ECE 656: Electronic Transport in Semiconductors Fall 2017

Electron-Electron Scattering

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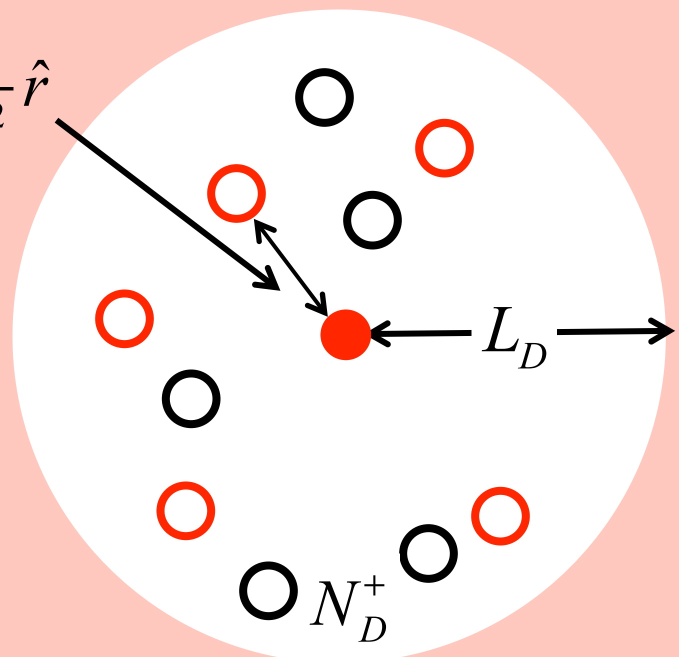
Long and short range Coulomb effects

$$\vec{F}_e(\vec{r},t) = -q\vec{\mathcal{E}}(\vec{r},t)$$

$$\nabla \bullet \left[\kappa_S \varepsilon_0 \vec{\mathcal{E}}(\vec{r},t)\right] = \rho(\vec{r},t)$$

$$\vec{F}_e(\vec{r},t) = \frac{q_1 q_2}{4\pi\kappa_S\varepsilon_0 r^2}\hat{r}$$



n_0 cm^{-3}

$$\rho = q\left(N_D^+ - n_0\right) \text{ C/cm}^{-3}$$

Long vs. short range e-e scattering

An electron can scatter by exciting or absorbing an oscillation in the "plasma" of free electrons (or holes).

$$\omega_p = \left(\frac{e^2 n}{\kappa_S \varepsilon_0 m^*} \right)^{1/2} \quad \text{plasma frequency}$$

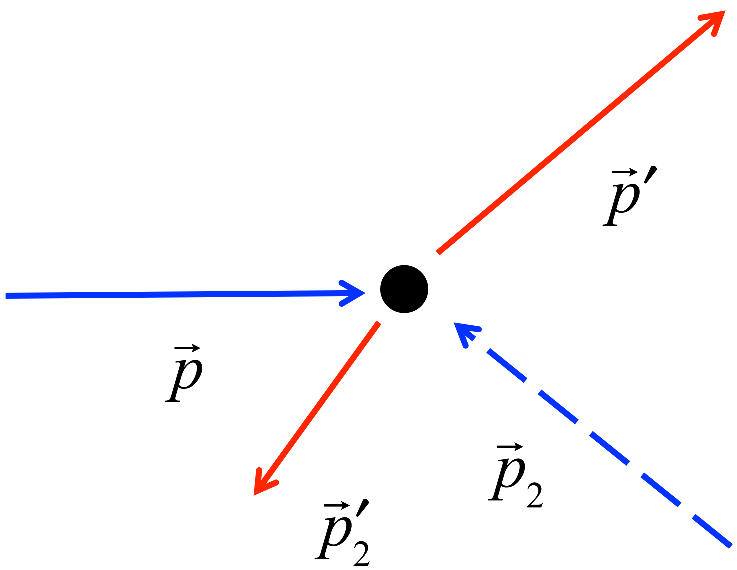
This is "collective e-e scattering". See Lundstrom, FCT, Sec. 2.10.2 for a discussion.

Outline

1) Introduction
2) **Binary e-e scattering**
3) Effects of e-e scattering
4) Summary

Binary e-e scattering

Important when $n > \sim 10^{17}$ cm^{-3}



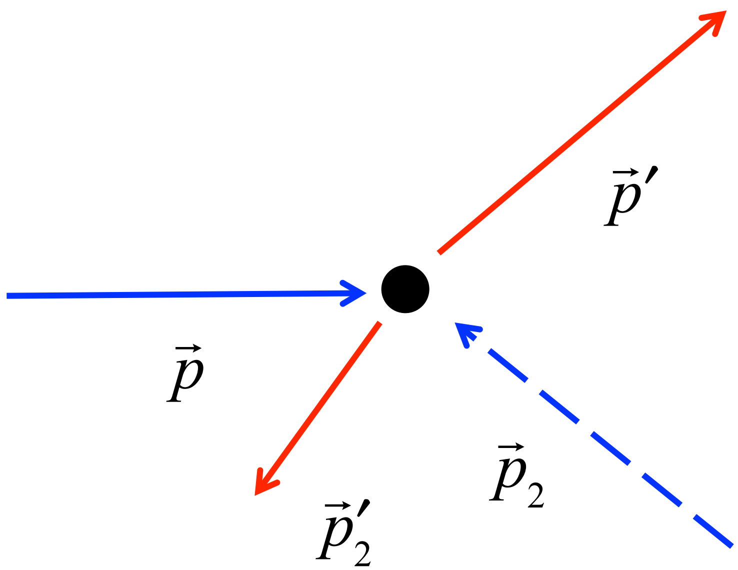
$$\vec{p} + \vec{p}_2 = \vec{p}' + \vec{p}_2'$$

$$E + E_2 = E' + E_2'$$

$$S\left(\vec{p}, \vec{p}'\right) \rightarrow S\left(\vec{p}, \vec{p}_2; \vec{p}', \vec{p}_2'\right)$$

$$S\left(\vec{p}, \vec{p}_2; \vec{p}', \vec{p}_2'\right) \propto \delta\left(\vec{p} + \vec{p}_2 - \vec{p}' - \vec{p}_2'\right)\delta\left(E' + E_2' - E - E_2\right)$$

binary e-e scattering rate



$$\frac{1}{\tau} = \sum_{\vec{p}'} S\left(\vec{p}, \vec{p}'\right) \rightarrow$$

Important when $n > \sim 10^{17}$ cm^{-3}

$$\frac{1}{\tau^{e-e}\left(\vec{p}\right)} = \sum_{\vec{p}', \vec{p}_2} S\left(\vec{p}, \vec{p}_2; \vec{p}', \vec{p}_2'\right) f\left(\vec{p}_2\right)\left[1 - f\left(\vec{p}\right)\right]\left[1 - f\left(\vec{p}_2'\right)\right]$$

(a **very** difficult problem to solve)

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Effects of e-e scattering

$$f\left(\vec{r},\vec{p},t\right)$$

The probability that a state at position $\vec{r}$ with crystal momentum $\vec{p}=\hbar\vec{k}$ is occupied at time, t.

How does *f* change with time due to collisions?

$$\left.\frac{\partial f}{\partial t}\right|_{coll}=?$$

Collision integral

$$\left.\frac{\partial f(\vec{p})}{\partial t}\right|_{coll} = \sum_{\vec{p}',\vec{p}_2'} S(\vec{p}',\vec{p}_2';\vec{p},\vec{p}_2) f(\vec{p}') f(\vec{p}_2') (1-f(\vec{p}))(1-f(\vec{p}_2))$$
$$- S(\vec{p},\vec{p}_2;\vec{p}',\vec{p}_2') f(\vec{p}) f(\vec{p}_2) (1-f(\vec{p}'))(1-f(\vec{p}_2'))$$

Now assume a non-degenerate semiconductor (just to keep the math simple)

$$\left.\frac{\partial f(\vec{p})}{\partial t}\right|_{coll} = \sum_{\vec{p}',\vec{p}_2'} S(\vec{p}',\vec{p}_2';\vec{p},\vec{p}_2) f(\vec{p}') f(\vec{p}_2') - S(\vec{p},\vec{p}_2;\vec{p}',\vec{p}_2') f(\vec{p}) f(\vec{p}_2)$$

Collision integral (ii)

The probability for a transition and its inverse are equal (see. eqn. (2.100) p. 90 of Lundstrom)

$$S(\vec{p},\vec{p}_2;\vec{p}',\vec{p}_2') = S(\vec{p}',\vec{p}_2';\vec{p},\vec{p}_2)$$

$$\left.\frac{\partial f}{\partial t}\right|_{coll} = \sum_{\vec{p}',\vec{p}_2'} S(\vec{p},\vec{p}_2;\vec{p}',\vec{p}_2')\left[f(\vec{p})f(\vec{p}_2) - f(\vec{p}')f(\vec{p}_2')\right]$$

Eventually the electron system comes into equilibrium – every in-scattering is balanced by a corresponding out-scattering and…

$$\left.\frac{\partial f}{\partial t}\right|_{coll} = 0 \rightarrow f(\vec{p})f(\vec{p}_2) = f(\vec{p}')f(\vec{p}_2')$$

"equilibrium" solution

$$f(\vec{p})f(\vec{p}_2)=f(\vec{p}\,')f(\vec{p}_2\,')$$

For a solution, try: $f(\vec{p})=e^{Kp^2}$

$$f(\vec{p})f(\vec{p}_2)=f(\vec{p}\,')f(\vec{p}_2\,') \rightarrow e^{K\left(p^2+p_2^2\right)}=e^{K\left(p'^2+p_2'^2\right)}$$

True because energy is conserved →

$$f(\vec{p}) \propto e^{p^2/2mk_BT_e}$$

“equilibrium” solution”

The average energy per carrier for an equilibrium Maxwellian is

$$3k_BT_L/2$$

When the electron system is out of equilibrium with the lattice, then the average energy per carrier can be written as:

$$3k_BT_e/2$$

$$T_e > T_L$$

e-e scattering

To first order, electron-electron scattering does not lower the mobility, because the total momentum of the electron system is conserved, but there can be an indirect effect because the shape of the distribution affects the average scattering rate for other scattering processes.

Note: For those familiar with phonon scattering, this is analogous to the “N-processes” in phonon transport.

Questions?

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