{1}{\tau_m} = \frac{\Omega}{(2\pi)^3}\int_0^{2\pi} d\phi \int_{\alpha_{\min}}^{\pi} S(\vec{p},\vec{p}')(1-\cos\alpha)\sin\alpha\, d\alpha\, p'^2 dp' =$$

$$b_{\max} = \frac{q^2}{8\pi\kappa_S\varepsilon_0 E(p)}\cot(\alpha_{\min}/2)$$ (Rutherford)

$$\tau_m(E) = \frac{16\pi\sqrt{2m^*}\kappa_S^2\varepsilon_0^2}{N_I q^4}\left[\frac{1}{\ln(1+\gamma_{CW}^2)}\right]E^{3/2} \qquad \gamma_{CW}^2 = b_{\max}\Big/\left(q^2/8\pi\kappa_S\varepsilon_0 E\right)$$

CW Momentum relaxation time (ii)

$$\tau_m(E) = \frac{16\pi\sqrt{2m^*}\kappa_S^2\varepsilon_0^2}{N_I q^4}\left[\frac{1}{\ln\left(1+\gamma_{CW}^2\right)}\right]E^{3/2} \qquad \gamma_{CW}^2 = b_{\max}\Big/\left(q^2/8\pi\kappa_S\varepsilon_0 E\right)$$

$$\tau_m(E) \sim E^{3/2}$$

$$\tau_m(E) \approx \tau_0\left(E/k_B T\right)^s \qquad \tau_0 \sim T^{3/2} \qquad s = 3/2$$

Just like the Brooks-Herring result.

Questions?

1) Introduction

2) Transition rate

3) Characteristic times