ECE 656: Electronic Transport in Semiconductors Fall 2017

Phonon Scattering

in Polar Semiconductors

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9/12/17

Outline

1) Quick re-cap
2) POP coupling constant
3) POP Transition rate and scattering time
4) PZ scattering
5) Summary

Transition rate for phonon scattering

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}\left|H_{p,p'}\right|^2 \delta\left(E' - E \mp \hbar\omega\right)$$

$$H_{p',p} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\vec{p}\,'\bullet\vec{r}/\hbar}\, U_S(\vec{r})\, e^{i\vec{p}\bullet\vec{r}/\hbar}\, d\vec{r}$$

$$U_S(\vec{r}) = K_q u_q \qquad u_q(\vec{r}) = A_q e^{\pm i\vec{q}\bullet\vec{r}}$$

$$\left|H_{p',p}\right|^2 = \left|K_q\right|^2 \left|A_q\right|^2 \delta_{\vec{p}\,',\vec{p}\pm\hbar\vec{q}}$$

$$\left|A_q\right|^2 \rightarrow \frac{\hbar}{2\rho\Omega\omega}\left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right)$$

$$S(\vec{p},\vec{p}\,') = \frac{\pi}{\Omega\rho\,\omega}\left|K_q\right|^2 \left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right)\delta\left(\vec{p}\,' - \vec{p} \mp \hbar\vec{q}\right)\delta\left(E' - E \mp \hbar\omega\right)$$

Scattering times

$$S(\vec{p},\vec{p}\,') = \frac{\pi}{\Omega\rho\,\omega}\left|K_q\right|^2\left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right)\delta\left(\vec{p}\,' - \vec{p} \mp \hbar\vec{q}\right)\delta\left(E' - E \mp \hbar\omega\right)$$

Coupling constant has a **strong q-dependence.**

$$\frac{1}{\tau(\vec{p})} == \sum_{\vec{p}\,',\uparrow} S(\vec{p},\vec{p}\,')$$

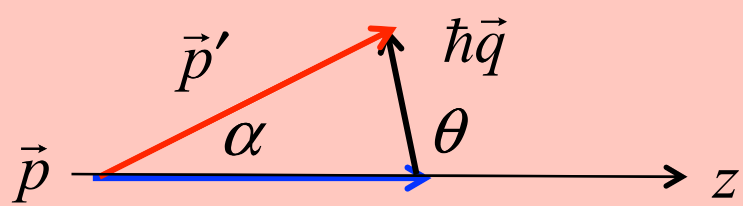
$$\frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}\,',\uparrow} S(\vec{p},\vec{p}\,')\frac{\Delta p_z}{p_{z0}}$$

Including E-M conservation

$$S(\vec{p},\vec{p}\,') = \frac{2\pi}{\hbar}|K_q|^2 \frac{\hbar}{2\rho\Omega\omega_0}\frac{1}{\hbar\upsilon q}\left(N_q + \frac{1}{2} \mp \frac{1}{2}\right)\delta\left(\pm\cos\theta + \frac{\hbar q}{2p} \mp \frac{\omega_0}{\upsilon q}\right)$$

$$S(\vec{p},\vec{p}\,') = \frac{1}{\Omega}C_q\left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right)\delta\left(\pm\cos\theta + \frac{\hbar q}{2p} \mp \frac{\omega_0}{\upsilon q}\right)$$

$$C_q = \frac{\pi}{\hbar\rho\upsilon\omega_0 q}|K_q|^2 \qquad N_\omega = \frac{1}{e^{\hbar\omega_0/k_B T} - 1}$$



General expression for scattering time

$$\frac{1}{\tau} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}') = \sum_{\vec{q},\uparrow} S(\vec{p},\vec{p}') \qquad \vec{p}' = \vec{p} \pm \hbar\vec{q}$$

Integration of the delta function simply restricts q to those values that satisfy energy and momentum conservation.

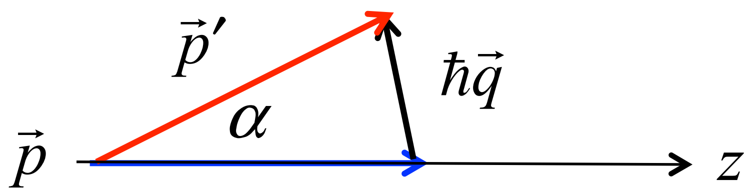
$$q_{\min} < q < q_{\max}$$

$$\frac{1}{\tau} = \frac{1}{4\pi^2} \int_{q_{\min}}^{q_{\max}} C_q \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) q^2 \, dq$$

$$C_q = \frac{\pi}{\hbar \rho \upsilon \omega_0 q} \left| K_q \right|^2 \qquad N_\omega = \frac{1}{e^{\hbar\omega_0 / k_B T} - 1}$$

Momentum relaxation time

$$\frac{1}{\tau_m} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}') \frac{(\Delta p_z)}{p_z} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}') \left(1 - \frac{p'}{p}\cos\alpha\right)$$



$$\frac{1}{\tau_m} = \frac{1}{4\pi^2} \int_{q_{\min}}^{q_{\max}} C_q \left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right)\left(\frac{\hbar q}{2p} \mp \frac{\omega_q}{\upsilon q}\right) \frac{\hbar q^3}{p}\, dq$$

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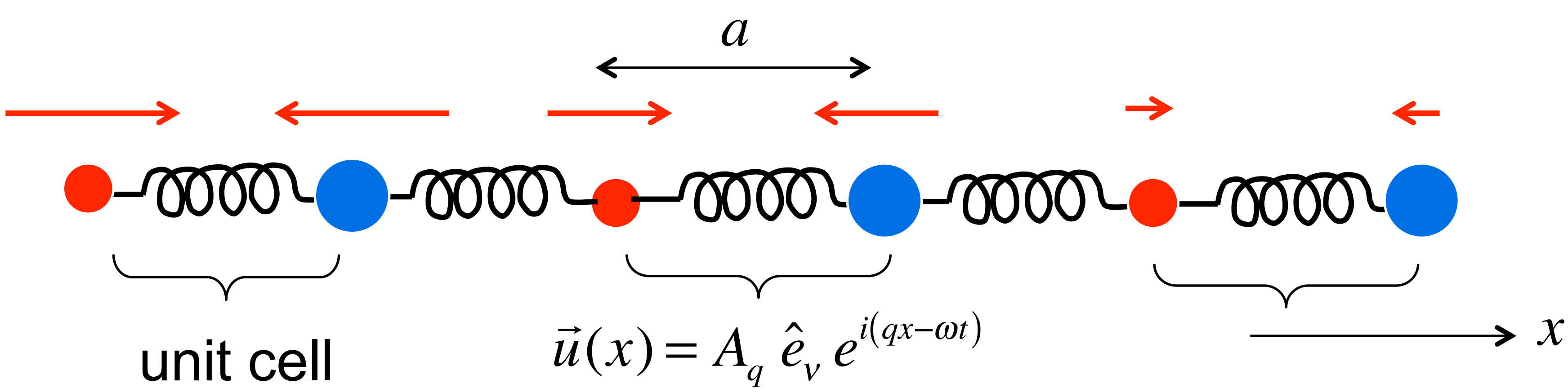
LO phonons



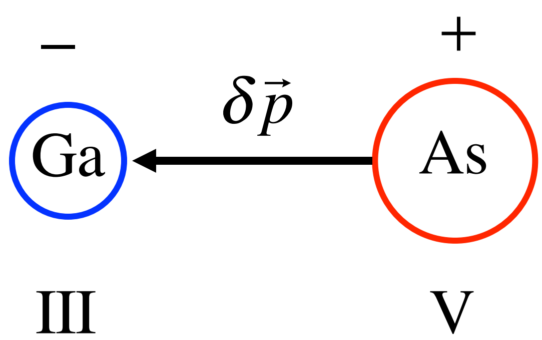
$$\vec{u}(x) = A_q\, \hat{e}_\nu\, e^{i(qx-\omega t)}$$

In covalent semiconductors (e.g. Si, Ge), all atoms are the same.

In II-VI and III-V semiconductors, the bonds are partially ionic, which gives rise to internal dipole moments.

Oscillating dipole moments produce strong scattering potentials.

POP coupling constant

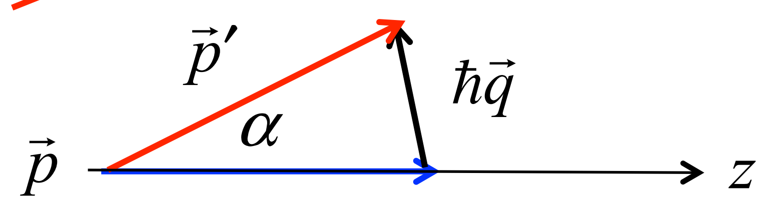


$$U_s = K_q u_q$$

$$u_q(x,t) = A_q e^{\pm i(qz-\omega t)}$$

$$\left|K_q\right|^2 = \frac{\rho e^2 \omega_0^2}{q^2 \kappa_0 \varepsilon_0}\left(\frac{\kappa_0}{\kappa_\infty} - 1\right)$$

small angle scattering dominates



See Lundstrom, FCT, Sec. 2.2.2

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Transition rate

$$S(\vec{p},\vec{p}\,') = \frac{\pi}{\Omega\rho\,\omega}\left|K_q\right|^2\left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right)\delta\left(\vec{p}\,' - \vec{p} \mp \hbar\vec{q}\right)\delta\left(E' - E \mp \hbar\omega_0\right)$$

scattering is inelastic

scattering is anisotropic

$$\left|K_q\right|^2 = \frac{\rho\, e^2\omega_0^2}{q^2\kappa_0\varepsilon_0}\left(\frac{\kappa_0}{\kappa_\infty} - 1\right)$$

Scattering times

$$\frac{1}{\tau} = \frac{1}{4\pi^2} \int_{q_{\min}}^{q_{\max}} C_q \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) q^2 \, dq \qquad C_q = \frac{\pi}{\hbar \rho \upsilon \omega_0 q} \left| K_q \right|^2$$

$$\frac{1}{\tau_m} = \frac{1}{4\pi^2} \int_{q_{\min}}^{q_{\max}} C_q \left(N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \left(\frac{\hbar q}{2p} \mp \frac{\omega_q}{\upsilon q} \right) \frac{\hbar q^3}{p} \, dq$$

$$q_{\min} = \frac{p}{m^*} \left[\mp 1 \pm \left(1 \pm \hbar\omega_0 / E_k \right)^{1/2} \right]$$ (ABS/EMS)

$$q_{\max} = \frac{p}{m^*} \left[1 + \left(1 \pm \hbar\omega_0 / E_k \right)^{1/2} \right]$$ (For EMS: $E_k > \hbar\omega_0$)

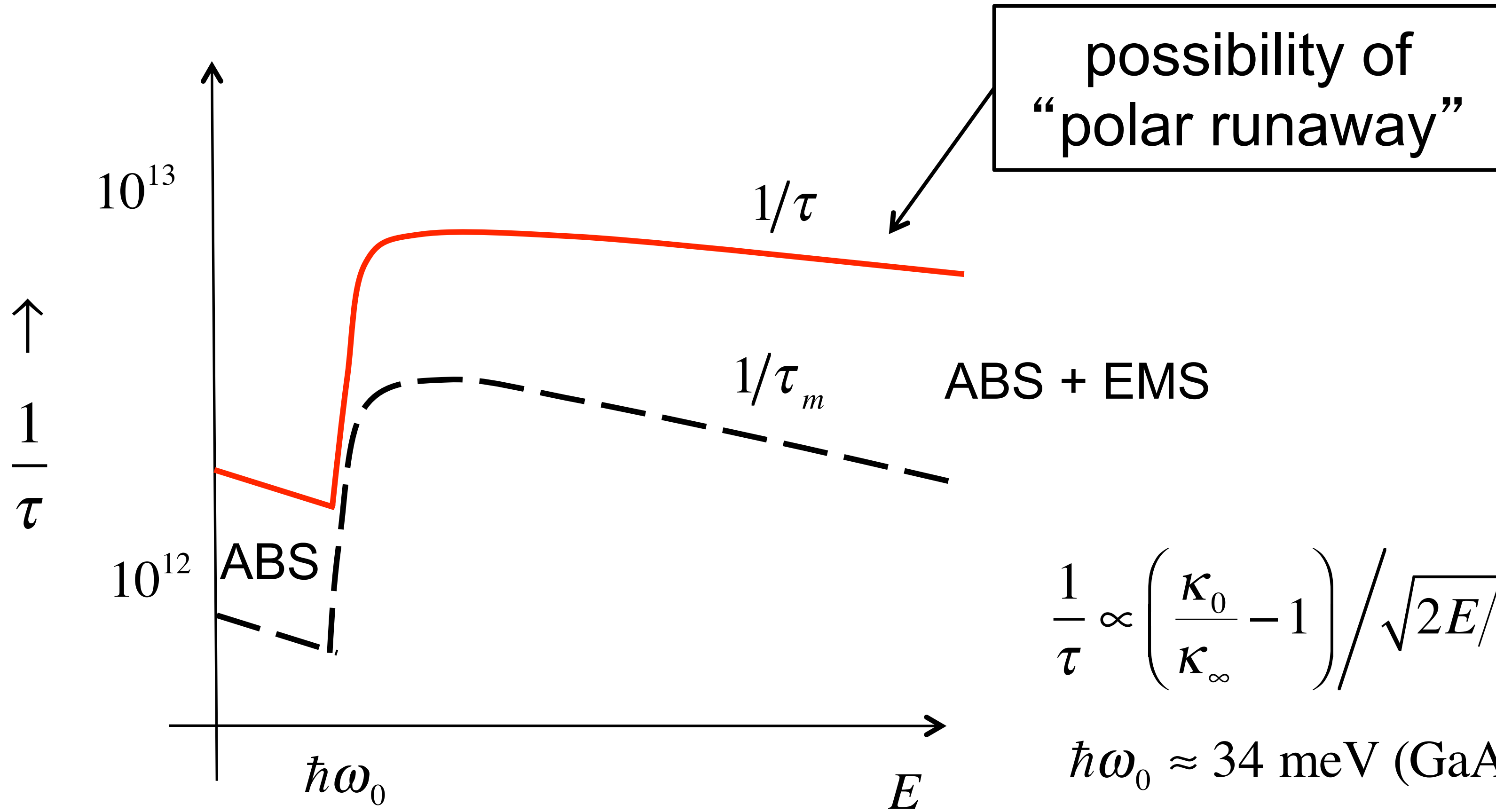
Scattering rate

$$\frac{1}{\tau}=\frac{1}{\tau_{abs}}+\frac{1}{\tau_{ems}}=\frac{q^2\omega_0^2\left(\frac{\kappa_0}{\kappa_\infty}-1\right)}{2\pi\kappa_0\varepsilon_0\hbar\sqrt{2E/m^*}}\left[N_0\sinh^{-1}\left(\frac{E}{\hbar\omega_0}\right)+\left(N_0+1\right)\sinh^{-1}\left(\frac{E}{\hbar\omega_0}-1\right)\right]$$

$$\frac{1}{\tau}>\frac{1}{\tau_m}>\frac{1}{\tau_E}$$

See Lundstrom (FCT) , pp. 84 – 86 for momentum and energy relaxation rates.

POP scattering



$$\frac{1}{\tau} \propto \left(\frac{\kappa_0}{\kappa_\infty} - 1\right) \Big/ \sqrt{2E/m^*}$$

$$\hbar\omega_0 \approx 34 \text{ meV (GaAs)}$$

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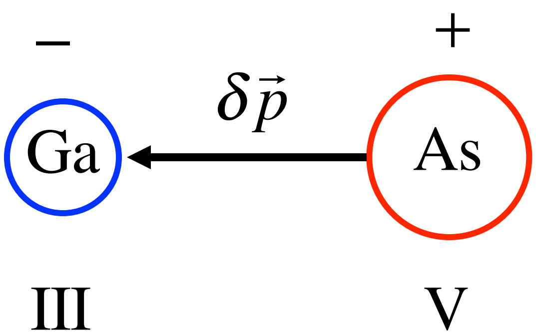
Acoustic phonons in polar materials



$$u_q(x,t) = A_q e^{\pm i(qx - \omega t)}$$

$$U_S = K_q u_q$$

$$|K_q|^2 = \frac{e^2 e_{PZ}^2}{\kappa_0^2 \varepsilon_0^2}$$

Isotropic and elastic near room temperature.

See Lundstrom, FCT, Sec. 2.2.2

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Intra-band phonon scattering

$$u_q(\vec{r},t) = A_q e^{\pm i(\vec{q} \bullet \vec{r} - \omega_q t)} \qquad U_S = K_q u_q$$

ADP $\quad |K_q|^2 = q^2 D_A^2$

ODP $\quad |K_q|^2 = D_0^2$

PZ $\quad |K_q|^2 = (e e_{PZ} / \kappa_S \varepsilon_0)^2$

POP $\quad |K_q|^2 = \dfrac{\rho e^2 \omega_0^2}{q^2 \kappa_0 \varepsilon_0} \left(\dfrac{\kappa_0}{\kappa_\infty} - 1 \right)$

Questions?

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