Ionized Impurity Scattering:

Wrap-up

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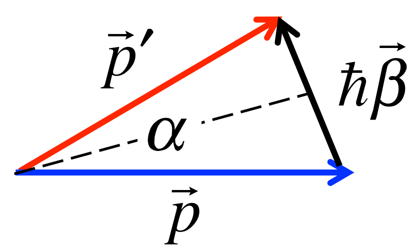
(revised 9/5/17)

BH vs. CW

Brook-Herring means "screened Coulomb scattering."

$$U_S(r) = -\frac{q^2}{4\pi\kappa_S\varepsilon_0 r}e^{-r/L_D}$$

$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar\kappa_S^2\varepsilon_S^2\Omega}\frac{\delta(E'-E)}{\left(\frac{4p^2}{\hbar^2}\sin^2\alpha/2 + 1/L_D^2\right)^2}$$



Conwell-Weisskopf means "unscreened Coulomb scattering."

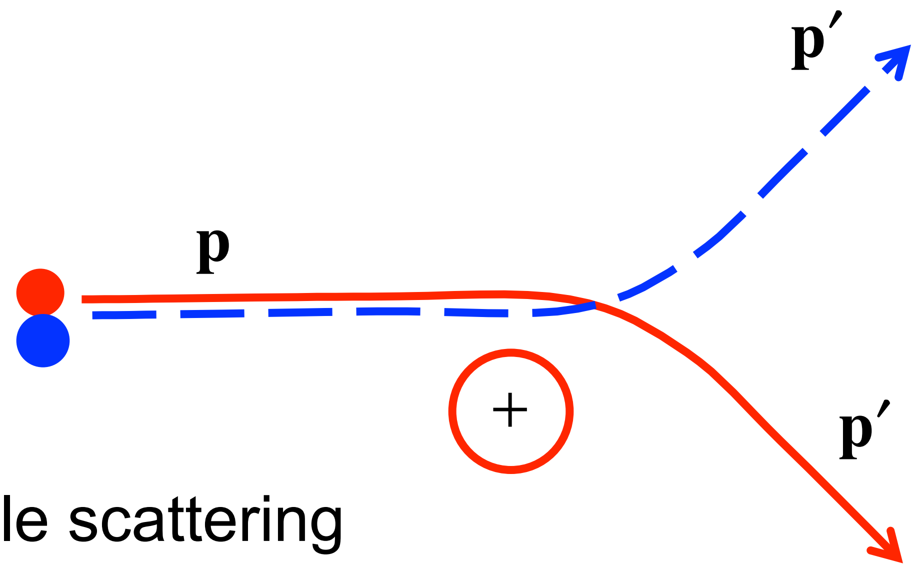
$$U_S(r) = +\frac{q^2}{4\pi\kappa_S\varepsilon_0 r}$$

$$S(\vec{p},\vec{p}\,') = \frac{2\pi q^4 N_I}{\hbar\kappa_S^2\varepsilon_S^2\Omega}\frac{\delta(E'-E)}{\left(\frac{4p^2}{\hbar^2}\sin^2\alpha/2\right)^2}$$

BH and CW

1) $S(\vec{p},\vec{p}') \sim N_I$

2) $S(\vec{p},\vec{p}') \sim q^4$



3) $S(\vec{p},\vec{p}') \sim 1/E^2$

4) Both favor small angle scattering

BH vs. CW momentum relaxation time

Brooks-Herring:

$$\tau_m(E) = \frac{16\sqrt{2m^*}\pi\kappa_S^2\varepsilon_0^2}{N_I q^4}\left[\ln\left(1+\gamma_{BH}^2\right) - \frac{\gamma_{BH}^2}{1+\gamma_{BH}^2}\right]E^{3/2} >> \tau(E)$$

$$\gamma_{BH}^2 = 8m^* E L_D^2 / \hbar^2$$

Conwell-Weisskopf:

$$\tau_m(E) = \frac{16\pi\sqrt{2m^*}\kappa_S^2\varepsilon_0^2}{N_I q^4}\left[\frac{1}{\ln\left(1+\gamma_{CW}^2\right)}\right]E^{3/2} \qquad \gamma_{CW}^2 = b_{\max} / \left(q^2 / 8\pi\kappa_S\varepsilon_0 E\right)$$

BH and CW momentum relaxation time

$$\tau_m(E) \sim E^{3/2}$$

"power law scattering"

$$\tau_m(E) \approx \tau_0 \left(E/k_B T \right)^s \qquad \tau_0 \sim T^{3/2} \qquad s = 3/2$$

BH or CW ?

Compare b_{MAX} to L_D

Use BH if:

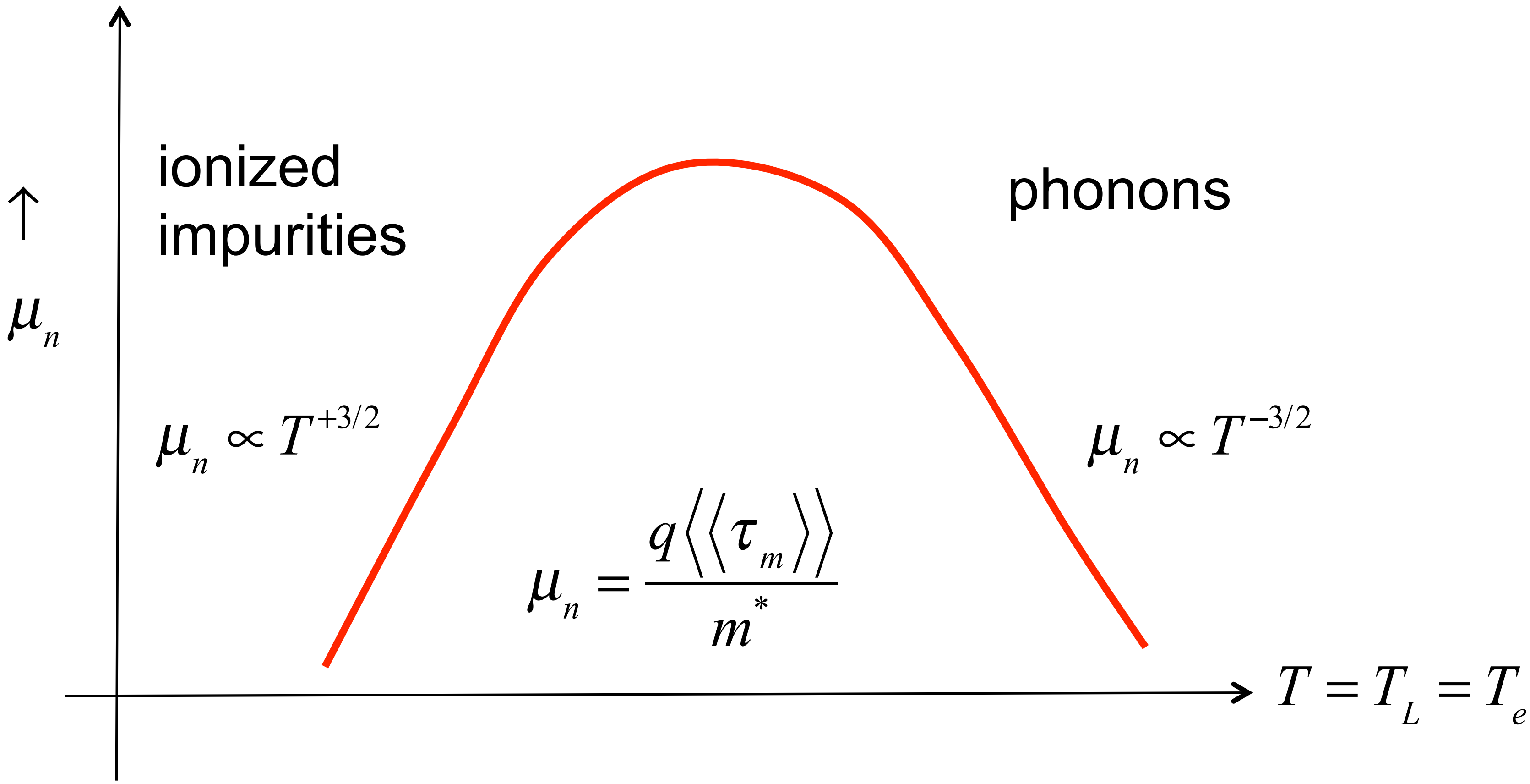
$$b_{\max} > L_D$$

$$b_{\max} = \frac{1}{2} N_I^{-1/3}$$

$$L_D = \sqrt{\frac{\kappa_S \varepsilon_0 k_B T}{q^2 n_0}}$$

B. K. Ridley, "Reconciliation of the Conwell-Weisskopf and Brooks-Herring formulae for charged-impurity scattering in semiconductors: Third-body interference," *J. Phys. C: Solid State Phys*. **10,** p. 1589 doi:10.1088/0022-3719/10/10/003, 1977.

Temperature-dependent mobility



Observation of a $T^{+3/2}$ temperature dependent mobility is generally taken as the “signature” of ionized impurity scattering.

Mobility due to BH II scattering

$$\tau_m(E) = \frac{16\sqrt{2m^*}\pi\kappa_S^2\varepsilon_0^2}{N_I q^4}\left[\ln\left(1+\gamma_{BH}^2\right) - \frac{\gamma_{BH}^2}{1+\gamma_{BH}^2}\right]E^{3/2}$$

$$\gamma_{BH}^2 = 8m^* E L_D^2 \big/ \hbar^2$$

(solve the BTE)

$$\mu_n = q\frac{\langle\langle \tau_m(E)\rangle\rangle}{m^*} \qquad \tau_m(E) = \tau_0\left(E/k_B T_L\right)^s \qquad \langle\langle \tau_m \rangle\rangle = \tau_0\frac{\Gamma(s+5/2)}{\Gamma(5/2)}$$

Mobility due to BH II scattering (ii)

$$\tau_m(E) = \frac{16\sqrt{2m^*}\pi\kappa_S^2\varepsilon_0^2\left(k_B T_L\right)^{3/2}}{N_I q^4}\left[\ln\left(1+\gamma_{BH}\right) - \frac{\gamma_{BH}}{1+\gamma_{BH}}\right]\left(\frac{E}{k_B T_L}\right)^{3/2}$$

$$\gamma_{BH}^2 = 8m^* \hat{E} L_D^2 / \hbar^2 \qquad \tau_m\left(E\right) = \tau_0\left(\hat{E}\right)\left(E/k_B T\right)^s \qquad \hat{E} = \left(s + 3/2\right)k_B T_L$$

$$\mu_n = q\frac{\left\langle\left\langle \tau_m(E)\right\rangle\right\rangle}{m^*}$$

$$\left\langle\left\langle \tau_m \right\rangle\right\rangle = \tau_0\left(\hat{E}\right)\frac{\Gamma\left(s+5/2\right)}{\Gamma\left(5/2\right)}$$

$$\mu_{BH} = \frac{128\sqrt{2\pi}\kappa_S^2\varepsilon_0^2\left(k_B T_L\right)^{3/2}}{q^3\sqrt{m^*}N_I\left[\ln\left(1+\gamma_{BH}^2\right) - \gamma_{BH}^2/\left(1+\gamma_{BH}^2\right)\right]}$$

(Lundstrom, FCT, Sec. 4.8.1)

Mobility due to CW II scattering

$$\tau_m(E) = \frac{16\pi\sqrt{2m^*}\kappa_S^2\varepsilon_0^2\left(k_BT_L\right)^{3/2}}{N_I q^4}\left[\frac{1}{\ln\left(1+\gamma_{CW}^2\right)}\right]\left(\frac{E}{k_BT_L}\right)^{3/2}$$

$$\gamma_{CW}^2 = b_{\max}\Big/\left(q^2/8\pi\kappa_S\varepsilon_0\hat{E}\right) \qquad b_{\max} = \frac{1}{2}N_I^{-1/3} \qquad \tau_m\left(E\right) = \tau_0\left(\hat{E}\right)\left(E/k_BT_L\right)^s$$

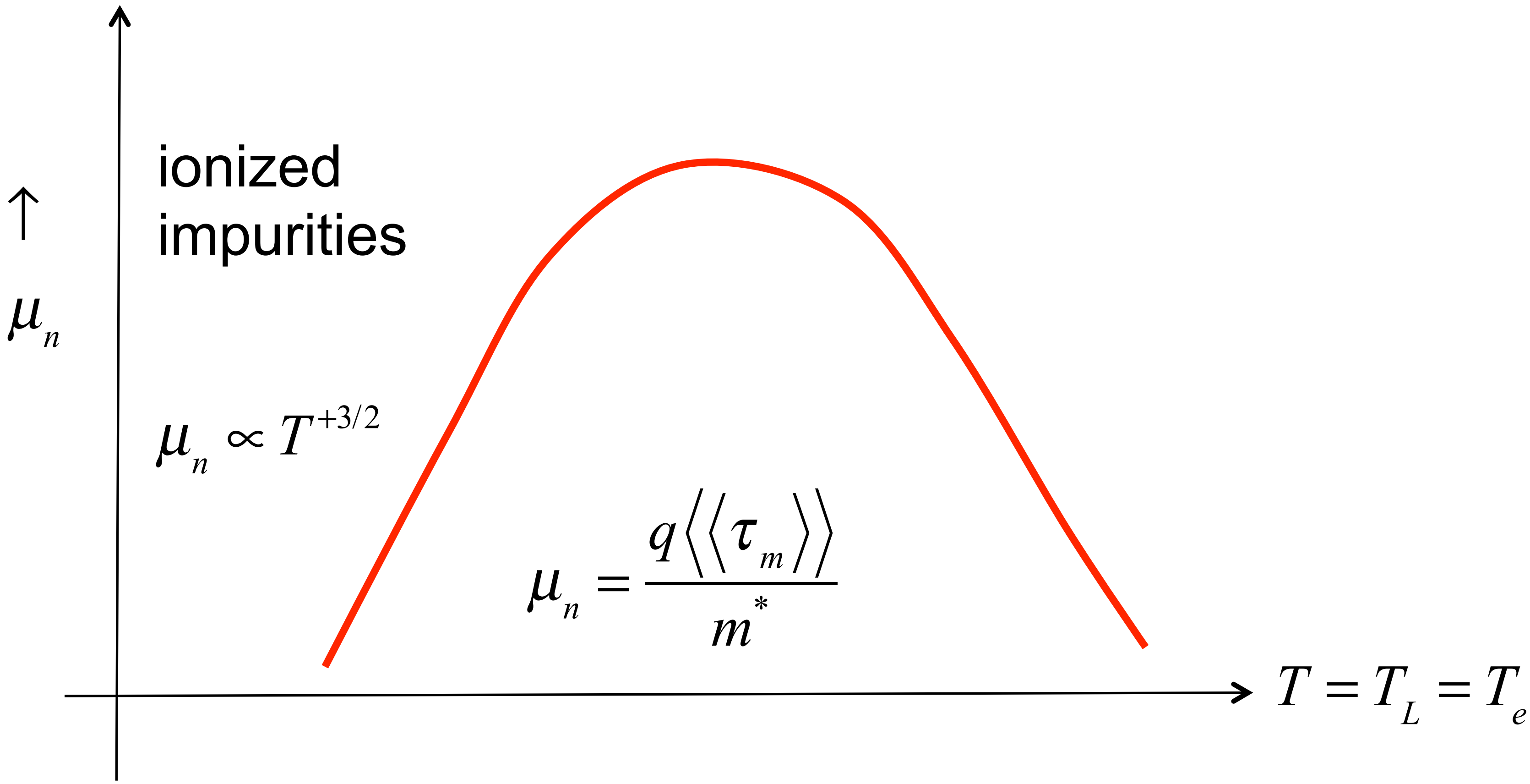
$$\mu_n = q\frac{\left\langle\left\langle\tau_m(E)\right\rangle\right\rangle}{m^*}$$

$$\left\langle\left\langle\tau_m\right\rangle\right\rangle = \tau_0\left(\hat{E}\right)\frac{\Gamma\left(s+5/2\right)}{\Gamma\left(5/2\right)}$$

$$\mu_{CW} = \frac{128\sqrt{2\pi}\kappa_S^2\varepsilon_0^2\left(k_BT_L\right)^{3/2}}{q^3\sqrt{m^*}N_I\left[\ln\left(1+\gamma_{CW}^2\right)\right]}$$

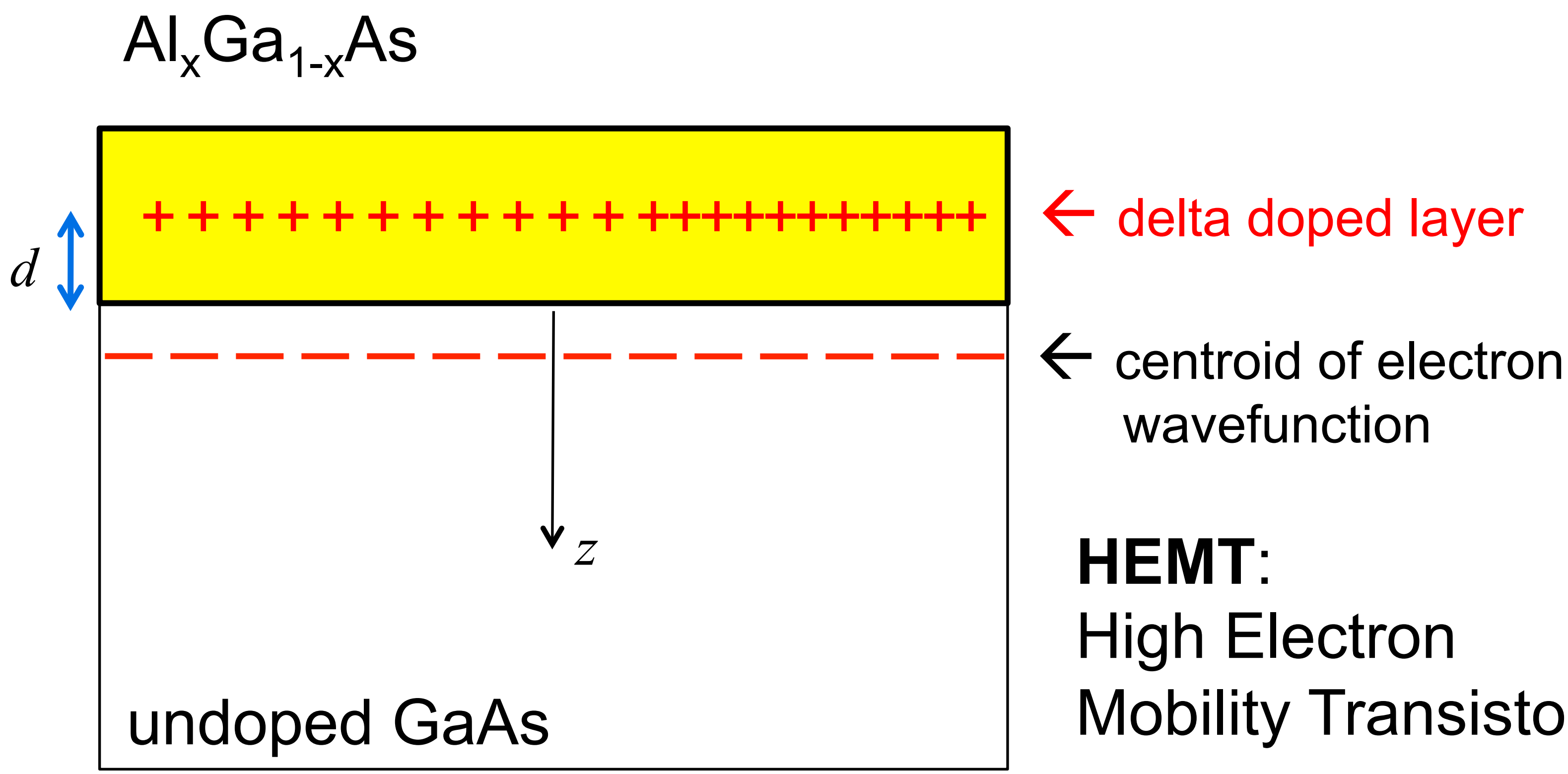
(Lundstrom, FCT, prob. 4.17)

Temperature-dependent mobility



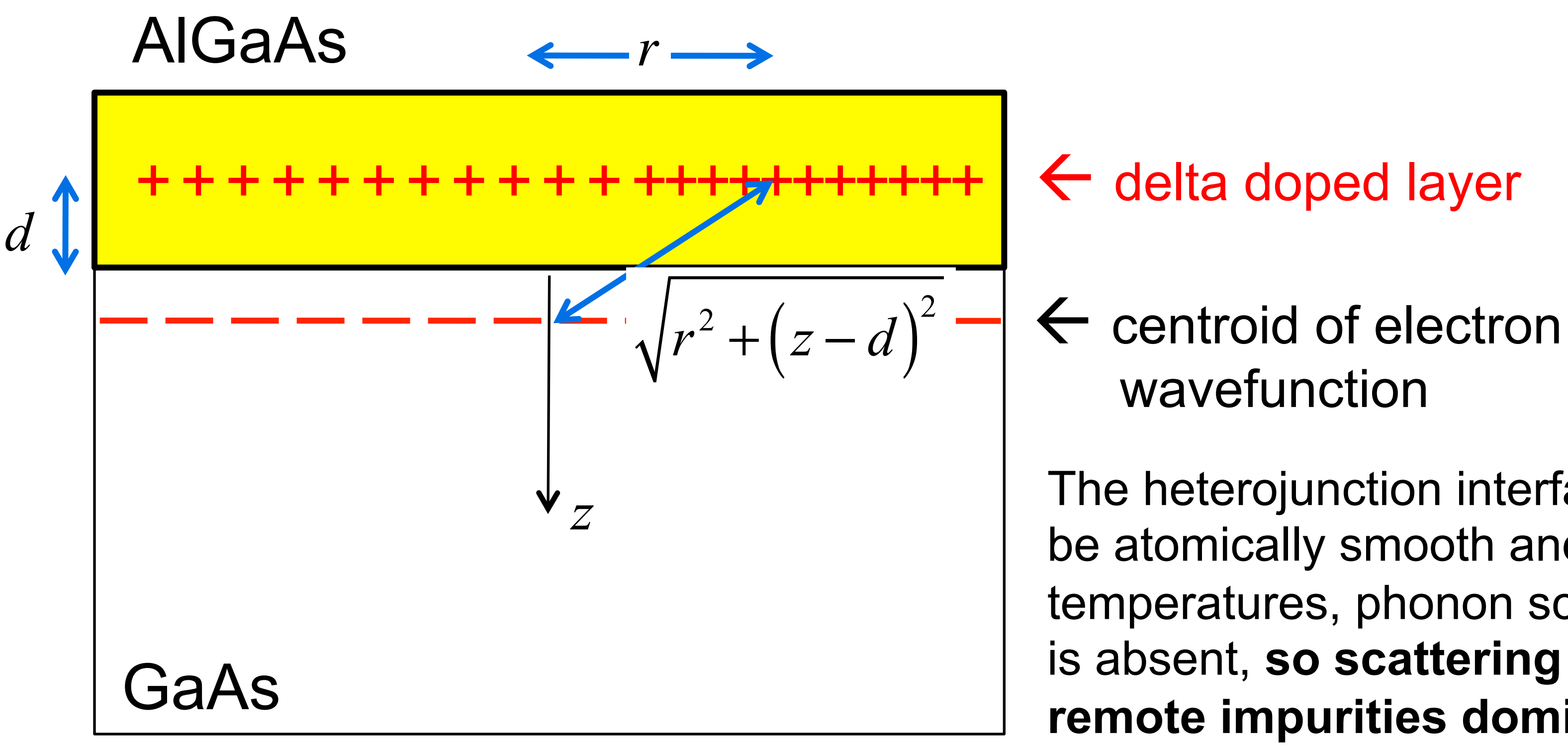
If the mobility increases at $T^{3/2}$, then we can analyze it with the BH or CW approach.

Modulation doped structures



HEMT:
High Electron
Mobility Transistor

Remote impurity scattering



The heterojunction interface can be atomically smooth and at low temperatures, phonon scattering is absent, **so scattering by remote impurities dominates**. Extraordinarily high mobilities (e.g. > 10^6 cm^2/V-s) can be achieved at about T_L = 1K.

Modulation doped structures

For a discussion of modulation doping, screening in 2D, and remote impurity scattering in 2D, see:

J.H. Davies, *The Physics of Low-Dimensional Semiconductors*, Chapter 8, Cambridge Univ. Press, 1998.

Summary

1) The two classic treatments of II scattering are Brooks-Herring and Conwell-Weisskopf

2) II scattering is actually difficult to treat properly because:

   FGR does not account for the difference in sign of the scattering potential

   "multiple scattering" occurs at heavy doping.

Summary

3) For a bulk semiconductor, the II scattering time can be described (approximately) in a power law form with a characteristic exponent of 3/2.

4) A mobility that increases as $T^{3/2}$ is the "signature" of II scattering.

5) The low temperature mobility is often used as a measure of the total ionized impurity concentration in a sample.

Questions?