Ionized Impurity Scattering:
Conwell-Weisskoph Approach

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BH summary

\[ S(\vec{p}, \vec{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa_s^2 \varepsilon_s^2 \Omega} \frac{\delta(E' - E)}{\left(\frac{4p^2}{\hbar^2} \sin^2 \alpha/2 + 1/L_D^2\right)^2} \]

1) \( S(\vec{p}, \vec{p}') \sim N_I \)

2) \( S(\vec{p}, \vec{p}') \sim q^4 \)

3) \( S(\vec{p}, \vec{p}') \sim 1/E^2 \)

4) Favors small angle scattering

All true for CW too.
Momentum relaxation time:

\[
\tau_m(E) = \frac{16\sqrt{2m^*\pi \kappa_s^2 \epsilon_0^2}}{N_I q^4} \left[ \ln(1+\gamma^2) - \frac{\gamma^2}{1+\gamma^2} \right] E^{3/2} \gg \tau(E)
\]

\[
\gamma^2 = 8m^* EL_D^2 / \hbar^2
\]

\[
\tau_m(E) \sim E^{3/2}
\]

\[
\tau_m(E) \approx \tau_0 \left( E/k_B T \right)^s, \quad \tau_0 \sim T^{3/2}, \quad s = 3 / 2
\]

(True for CW too)
Screened vs. unscreened

CW approach treats unscreened Coulomb scattering.
Outline

1) Introduction

2) Transition rate

3) Characteristic times
Fermi’s Golden Rule

\[ S(\tilde{p}, \tilde{p}') = S(\tilde{p} \rightarrow \tilde{p}') \]

\[ E(\tilde{p}') \]

\[ \tilde{p} = \hbar \tilde{k} \]

\[ \tilde{p}' = \hbar \tilde{k}' \]

\[ S(\tilde{p} \rightarrow \tilde{p}') = \frac{2\pi}{\hbar} |H_{\tilde{p}', \tilde{p}}|^2 \delta(E' - E) \]

\[ H_{\tilde{p}', \tilde{p}} = \int_{-\infty}^{+\infty} \psi_f^* U_S(\vec{r}) \psi_i \, d\vec{r} \]

\[ U_S(\vec{r}) = \pm \frac{q^2}{4\pi \kappa_S \epsilon_0 r} \]

Unscreened Coulomb potential
Recall BH transition rate

\[ S(\bar{p}, \bar{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa_s^2 \epsilon_s^2 \Omega} \frac{\delta(E' - E)}{(\beta^2 + 1/L_D^2)^2} \]

\[ S(\bar{p}, \bar{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa_s^2 \epsilon_s^2 \Omega} \frac{\delta(E' - E)}{\left(\frac{4p^2}{\hbar^2} \sin^2 \alpha/2 + 1/L_D^2\right)^2} \]

\[ \bar{p}' = \bar{p} + \hbar \beta \]

\[ \frac{\hbar \beta}{2} = p \sin \alpha/2 \]
CW transition rate

\[ L_D = \sqrt{\frac{\kappa_S \varepsilon_0 k_B T_L}{q^2 n_0}} \rightarrow \infty \]

\[ S(\vec{p}, \vec{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_0^2 \Omega} \frac{\delta(E' - E)}{\left(\beta^2 + 1/L_D^2\right)^2} \rightarrow S(\vec{p}, \vec{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_0^2 \Omega} \frac{\delta(E' - E)}{\beta^4} \]

\[ S(\vec{p}, \vec{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa_S^2 \varepsilon_s^2 \Omega} \frac{\delta(E' - E)}{\left(\frac{4p^2}{\hbar^2} \sin^2 \alpha/2\right)^2} \]
Question about impact parameter

Is there a maximum $b$?
Outline

1) Introduction

2) Transition rate

3) Characteristic times
Momentum relaxation time

\[ S(\vec{p}, \vec{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa_s^2 \varepsilon_0^2 \Omega} \frac{\delta(E' - E)}{\left( \frac{4p^2}{\hbar^2} \sin^2 \alpha/2 \right)^2} \]

unscreened Coulomb potential

\[ S(\vec{p}, \vec{p}') \to \infty \quad \text{as} \quad \alpha \to 0 \]

\[ \frac{1}{\tau_m} = \sum_{\vec{p}'} S(\vec{p}, \vec{p}') \left( 1 - \cos \alpha \right) \]

Can we specify a minimum angle, so that the integral does not blow up?
Maximum impact factor

\[
S(\vec{p}, \vec{p}') = \frac{2\pi q^4 N_I}{\hbar \kappa^2 \varepsilon_0 \Omega} \frac{\delta(E' - E)}{\left(\frac{4p^2}{\hbar^2} \sin^2 \alpha/2\right)^2}
\]

As the impact parameter increases, the deflection angle decreases.

But there is a maximum impact parameter?

\[b_{\text{max}} = \frac{1}{2} N_I^{-1/3}\]
CW Momentum relaxation time

\[
\frac{1}{\tau_m} = \sum_{\tilde{p}'} S(\tilde{p}, \tilde{p}')(1 - \cos \alpha)
\]

\[
\frac{1}{\tau_m} = \frac{\Omega}{(2\pi)^3} \int_0^{2\pi} d\phi \int_0^\pi d\alpha \int_0^{\alpha_{\text{min}}} d\alpha' S(\tilde{p}, \tilde{p}')(1 - \cos \alpha) \sin \alpha d\alpha' p'^2 dp' =
\]

\[
b_{\text{max}} = \frac{q^2}{8\pi\kappa S \varepsilon_0 E(p)} \cot(\alpha_{\text{min}}/2) \quad \text{(Rutherford)}
\]

\[
\tau_m(E) = \frac{16\pi\sqrt{2m^*\kappa_s^2}\varepsilon_0^2}{N_f q^4} \left[ \frac{1}{\ln\left(1 + \gamma_{\text{CW}}^2\right)} \right] E^{3/2} \quad \gamma_{\text{CW}} = b_{\text{max}}/\left(q^2/8\pi\kappa S \varepsilon_0 E\right)
\]
CW Momentum relaxation time (ii)

\[ \tau_m(E) = \frac{16\pi \sqrt{2m^* \kappa_s^2 \varepsilon_0^2}}{N_1 q^4} \left[ \frac{1}{\ln(1 + \gamma_{CW}^2)} \right] E^{3/2} \]

\[ \gamma_{CW}^2 = b_{\text{max}} \left/ \left( q^2 / 8\pi \kappa_s \varepsilon_0 E \right) \right. \]

\[ \tau_m(E) \sim E^{3/2} \]

\[ \tau_m(E) \approx \tau_0 \left( E/k_B T \right)^s \quad \tau_0 \sim T^{3/2} \quad s = 3/2 \]

Just like the Brooks-Herring result.
Questions?

1) Introduction
2) Transition rate
3) Characteristic times