Electron Scattering
in Common Semiconductors

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Outline

1) Review of Approach
2) II Scattering
3) Phonon Scattering
4) Other scattering mechanisms
5) Summary
Fermi’s Golden Rule

\[ S(\bar{p}, \bar{p}') = \frac{2\pi}{\hbar} |H_{\bar{p}', \bar{p}}|^2 \delta(E' - E - \Delta E) \]

\[ H_{\bar{p}', \bar{p}} = \frac{1}{\Omega} \int_{-\infty}^{+\infty} e^{-i\bar{p}' \cdot \bar{r}/\hbar} U_S(\bar{r}) e^{i\bar{p} \cdot \bar{r}/\hbar} d\bar{r} \]

\[ \Delta E = \pm \hbar \omega \]

\[ \psi_i = \frac{1}{\sqrt{\Omega}} e^{i\bar{p} \cdot \bar{r}/\hbar} \]

\[ \psi_f = \frac{1}{\sqrt{\Omega}} e^{i\bar{p}' \cdot \bar{r}/\hbar} \]

\[ \bar{p}' = \bar{p} \pm \hbar \bar{q} \]
Characteristic times

\[ \frac{1}{\tau_m(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}') \frac{\Delta p_z}{p_{z0}} \]

(\(\tau\), single particle lifetime)

\[ \frac{1}{\tau_E(\vec{p})} = \sum_{\vec{p}',\uparrow} S(\vec{p},\vec{p}') \frac{\Delta E}{E_0} \]

\[ \sum_{t=\tau_m} \approx \geq 1 \tau_m \]

\[ \sum_{t=\tau_E} \approx \geq 1 \tau_E \]

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Characteristic times

$$\frac{1}{\tau(\bar{p})} = \sum_{\bar{p}', \uparrow} S(\bar{p}, \bar{p}')$$

(\(\tau\), single particle lifetime)

Note that these are “out-scattering” times – they assume that the final state is empty. To actually compute the rate, we need to weight by the probability that the final state is empty.

$$\frac{1}{\tau(\bar{p})} = \sum_{\bar{p}', \uparrow} S(\bar{p}, \bar{p}') \left[ 1 - f(\bar{p}') \right]$$
General observations

Expect:

\[ \frac{1}{\tau(E_i)} \propto D(E_f) \quad E_f = E_i + \Delta E \]

Isotropic scattering:

\[ \frac{1}{\tau(E_i)} = \frac{1}{\tau_m(E_i)} \]

Anisotropic scattering selects certain preferred final states, e.g. electrostatic interactions emphasize small angle scattering.

\[ \frac{1}{\tau(E_i)} > \frac{1}{\tau_m(E_i)} \]
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II summary

CW: \[ U_s(\vec{r}) = \pm \frac{q^2}{4\pi \kappa_s \varepsilon_0 r} \]

BH: \[ U_s(r) = -\frac{q^2}{4\pi \kappa_s \varepsilon_0 r} e^{-r/L_D} \]

1) \[ S(\vec{p}, \vec{p}') \sim N_I \]

2) \[ S(\vec{p}, \vec{p}') \sim e^4 \]

3) favors small angle scattering \[ \tau_m(E) > \tau(E) \]

4) \[ \tau_m(E) \approx \tau_0 \left( E - E_C \right)^{3/2} \quad \tau_0 \propto T^{3/2} \quad \text{(power law scattering)} \]
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Phonon dispersion

$\omega(q) = \omega_0 + \nu_S q$

These phonons are involved in intra-valley scattering.

Longitudinal modes couple most strongly to electrons.

These phonons are involved in inter-valley scattering.

LO (1)  
LA (1)
Transition rate for phonon scattering

\[ S(\vec{p}, \vec{p}') = \frac{2\pi}{\hbar} |H_{p',p}|^2 \delta(E' - E \mp \hbar \omega) \]

\[ H_{p',p} = \frac{1}{\Omega} \int_{-\infty}^{+\infty} e^{-i\vec{p}' \cdot \vec{r} / \hbar} U_S(\vec{r}) e^{i\vec{p} \cdot \vec{r} / \hbar} d\vec{r} \]

\[ U_S(\vec{r}) = K_q u_q \quad u_q(\vec{r}) = A_q e^{\pm i\vec{q} \cdot \vec{r}} \]

\[ |H_{p',p}|^2 = |K_q|^2 |A_q|^2 \delta(\vec{p}' - \vec{p} \mp \hbar \vec{q}) \]

\[ |A_q|^2 \rightarrow \frac{\hbar}{2\rho \Omega \omega} \left( N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \]

\[ S(\vec{p}, \vec{p}') = \frac{\pi}{\Omega \rho \omega} |K_q|^2 \left( N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \delta(\vec{p}' - \vec{p} \mp \hbar \vec{q}) \delta(E' - E \mp \hbar \omega) \]

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Electron-phonon coupling

\[ u_q(\vec{r},t) = A_q e^{\pm i(\vec{q} \cdot \vec{r} - \omega_q t)} \]

\[ U_S = K_q u_q \]

| \text{ADP} | |K_q|^2 = q^2 D_A^2 |
| \text{ODP} | |K_q|^2 = D_0^2 |
| \text{PZ} | |K_q|^2 = \left(\frac{q e_{PZ}}{\kappa_S \varepsilon_0}\right)^2 |
| \text{POP} | |K_q|^2 = \frac{\rho e^2 \omega_0^2}{q^2 \kappa_0 \varepsilon_0} \left(\frac{\kappa_0}{\kappa_\infty} - 1\right) |
Simplified Transition Rate

\[ S(\tilde{p}, \tilde{p}') = \frac{2\pi}{\hbar} |K_q|^2 |A_{q}^{a,e}|^2 \delta(p' - \tilde{p} + \hbar \tilde{q}) \delta(E' - E \mp \hbar \omega_0) \]

\[ |K_q| \quad |A_{q}^{a,e}|^2 = \left( N_{\omega} + \frac{1}{2} \mp \frac{1}{2} \right) \frac{\hbar}{2\Omega \rho \omega_0} \]

When there is no preference for one momentum or another, then there is no need to explicitly consider momentum conservation.
Isotropic scattering times

\[ S(\bar{p}, \bar{p}') = \frac{2\pi}{\hbar} |K_q|^2 |A_q^{a,e}|^2 \delta(E' - E \mp \hbar \omega_0) \]

\[ \left| K_q \right|^2 \left| A_q^{a,e} \right|^2 = \left( N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) \frac{\hbar}{2\Omega \rho \omega_0} \]

\[ \frac{1}{\tau}\left(\bar{p}\right) = \frac{1}{\tau_m\left(\bar{p}\right)} = \sum_{\bar{p}',\uparrow} S(\bar{p}, \bar{p}') \]

\[ \frac{1}{\tau_m(E)} \propto \frac{1}{\Omega} \sum_{\bar{p}',\uparrow} \delta(E' - E \mp \hbar \omega_0) \]

\[ \frac{1}{\tau_m(E)} \propto \left( N_\omega + \frac{1}{2} \mp \frac{1}{2} \right) D\left( E \pm \hbar \omega_0 \right) \frac{\hbar}{2} \]

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Acoustic phonon scattering

\[ N_\omega \approx N_\omega + 1 \approx \frac{k_B T}{\hbar \omega} \]  
(elastic)

\[ \frac{1}{\tau} = \frac{1}{\tau_{\text{abs}}} + \frac{1}{\tau_{\text{ems}}} = \frac{2\pi}{\hbar} \left( \frac{D_A^2 k_B T}{c_l} \right) \frac{D_{3D}}{2} \left( E - E_C \right) \]

(isotropic)

\[ \frac{1}{\tau} = \frac{1}{\tau_m} \]

\[ \tau_m (E) = \tau_0 \left( \frac{E - E_C}{k_B T} \right)^{-1/2} \]

\[ \tau_0 \propto T^{-3/2} \]
Mobility vs. temperature

\[ \mu_n = \frac{q \langle \tau_m \rangle}{m^*} \]

\[ \mu_n \propto T^{+3/2} \]

\[ \mu_n \propto T^{-3/2} \]

\[ T_L = T_e = T \]
ODP scattering

$$N_0 = \frac{1}{e^{\hbar \omega_0 / k_B T} - 1}$$

$$\frac{1}{\tau_{\text{abs}}} \neq \frac{1}{\tau_{\text{ems}}}$$

isotropic inelastic

$$\frac{1}{\tau(E)} = \frac{1}{\tau_m(E)} = \frac{2\pi}{\hbar} \left( \frac{\hbar D_0^2}{2 \rho \omega_0} \right) \left( N_0 + \frac{1}{2} \mp \frac{1}{2} \right) \frac{D_{3D}(E \pm \hbar \omega_0)}{2}$$
POP scattering

possibility of “polar runaway”

\[
\frac{1}{\tau} \propto \left( \frac{\kappa_0}{\kappa_\infty} - 1 \right) / \sqrt{2E/m^*}
\]

\[\hbar\omega_0 \approx 34 \text{ meV (GaAs)}\]
IV scattering can be treated like ODP scattering

Isotropic:

\[
\frac{1}{\tau} = \frac{1}{\tau_m}
\]

Number of final valleys: \( Z_f \)

\[
\frac{1}{\tau} = \frac{2\pi}{\hbar} \left( \frac{\hbar D_{if}^2 Z_f}{2 \rho \omega_{if}} \right) \left( N_{if} + \frac{1}{2} \pm \frac{1}{2} \right) \frac{D_f}{2} \left( \frac{E \pm \hbar \omega_{if} - \Delta E_{fi}}{2} \right)
\]
equivalent IV scattering (Si)

- **Si conduction band**
- **g-type scattering**
  - \( m^*_i = 0.9 m_0 \)
  - \( m^*_f = 0.19 m_0 \)
- **f-type scattering**

**g-type**:
- \( Z_f = 1 \)
- \( \Delta E_{fi} = 0 \text{ eV} \)

**f-type**:
- \( Z_f = 4 \)
- \( \Delta E_{fi} = 0 \text{ eV} \)
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Scattering rate in common semiconductors

1) What is the total scattering rate vs. energy for common semiconductors?

\[ \Gamma = \sum_i \frac{1}{\tau_i} \]

2) How do covalent semiconductors (e.g. Si, Ge) differ from polar semiconductors (e.g. GaAs, InP, InGaAs, ZnSe)?
model bandstructure (for analytical calculations)

Si, covalent, indirect BG

\[ m_{lh}^* = 0.16 m_0 \]

\[ m_{lh}^* = 0.49 m_0 \]

\[ m_{hh}^* = 0.92 m_0 \]

\[ m_{lt}^* = 0.19 m_0 \]

GaAs, polar, direct BG

\[ m_{lh}^* = 0.063 m_0 \]

\[ m_{hh}^* = 0.16 m_0 \]

\[ m_{lt}^* = 0.39 m_0 \]

\[ 1.12 \text{eV} \]

\[ 1.42 \text{eV} \]
“full band” scattering rates

For a good, general reference on the numerical evaluation of scattering rates in common semiconductors, see:

Electrons and holes in Si

Electrons and holes in GaAs

Valence band is complex (warped) and can be engineered by strain.

Intra-band:

\[
\frac{1}{\tau_{hh\rightarrow hh}} \quad \frac{1}{\tau_{lh\rightarrow lh}} \quad \frac{1}{\tau_{so\rightarrow so}}
\]

Inter-band:

\[
\frac{1}{\tau_{hh\rightarrow so}} \quad \frac{1}{\tau_{so\rightarrow hh}} \quad \frac{1}{\tau_{lh\rightarrow so}} \quad \frac{1}{\tau_{so\rightarrow lh}}
\]
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Other scattering mechanisms

1) Neutral impurity
2) Alloy scattering
3) Surface / edge roughness scattering
4) Plasmon scattering
5) Electron-electron scattering
6) Electron-hole
7) Crystal defects, etc.
Scattering in semiconductors

- ionized impurities
- neutral impurities
- dislocations
- surface roughness
- alloy

- electron-electron
- electron-plasmon
- electron-hole

- intravalley
  - ADP
  - ODP
  - POP
  - PZ
- intervalley
  - acoustic
  - optical
Summary

1) Characteristic times are derived from the transition rate, $S(p,p')$
2) $S(p,p')$ is obtained from Fermi’s Golden Rule
3) The scattering rate is often proportional to the final DOS
4) Static potentials lead to elastic scattering
5) Time varying potentials lead to inelastic scattering
6) The general features of scattering in common semiconductors are readily understood.
Questions?

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