ECE 656: Electronic Transport in Semiconductors Fall 2017

Electron-Phonon Scattering in 1D, 2D, and 3D

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outline

1) **Review of ADP Scattering in 3D**
2) ADP Scattering in 2D: MCA
3) ADP Scattering in 2D: FGR
4) ADP Scattering in 1D: FGR
5) Summary

ADP scattering: 3D review

$$S(\vec{p},\vec{p}') = \frac{2\pi}{\hbar}\left|H_{p',p}\right|^2 \delta\left(E' - E \mp \hbar\omega\right)$$

$$H_{p',p} = \frac{1}{\Omega}\int_{-\infty}^{+\infty} e^{-i\vec{p}'\bullet\vec{r}/\hbar} U_S\left(\vec{r}\right) e^{i\vec{p}\bullet\vec{r}/\hbar} d\vec{r}$$

$$U_S = \sum_{\vec{q}} K_q u_q$$

Note the sum over q

$$\left|K_q\right|^2 = q^2 D_A^2$$

$$u_q\left(\vec{r},t\right) = A_q e^{\pm i\left(\vec{q}\bullet\vec{r} - \omega_q t\right)}$$

$$\left|A_q\right|^2 = \frac{\hbar}{2\rho\Omega\,\omega}\left(N_\omega + \frac{1}{2} \mp \frac{1}{2}\right)$$

$$U_S = K_q u_q$$

$$N_\omega = \frac{1}{e^{\hbar\omega/k_B T} - 1} \approx \frac{k_B T}{\hbar\omega}$$

$$N_\omega \approx N_\omega + 1$$

ADP scattering: 3D review

$$H_{p',p} = \frac{1}{\Omega} \int_{-\infty}^{+\infty} e^{-i\vec{p}'\bullet\vec{r}/\hbar} \left(\sum_{\vec{q}} K_q u_q \right) e^{i\vec{p}\bullet\vec{r}/\hbar} d\vec{r}$$

$$\left| H_{p',p} \right|^2 = \frac{1}{\Omega} \sum_{\vec{q}} U_{ac} \frac{1}{\Omega} \left| \int_{-\infty}^{+\infty} e^{-i\vec{p}'\bullet\vec{r}/\hbar} \left(e^{\pm i\vec{q}\bullet\vec{r}} \right) e^{i\vec{p}\bullet\vec{r}/\hbar} d\vec{r} \right|^2$$

$$U_{ac} = \Omega \left| K_q \right|^2 \left| A_q \right|^2 = \frac{D_A^2 k_B T_L}{2c_l}$$

$$\delta\left(\vec{p}' - \vec{p} \mp \hbar\vec{q} \right)$$

$$\left| H_{p',p} \right|^2 = \frac{1}{\Omega} U_{ac} \delta\left(\vec{p}' - \vec{p} \mp \hbar\vec{q} \right)$$

ADP scattering: 3D review

$$S(\vec{p},\vec{p}') = \frac{2\pi}{\hbar}\frac{U_{ac}}{\Omega}\delta(\vec{p}' - \vec{p} \mp \hbar\vec{q})\delta(E' - E \mp \hbar\omega) \qquad U_{ac} = \frac{D_A^2 k_B T}{2c_l}$$

$$\frac{1}{\tau} = \frac{1}{\tau_m} = \sum_{\vec{p}'} S(\vec{p},\vec{p}')$$

$$\frac{1}{\tau(E)} = \frac{2\pi}{\hbar} U_{ac} \frac{D_{3D}(E)}{2}$$

$$\tau = \tau_0 \left(E/k_B T\right)^{-1/2}$$

$$s = -1/2$$

$$\tau_0 \propto T^{-3/2}$$

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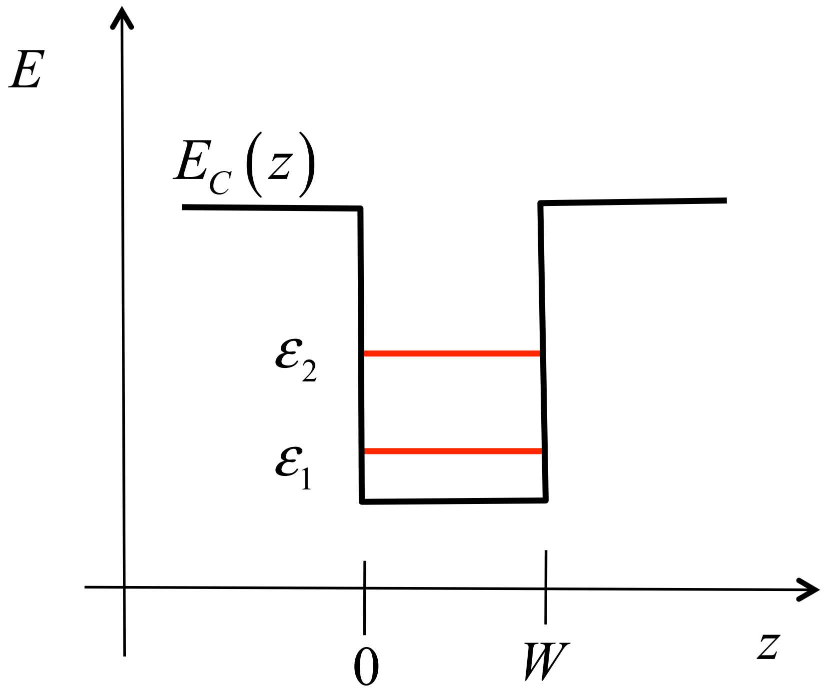
Quantum confined carriers



$$\psi_n(x,y,z) = F_n(z)\frac{1}{\sqrt{A}}e^{i\vec{k}_{\parallel}\bullet\vec{\rho}}$$

For an infinite well:

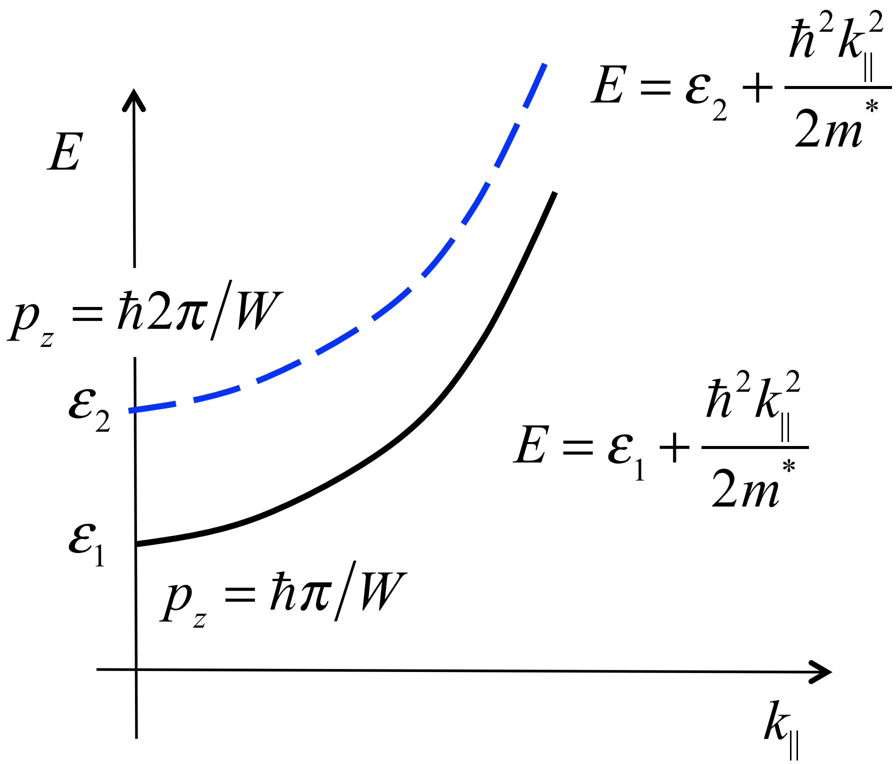
$$F_n(z) = \sqrt{\frac{2}{W}}\sin k_n z$$

$$k_n = \frac{n\pi}{W}$$

Electrons are <u>free to move</u> in the x-y plane

Note that $p_z = \hbar k_z$ is quantized.

Quantum confined carriers



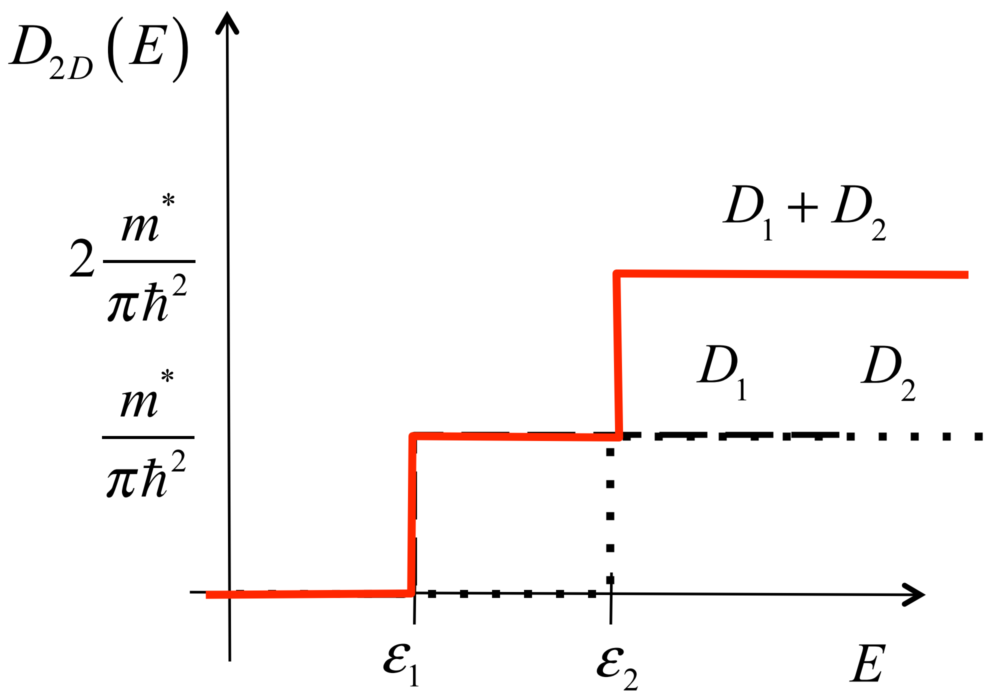
$$E = \varepsilon_n + \frac{\hbar^2 k_{\parallel}^2}{2m^*}$$

$$\varepsilon_n = \frac{\hbar^2 k_n^2}{2m^*}$$

$$k_n = \frac{n\pi}{W}$$

Electrons are free to move in the x-y plane

2D DOS

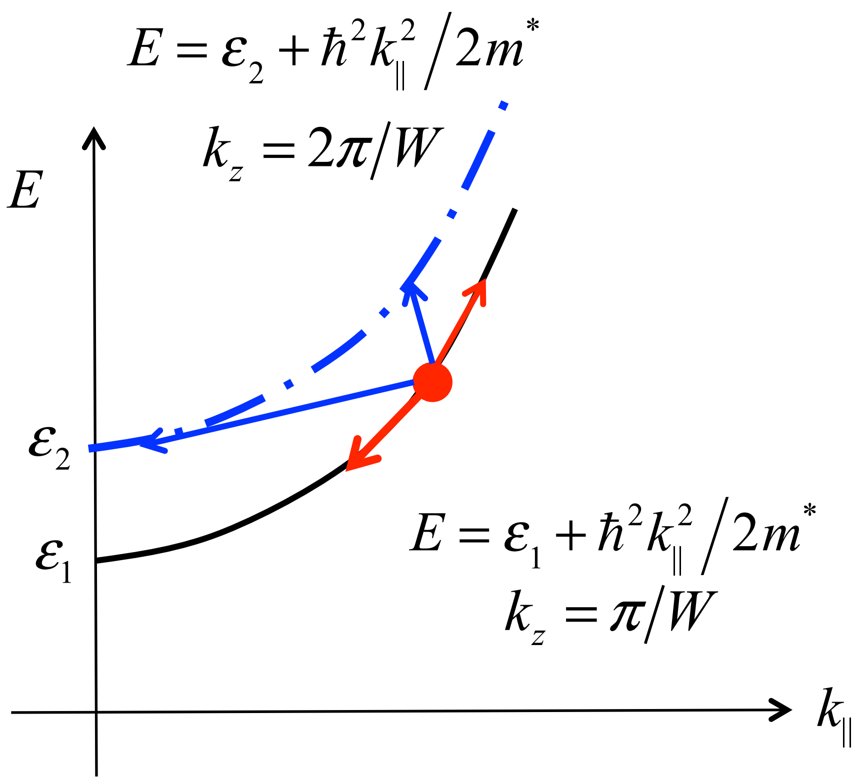


$$\varepsilon_n = \frac{\hbar^2 n^2 \pi^2}{2m^* W^2}$$

$$D_{2D}(E) = \frac{m^*}{\pi\hbar^2} \sum_{n=1} \Theta(E - \varepsilon_n)$$

(A valley degeneracy of 1 is assumed.)

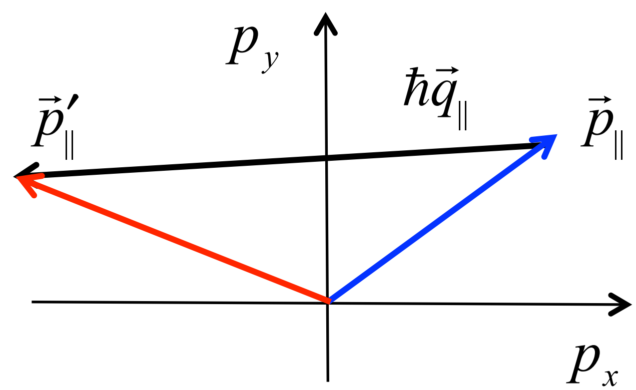
Momentum Conservation Approximation



in the plane

electron: $\vec{p} = \vec{p}_\parallel + p_z \hat{z}$

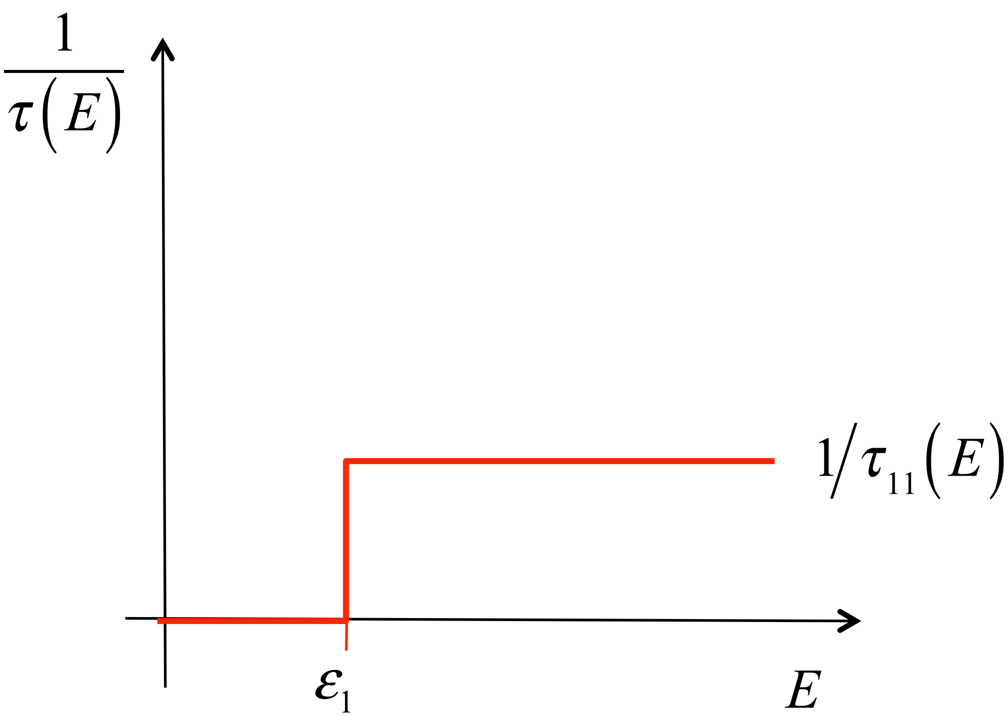
phonon: $\vec{q} = \vec{q}_\parallel + q_z \hat{z}$



Intra subband: $\Delta p_z = 0 \quad q_z = 0$

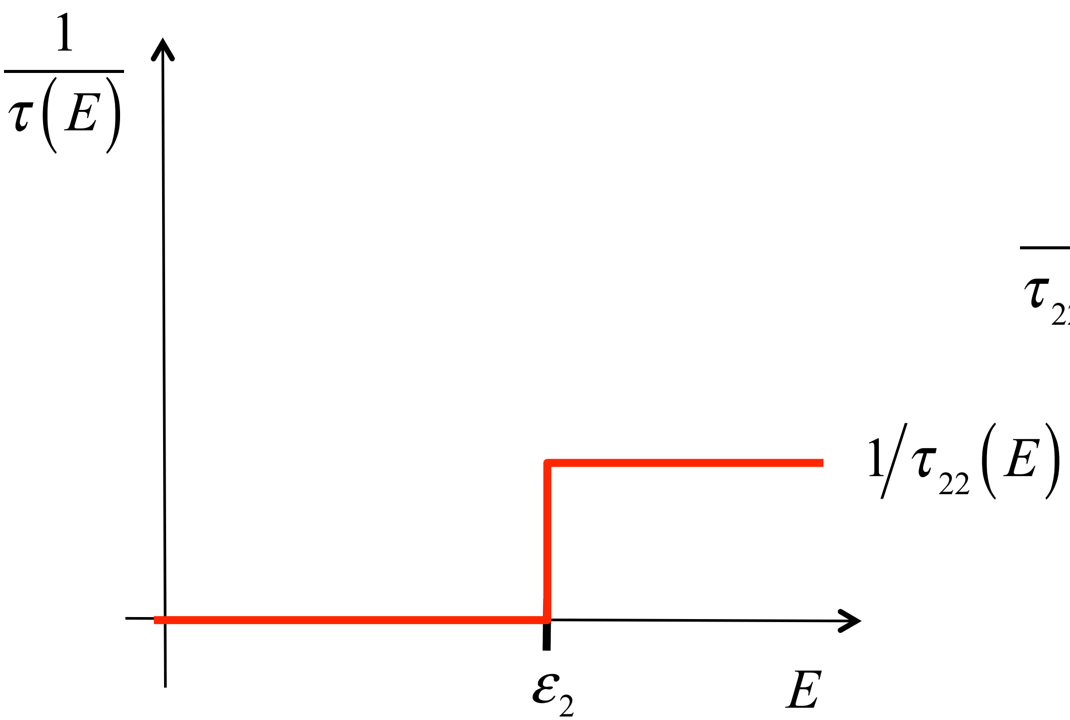
Inter subband: $\Delta p_z = p_{zi} - p_{zf} \quad \hbar q_z = \Delta p_z$

2D Scattering rate: subband 1



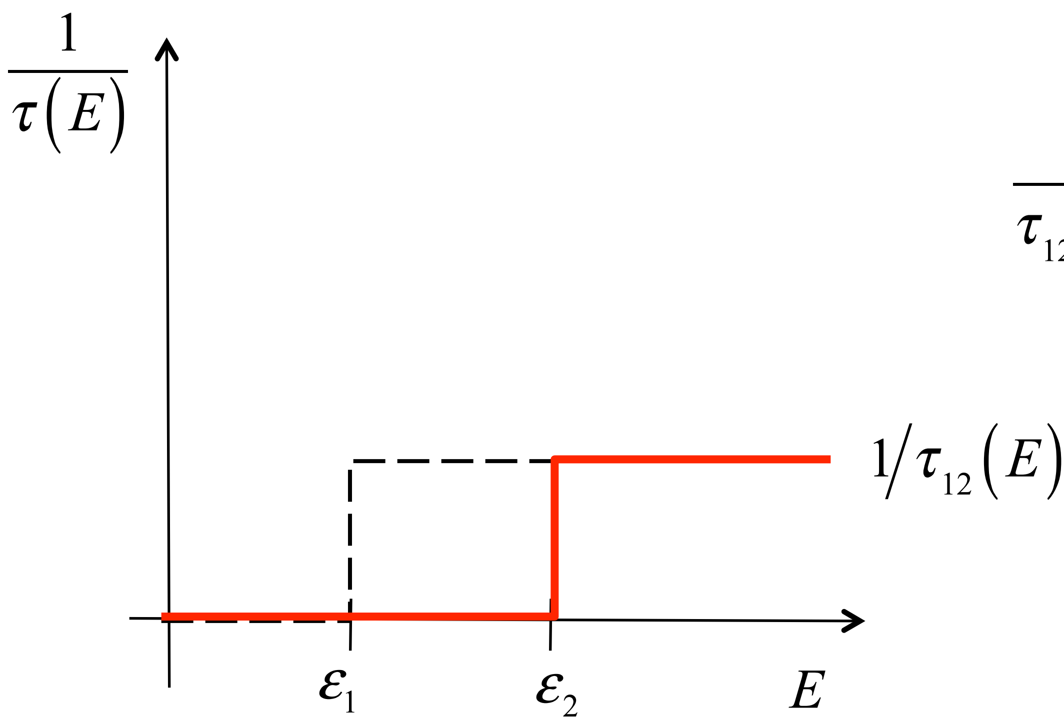
$$\frac{1}{\tau_{11}(E)} \propto D_{2D}^{1}\left(E-\varepsilon_1\right) \quad E > \varepsilon_1$$

2D Scattering rate: subband 2



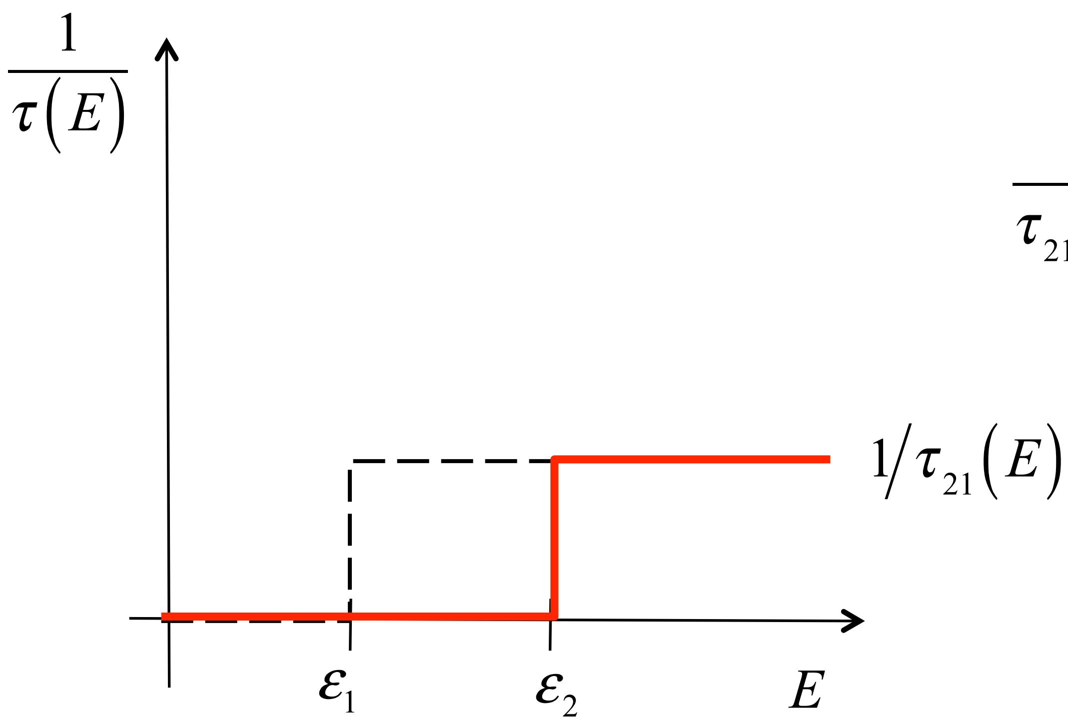
$$\frac{1}{\tau_{22}(E)} \propto D_{2D}^2 \left(E - \varepsilon_2\right) \quad E > \varepsilon_2$$

2D Scattering rate: subband 1 to 2



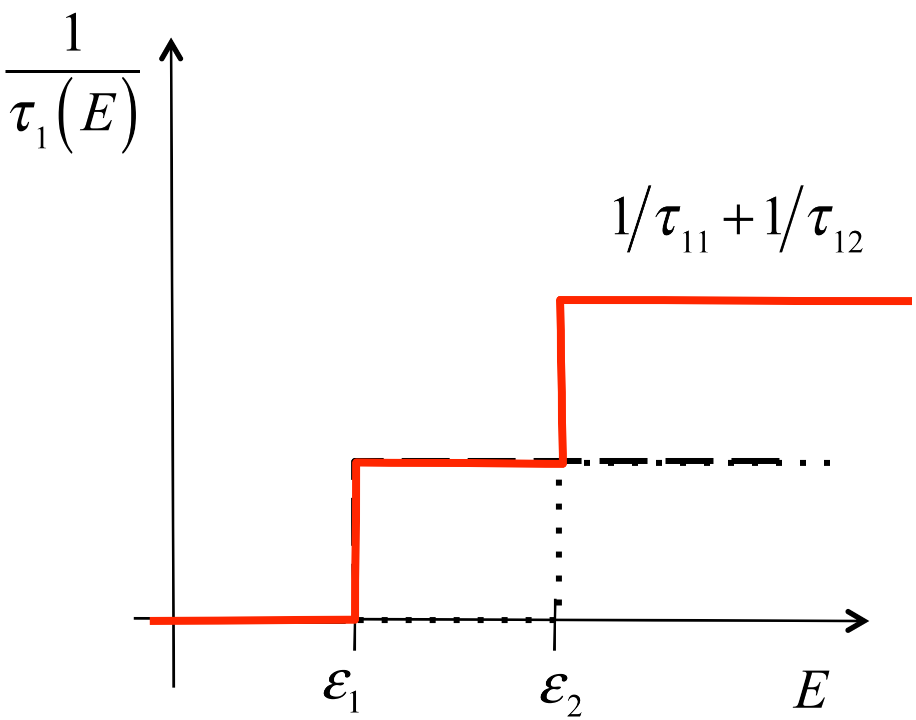
$$\frac{1}{\tau_{12}(E)} \propto D_{2D}^2(E-\varepsilon_2) \quad E > \varepsilon_2$$

2D Scattering rate: subband 2 to 1



$$\frac{1}{\tau_{21}(E)} \propto D_{2D}^{1}\left(E-\varepsilon_2\right) \quad E > \varepsilon_2$$

2D Total scattering rate



For an electron in subband 1

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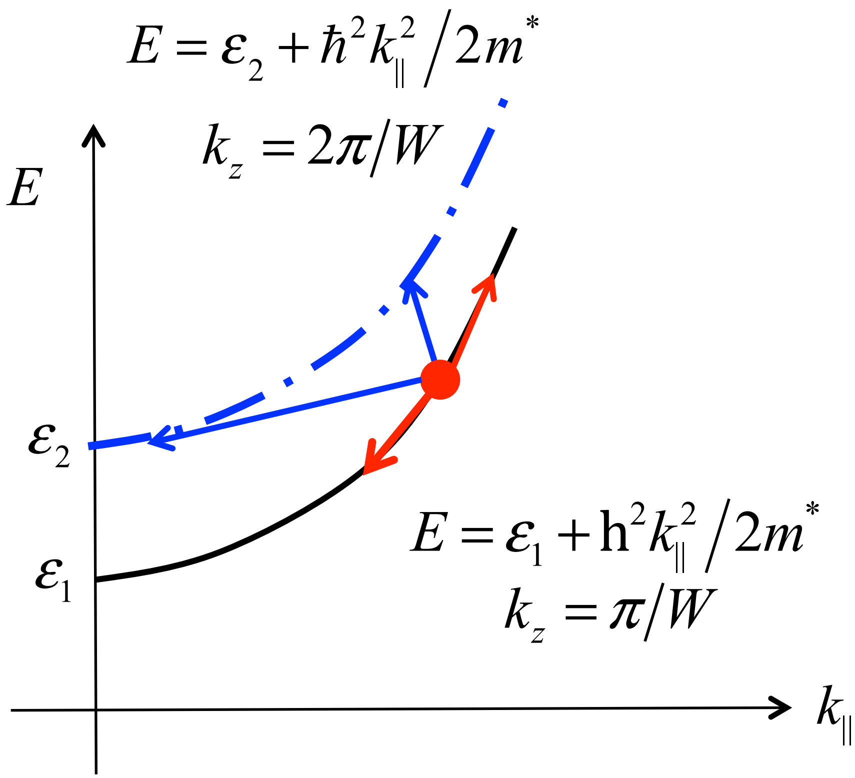
Momentum conservation is an approximation



$$\Delta p_z \Delta z \geq \frac{\hbar}{2}$$

Momentum does not need to be strictly conserved!

Recall that for short times, energy is not strictly conserved.

Momentum and energy conservation result from FGR in the appropriate limits.

2D electrons and 3D phonons

2D electrons:

$$\psi_{i,n}(\vec{\rho},z) = F_n(z)\frac{1}{\sqrt{A}}e^{i\vec{k}_{\parallel}\bullet\vec{\rho}} \qquad \psi_{f,n'}(\vec{\rho},z) = F_{n'}(z)\frac{1}{\sqrt{A}}e^{i\vec{k}'_{\parallel}\bullet\vec{\rho}}$$

3D phonons:

$$u_q(\vec{r}) = A_q e^{\pm i\vec{q}\bullet\vec{r}} = A_q\left(e^{\pm i\vec{q}_{\parallel}\bullet\vec{\rho}}e^{\pm iq_z z}\right)$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega}U_{ac}\sum_{\vec{q}}\left|\int_{-\infty}^{+\infty}\psi_f^*\left(e^{\pm i\vec{q}\bullet\vec{r}}\right)\psi_i d\vec{r}\right|^2$$

Matrix element for 2D electrons

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \sum_{\vec{q}} \left| \int_{-\infty}^{+\infty} \psi_f^* \left(e^{\pm i\vec{q}\cdot\vec{r}}\right) \psi_i d\vec{r} \right|^2$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \sum_{\vec{q}} \left| \int_{-\infty}^{+\infty} F_{n'}^*(z) \frac{1}{\sqrt{A}} e^{-i\vec{k}'_{\parallel}\cdot\vec{\rho}} \left(e^{\pm i\vec{q}_{\parallel}\cdot\vec{\rho}} e^{\pm i q_z z}\right) F_n(z) \frac{1}{\sqrt{A}} e^{i\vec{k}_{\parallel}\cdot\vec{\rho}} d\vec{\rho}\, dz \right|^2$$

2D 3D 2D

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \sum_{q_z} \left| \frac{1}{A} \int e^{-i\left(\vec{k}'_{\parallel}-\vec{k}_{\parallel}\mp\vec{q}_{\parallel}\right)\cdot\vec{\rho}}\, d\vec{\rho} \int_{-\infty}^{+\infty} F_{n'}^*(z) F_n(z) e^{\pm i q_z z} dz \right|^2$$

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \delta\left(\vec{p}'_{\parallel} - \vec{p}_{\parallel} \mp \hbar\vec{q}_{\parallel}\right) \sum_{q_z} \left| \int_{-\infty}^{+\infty} F_{n'}^*(z) F_n(z) e^{\pm i q_z z} dz \right|^2$$

"Form factor"

$$\frac{1}{\Omega} \to \frac{1}{A} \times \frac{1}{L}$$

$$\left|H_{p',p}\right|^2 = \frac{1}{A} U_{ac} \delta\left(\vec{p}'_{\parallel} - \vec{p}_{\parallel} \mp \hbar \vec{q}_{\parallel}\right) \left|F_{n',n}\right|^2$$

$$\left|F_{n',n}\right|^2 = \frac{1}{L} \sum_{q_z} \left| \int_{-\infty}^{+\infty} F_{n'}^*(z) F_n(z) e^{\pm i q_z z} dz \right|^2$$

3D → 2D

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \delta\left(\vec{p}' - \vec{p} \mp \hbar \vec{q}\right) \to \frac{1}{A} U_{ac} \delta\left(\vec{p}'_{\parallel} - \vec{p}_{\parallel} \mp \hbar \vec{q}_{\parallel}\right) \left|F_{n',n}\right|^2$$

Momentum conservation is replaced by momentum conservation in the plane times a "**form factor**."

Evaluation of the form factor

$$\left|F_{n',n}\right|^2 = \frac{1}{L}\sum_{q_z}\left|\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{\pm iq_z z}dz\right|^2$$

$$\left|F_{n',n}\right|^2 = \frac{1}{L}\frac{L}{2\pi}\int dq_z\left|\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{\pm iq_z z}dz\right|^2$$

$$\left|F_{n',n}\right|^2 = \frac{1}{2\pi}\int dq_z\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{iq_z z}dz\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{-iq_z z}dz$$

$$\left|F_{n',n}\right|^2 = \frac{1}{2\pi}\int dq_z\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)e^{iq_z z}dz\int_{-\infty}^{+\infty} F_{n'}^*(z')F_n(z')e^{-iq_z z'}dz'$$

$$\left|F_{n',n}\right|^2 = \frac{1}{2\pi}\int e^{iq_z(z-z')}\,d\beta_z\int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)dz\int_{-\infty}^{+\infty} F_{n'}^*(z')F_n(z')dz'$$

Evaluation of the form factor (ii)

$$\left|F_{n',n}\right|^2 = \frac{1}{2\pi}\int e^{iq_z(z-z')}\,dq_z \int_{-\infty}^{+\infty} F_{n'}^*(z)F_n(z)dz \int_{-\infty}^{+\infty} F_{n'}^*(z')F_n(z')dz'$$

Do the integral over q_z first and use: $$\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{iq_z(z-z')}\,dq_z = \delta(z-z')$$

$$\left|F_{n',n}\right|^2 = \int_{-\infty}^{+\infty} \left|F_{n'}(z)\right|^2 \left|F_n(z)\right|^2 dz$$

Assume an infinite barrier quantum well: $$F_n(z) = \sqrt{\frac{2}{W}}\sin(n\pi z/W)$$

$$\left|F_{n',n}\right|^2 = \frac{1}{2W}\left(2+\delta_{n,n'}\right)$$

= $3/2W$ for intra-subband scattering
= $1/W$ for inter-subband scattering

3D to 2D re-cap

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \delta\left(\vec{p}' - \vec{p} \mp \hbar\vec{q}\right) \rightarrow \frac{1}{A} U_{ac} \delta\left(\vec{p}'_{\parallel} - \vec{p}_{\parallel} \mp \hbar\vec{q}_{\parallel}\right) \left|F_{n',n}\right|^2$$

$$\left|F_{n',n}\right|^2 = \int_{-\infty}^{+\infty} \left|F_{n'}(z)\right|^2 dz \int_{-\infty}^{+\infty} \left|F_{n}(z)\right|^2 dz$$

$$\left|F_{n',n}\right|^2 = \frac{1}{2W}\left(2 + \delta_{n,n'}\right)$$ (infinite barrier well)

For intra-subband scattering, the scattering rate will be 50% greater than inter-subband scattering

$$U_{ac} = \frac{D_A^2 k_B T}{2 c_l}$$

2D scattering rate

$$S(\vec{p},\vec{p}')=\frac{2\pi}{\hbar}\frac{U_{ac}}{A}\delta\left(\vec{p}_{\parallel}'-\vec{p}_{\parallel}\mp\hbar\vec{q}_{\parallel}\right)\left|F_{n',n}\right|^{2}\delta\left(E'-E\mp\hbar\omega\right)\qquad U_{ac}=\frac{D_A^2 k_B T_L}{2c_l}$$

$$\frac{1}{\tau}=\frac{1}{\tau_m}=\sum_{\vec{p}_{\parallel}'}S\left(\vec{p}_{\parallel},\vec{p}_{\parallel}'\right)$$

$$\left|F_{n',n}\right|^{2}=\frac{1}{2W}\left(2+\delta_{n,n'}\right)$$

(infinite barrier well)

$$\frac{1}{\tau}=\frac{2\pi}{\hbar}U_{ac}\frac{D_{2D}(E)}{2}\frac{\left(2+\delta_{n,n'}\right)}{2W}$$

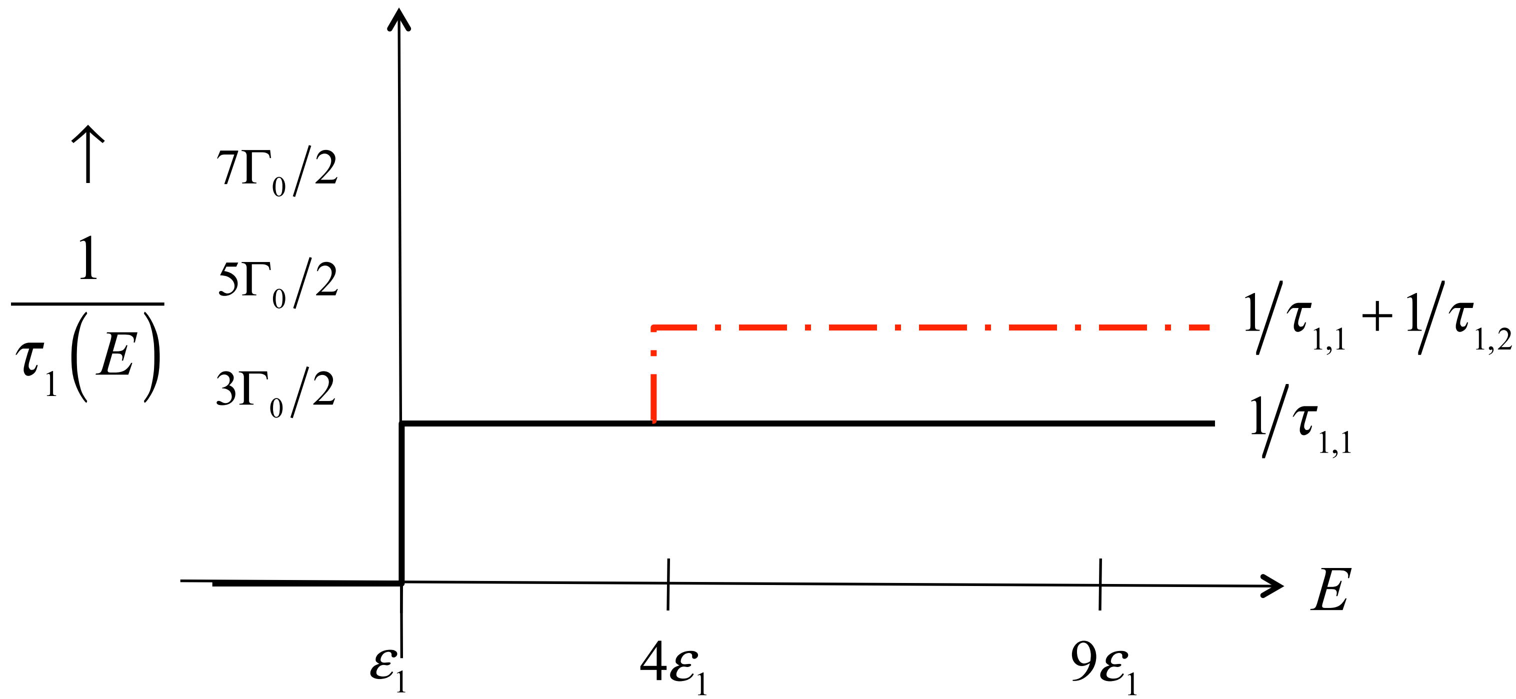
$$\tau=\tau_0\left(E/k_BT_L\right)^0$$

$$s=0$$

$$\tau_0=\frac{2c_l\hbar^3}{D_A^2 m^*}\frac{1}{k_BT_L}$$

2D scattering rate vs. energy

$$\frac{1}{\tau_{n,n'}} = \frac{2\pi}{\hbar} U_{ac} \frac{D_{2D}(E)}{2} \frac{\left(2+\delta_{n,n'}\right)}{2W} = \Gamma_0 \frac{\left(2+\delta_{n,n'}\right)}{2}$$



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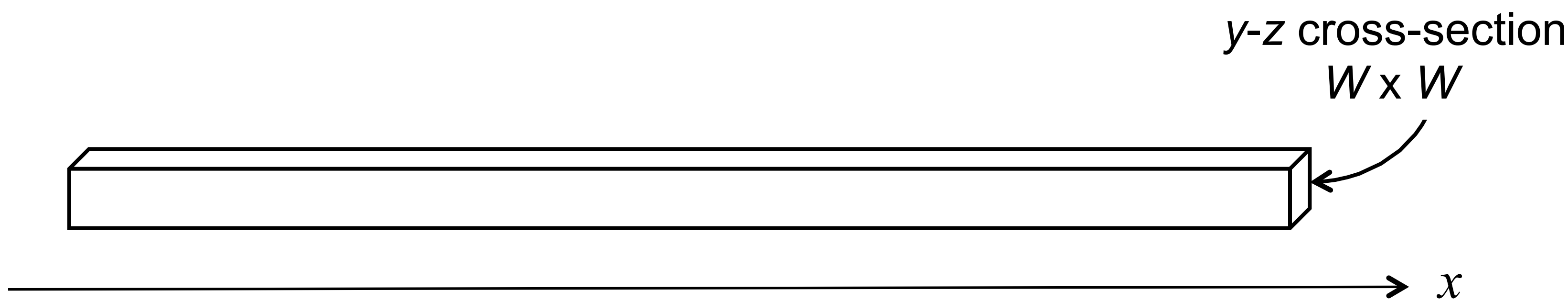
3D → 1D

Expect:

$$\left|H_{p',p}\right|^2 = \frac{1}{\Omega} U_{ac} \delta\left(\vec{p}' - \vec{p} \mp \hbar\vec{q}\right) \rightarrow \frac{1}{L} U_{ac} \delta\left(p'_x - p_x \mp \hbar q_x\right)\left|F_{l',l}\right|^2$$

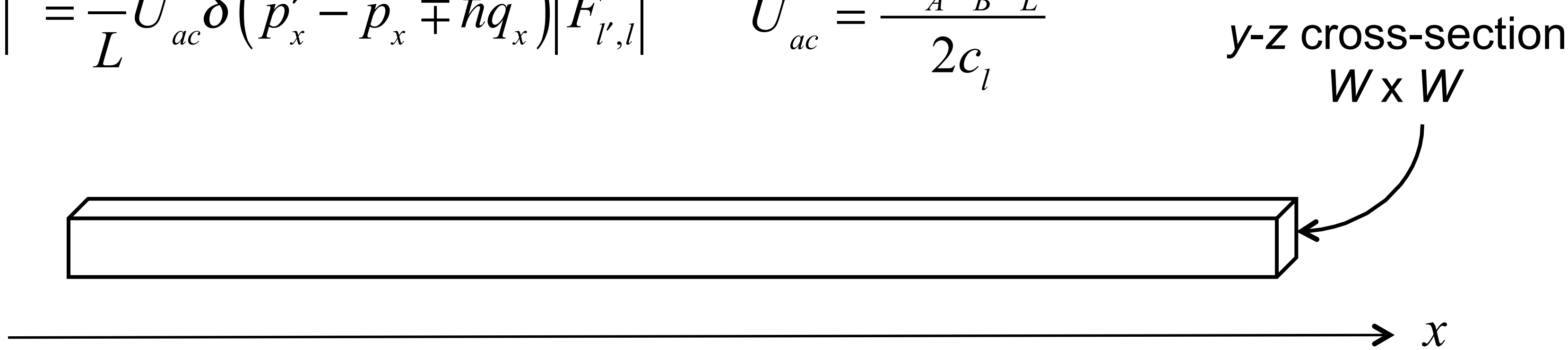


1D electrons:

$$\psi_l\left(x,y,z\right) = F_l\left(y,z\right)\frac{1}{\sqrt{L}} e^{i\vec{k}_x \bullet \hat{x}}$$

Form factor in 1D

$$\left|H_{p',p}\right|^2 = \frac{1}{L} U_{ac} \delta\left(p'_x - p_x \mp \hbar q_x\right) \left|F_{l',l}\right|^2 \qquad U_{ac} = \frac{D_A^2 k_B T_L}{2 c_l}$$



(infinite barrier well)

$$F_m(y) = \sqrt{2/W}\,\sin(m\pi y/W) \qquad F_n(z) = \sqrt{2/W}\,\sin(n\pi z/W)$$

$$F_{m'}(y) = \sqrt{2/W}\,\sin(m'\pi y/W) \qquad F_{n'}(z) = \sqrt{2/W}\,\sin(n'\pi z/W)$$

$$\left|F_{l',l}\right|^2 = \int_{-\infty}^{+\infty} F_{n'}^2(y)\,dy \int_{-\infty}^{+\infty} F_{n'}^2(z)\,dz \int_{-\infty}^{+\infty} F_{n}^2(y)\,dy \int_{-\infty}^{+\infty} F_{n}^2(z)\,dz$$

$$\left|F_{l',l}\right|^2 = \left[\frac{1}{2W}\left(2 + \delta_{l,l'}\right)\right]^2$$

1D scattering rate

$$S(p_x, p_x') = \frac{2\pi}{\hbar}\frac{U_{ac}}{L}\delta(p_x' - p_x \mp \hbar q_x)\left|F_{l',l'}\right|^2 \delta(E' - E \mp \hbar\omega) \qquad U_{ac} = \frac{D_A^2 k_B T_L}{2c_l}$$

$$\frac{1}{\tau_{l,l'}} = \frac{1}{\tau_m} = \sum_{p_x'} S(p_x, p_x')$$

$$\left|F_{l',l}\right|^2 = \left[\frac{1}{2W}\left(2 + \delta_{l,l'}\right)\right]^2$$

(infinite barrier well)

$$\frac{1}{\tau_{l,l'}} = \frac{2\pi}{\hbar}U_{ac}\frac{D_{1D}(E)}{2}\left[\frac{\left(2 + \delta_{l,l'}\right)}{2W}\right]^2$$

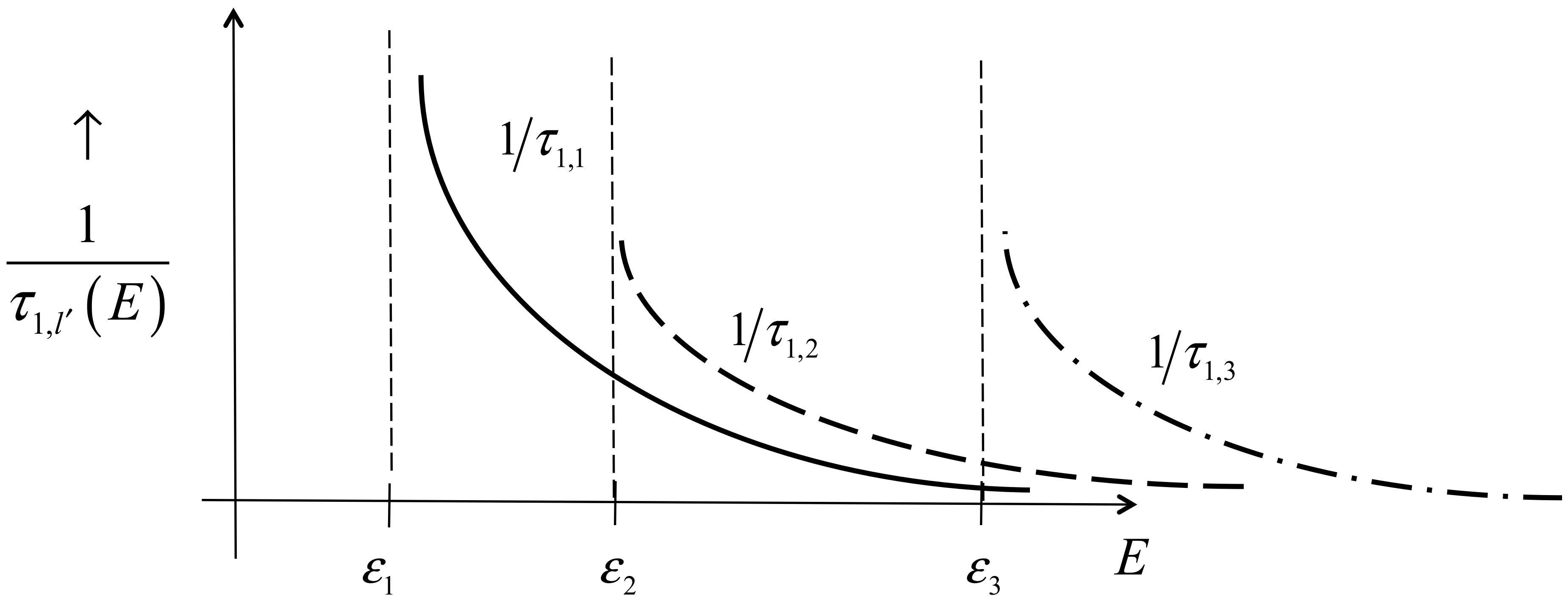
$$\tau_{11} = \tau_0\left(E/k_B T_L\right)^{1/2}$$

$$s = +1/2$$

$$\tau_0 = \frac{8c_l\hbar^2}{9D_A^2\sqrt{2m^*}}\frac{1}{\left(k_B T_L\right)^{1/2}}$$

1D scattering rate vs. energy

$$\frac{1}{\tau_{l,l'}} = \frac{2\pi}{\hbar} U_{ac} \frac{D_{1D}(E)}{2} \left[\frac{\left(2+\delta_{l,l'}\right)}{2W} \right]^2$$



Questions?

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